1C8 Advanced design of steel structures

prepared by

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List of lessons

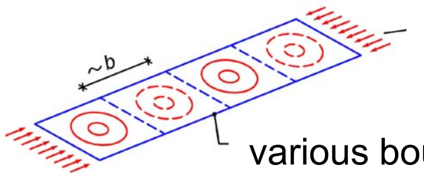
1) Lateral-torsional instability of beams.
2) Buckling of plates.
3) Thin-walled steel members.
4) Torsion of members.
5) Fatigue of steel structures.
6) Composite steel and concrete structures.
7) Tall buildings.
8) Industrial halls.
9) Large-span structures.
10) Masts, towers, chimneys.
11) Tanks and pipelines.
12) Technological structures.
13) Reserve.

1. Buckling of plates

- **Introduction (plate stability and strength).**
- **Buckling due to direct stresses.**
- **Effective width method.**
- **Shear buckling.**
- **Buckling under local loading.**

Introduction

Stability of an ideal (flat) plate:



Solution is based on linearized relation of a plate with „large deflections":

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+N_x^*\frac{\partial^2 w}{\partial x^2}+2N_{xy}^*\frac{\partial^2 w}{\partial x\partial y}+N_y^*\frac{\partial^2 w}{\partial y^2}=0$$

+ relevant boundary conditions

Thereof infinitely many solutions:

- **critical stresses σ^* (or N*) – take the lowest**
- **respective shapes of deflection *w* (modes of buckling)**

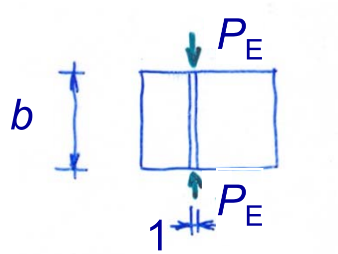
Introduction

Critical stresses are given as:

$$\sigma_{cr} = k_\sigma \cdot \sigma_E \quad \text{or} \quad \tau_{cr} = k_\tau \cdot \sigma_E$$

critical stress factor **Euler stress**

"Euler stress" σ_E



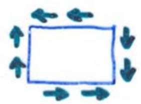
Auxiliary value, for a compression strut of width "1":

$$\sigma_E = \frac{P_E}{1 \cdot t} = \frac{\pi^2 D}{1 \cdot t \cdot b^2} = \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2 = 189800\left(\frac{t}{b}\right)^2$$

Critical stress factor:
(depends on loading and boundary conditions, see literature)

$k_\sigma = 4$

$k_\sigma = 23{,}9$



$k_\tau = 5{,}34 + 4\left(\frac{b}{a}\right)^2$ for $\frac{a}{b} \geq 1$

Introduction

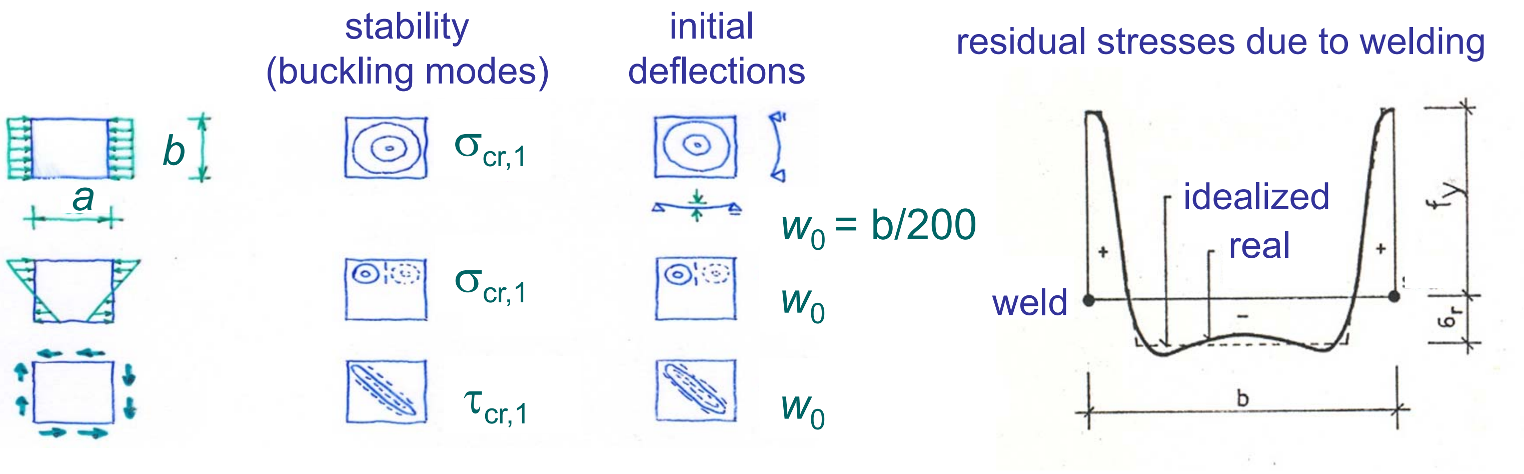
Strength of an actual (imperfect) plate:

Equations of a plate with „large deflections“ (Karman’s equations):

(1) $$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)-E\,t\left(\frac{\partial^2 \Phi}{\partial y^2}\frac{\partial^2 w}{\partial x^2}-2\frac{\partial^2 \Phi}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y}+\frac{\partial^2 \Phi}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right)=0$$

(2) $$\frac{\partial^4 \Phi}{\partial x^4}+2\frac{\partial^4 \Phi}{\partial x^2\partial y^2}+\frac{\partial^4 \Phi}{\partial y^4}+\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2=0$$

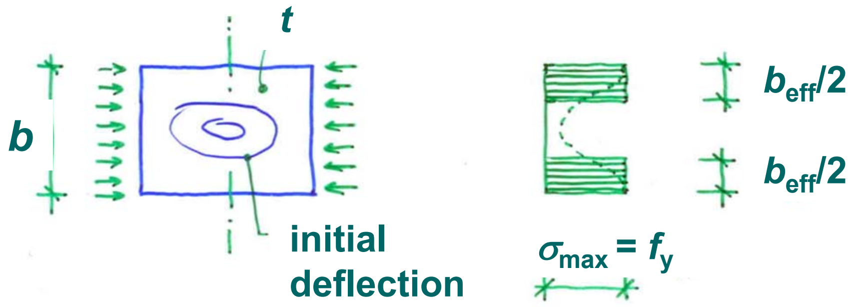
Plate imperfections



SUSTAINABLE STEEL AND TIMBER CONSTRUCTIONS

Introduction

Example of a compression plate with initial deflections and residual stresses:



Resulting strengths are used in the form of reduction (buckling) factors ρ :

$$\rho = \frac{\overline{\sigma}}{f_y} = \frac{b_{eff}}{b} \qquad \overline{\sigma} = \int_0^b \sigma \, db$$

SUSTAINABLE STEEL AND TIMBER CONSTRUCTIONS

Buckling due to direct stresses

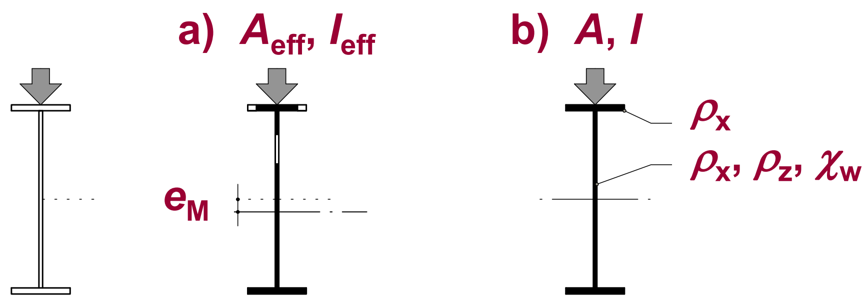
Eurocode 1993-1-5: Plated structural elements

1. Buckling due to direct stress (loading *N, M*):

Verification of class 4 cross sections:

a) ***effective width method***, in which the buckling parts of plates are excluded,

b) ***reduced stress method***, in which the stresses of full cross section are determined and limited by buckling reduction factors ρ_x, ρ_z, χ_w:



Note:
b) does not include stress redistribution after buckling among individual parts of cross section!!!

Buckling due to direct stresses

Reduction (buckling) factors:

Internal elements:

$$\rho = \frac{\bar{\lambda}_p - 0{,}055\,(3+\psi)}{\bar{\lambda}_p^2} \le 1{,}0 \qquad \bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} = \frac{b/t}{28{,}4\,\varepsilon\sqrt{k_\sigma}} \qquad \psi = \sigma_2/\sigma_1$$

For outstand compression elements similarly:

$$\rho = \frac{\bar{\lambda}_p - 0{,}188}{\bar{\lambda}_p^2} \le 1{,}0$$

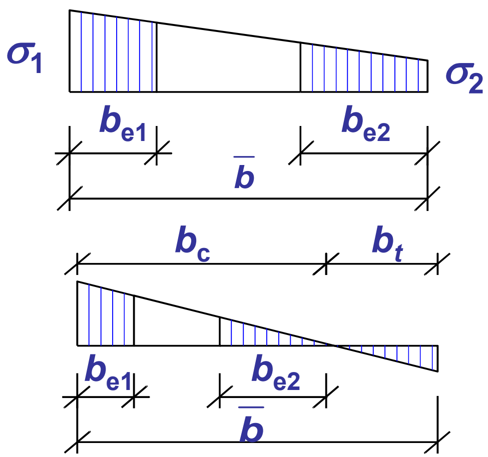
For k_σ see next tables or Eurocode.

Effective width method

Effective width method

The effective[p] area of the compression zone of a plate: $A_{c,eff} = \rho A_c$

- **internal elements:** $\psi = \sigma_1 / \sigma_2$



$1 > \psi \geq 0$: $b_{eff} = \rho\, \bar{b}$ $\qquad b_{e1} = \dfrac{2}{5-\psi} b_{eff}$

$b_{e2} = b_{eff} - b_{e1}$

$\psi < 0$: $b_{eff} = \rho\, b_c = \rho\, \bar{b} / (1-\psi)$ $\qquad b_{e1} = 0{,}4\, b_{eff}$

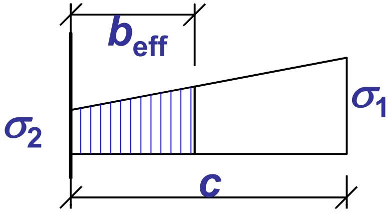
$b_{e2} = 0{,}6\, b_{eff}$

Factors k_σ

ψ	1	$1 > \psi > 0$	0	$0 > \psi > -1$	-1	$-1 > \psi > -3$
k_σ	4,0	$8{,}2/(1{,}05+\psi)$	7,81	$7{,}81-6{,}29\psi+9{,}78\psi^2$	23,9	$5{,}98(1-\psi)^2$

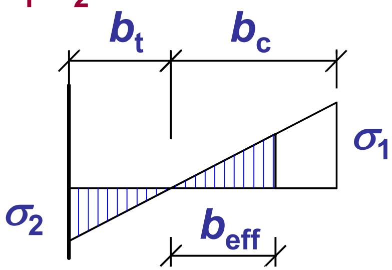
Effective width method

- **outstand elements:** $\psi = \sigma_1/\sigma_2$



$1 > \psi \geq 0$:

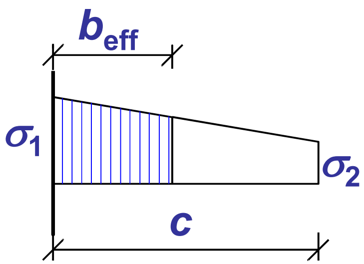
$b_{eff} = \rho c$



$\psi < 0$:

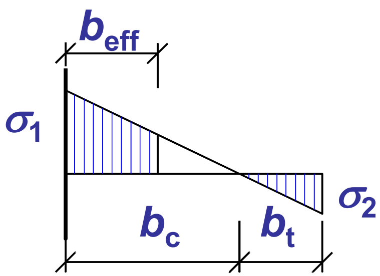
$b_{eff} = \rho b_c = \rho c /(1- \psi)$

ψ	1	0	-1	$1 \geq \psi \geq -3$
k_σ	0,43	0,57	0,85	$0,57-0,21\psi+0,078\psi^2$



$1 > \psi \geq 0$:

$b_{eff} = \rho c$



$\psi < 0$:

$b_{eff} = \rho b_c = \rho c /(1- \psi)$

Factors k_σ

ψ	1	$1 > \psi > 0$	0	$0 > \psi > -1$	-1
k_σ	0,43	$0,578/(\psi+0,34)$	1,70	$1,7-5\psi+17,1\psi^2$	23,8

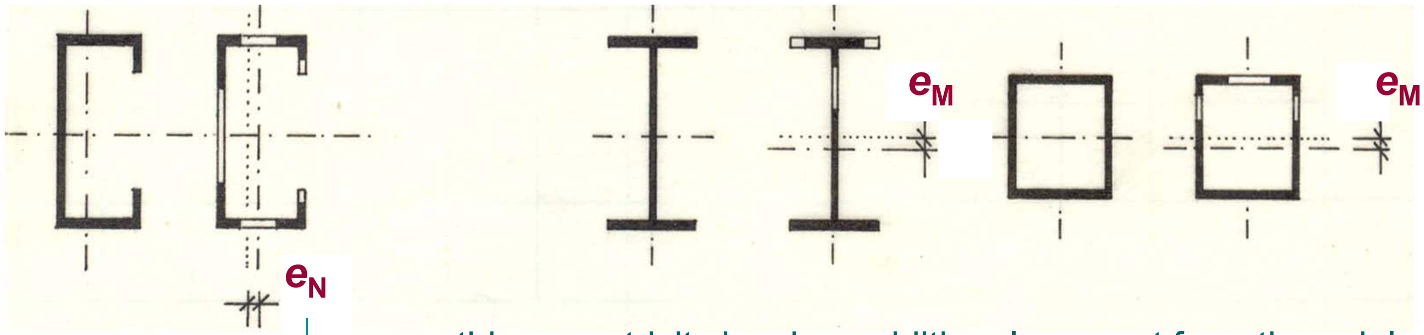
SUSTAINABLE STEEL AND TIMBER CONSTRUCTIONS

Effective width method

Effective cross sections (class 4 cross sections):

axial compression

moment



this eccentricity invokes additional moment from the axial force due to shift of neutral axis in interaction of M - N

Effective parameters of class 4 cross sections (A_{eff}, W_{eff}) are determined by common way.

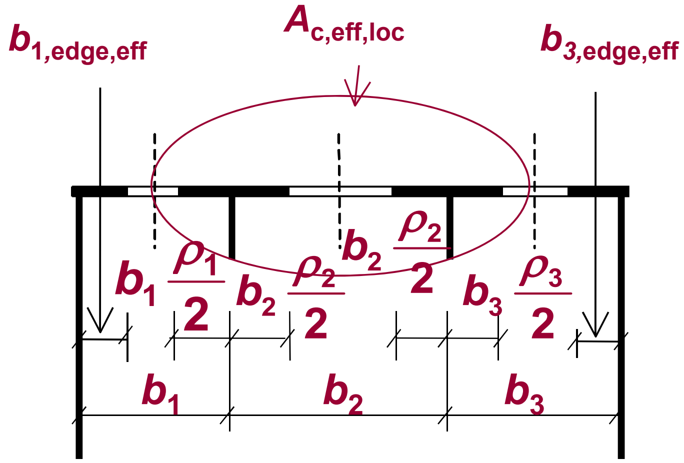
Verification of cross section in ULS:

$$\eta_1 = \frac{N_{Ed}}{\dfrac{f_y A_{eff}}{\gamma_{M0}}} + \frac{M_{Ed} + \overbrace{N_{Ed}\, e_N}}{\dfrac{f_y W_{eff}}{\gamma_{M0}}} \leq 1{,}0$$

(in stability checks: introduce χ, χ_{LT})

Effective width method

Stiffened plates:



Examples:
- **stiffened flange of a box girder,**
- **web of a deep girder.**

$$A_{c,eff} = \underbrace{\rho_c A_{c,eff,loc}}_{\text{middle part}} + \underbrace{\sum b_{edge,eff}\, t}_{\text{edges}}$$

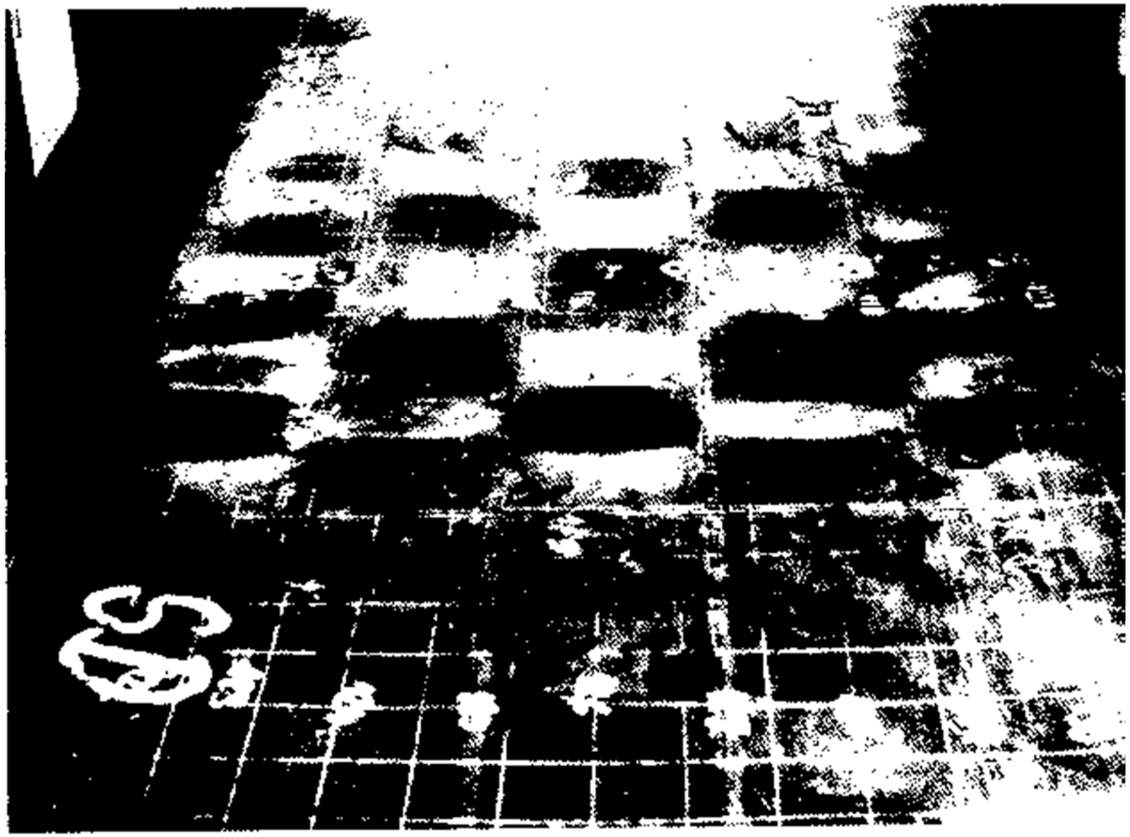
ρ_c ↓ global buckling reduction factor
(approx. given by reduction factor of the effective stiffener
- possible to calculate as a strut in compression)

[For more details see course: Stability of plates]

SUSCOS – SUSTAINABLE STEEL AND TIMBER CONSTRUCTIONS

Effective width method

Example of buckling of longitudinally and transversally stiffened flange of a box girder:



Shear buckling

2. Shear buckling (loading by shear force *V*):

Rotating stress field theory is used. Influence of stiffeners is included proportionally to higher critical stress – after modification agrees with tests.

Design resistance to shear (including shear buckling):

$$V_{b,Rd} = V_{bw,Rd} + V_{bf,Rd} \leq \frac{\eta\, f_y\, h_w\, t}{\sqrt{3}\, \gamma_{M1}}$$

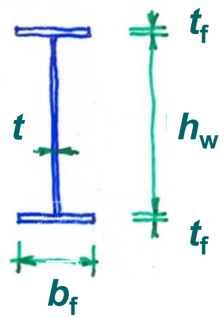
$V_{bf,Rd}$ → contribution from the flanges (can be ignored)

$V_{bw,Rd}$ → contribution from the web

η = 1,2 up to steels S460

$$\varepsilon = \sqrt{\frac{235}{f_y}}$$

Verification of ULS: $\eta_3 = \dfrac{V_{Ed}}{V_{b,Rd}} \leq 1{,}0$



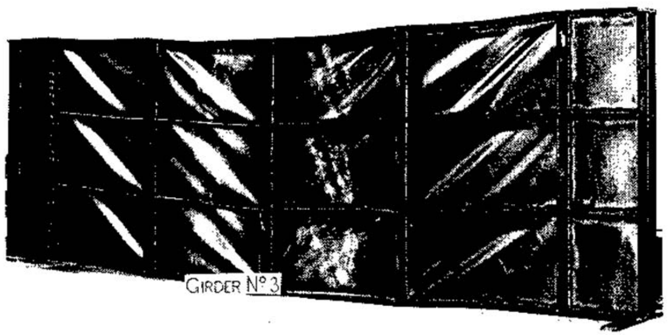
Shear buckling

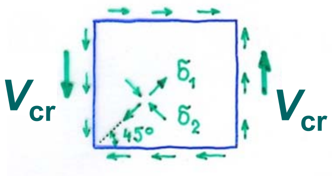
Shear buckling may be ignored for web slenderness:

unstiffened webs $\frac{h_w}{t} \leq \frac{72}{\eta}\varepsilon$ (i.e. 60 for S235)

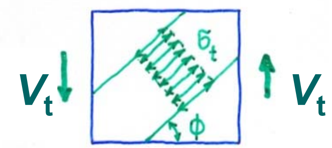
stiffened webs (transverse, longitudinal) $\frac{h_w}{t} \leq \frac{31}{\eta}\varepsilon\sqrt{k_\tau}$

Forming of tension diagonals in panels:

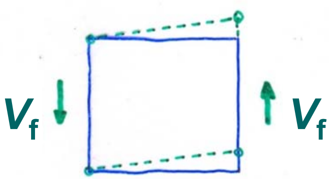




Phase 1
Beam behaviour



Phase 2
Truss behaviour



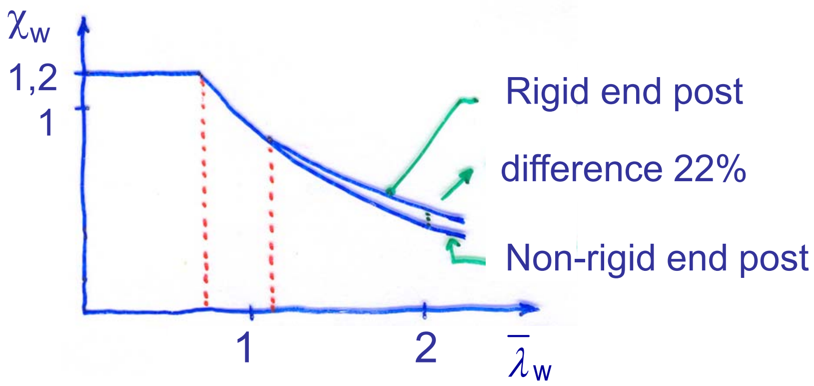
Phase 3
frame behaviour
(influence of several %)

Shear buckling

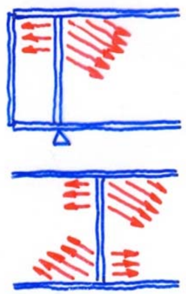
Contribution from the web $$V_{bw,Rd} = \frac{\chi_w f_{yw} h_w t}{\sqrt{3}\,\gamma_{M1}}$$

Factor χ_w for the contribution of the web to the shear buckling resistance may be (in acc. to tests) increased for rigid end post and internal panels:

Slenderness	Rigid end post	Non-rigid end post
$\bar{\lambda}_w < 0{,}83/\eta$	η	η
$0{,}83/\eta \le \bar{\lambda}_w < 1{,}08$	$0{,}83/\bar{\lambda}_w$	$0{,}83/\bar{\lambda}_w$
$\bar{\lambda}_w \ge 1{,}08$	$1{,}37/(0{,}7+\bar{\lambda}_w)$	$0{,}83/\bar{\lambda}_w$



Reason: anchorage of panels →



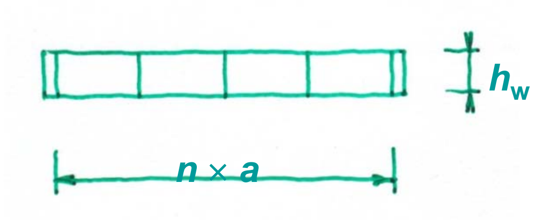
Shear buckling

Web slenderness $\bar{\lambda}_w$

- **unstiffened webs (with the exception at the beam ends):**

$$\bar{\lambda}_w = \sqrt{\frac{f_y / \sqrt{3}}{\tau_{cr}}} = \frac{h_w}{86{,}4\, t\, \varepsilon}$$

- **webs with transverse stiffeners in distance *a*:**



$$\bar{\lambda}_w = \frac{h_w}{37{,}4\, t\, \varepsilon \sqrt{k_\tau}}$$

Critical stress factor k_τ:

$$k_\tau = 5{,}34 + 4{,}00\,(h_w / a)^2 \qquad \text{as far as} \quad a / h_w \geq 1$$

$$k_\tau = 4{,}00 + 5{,}34\,(h_w / a)^2 \qquad \text{as far as} \quad a / h_w < 1$$

[For webs with longitudinal stiffeners see course: Stability of plates]

Objectives

Introduction

Buckling due to direct stresses

Effective width method

Shear buckling

Buckling under local loading

Interaction N+M+F

Assessment

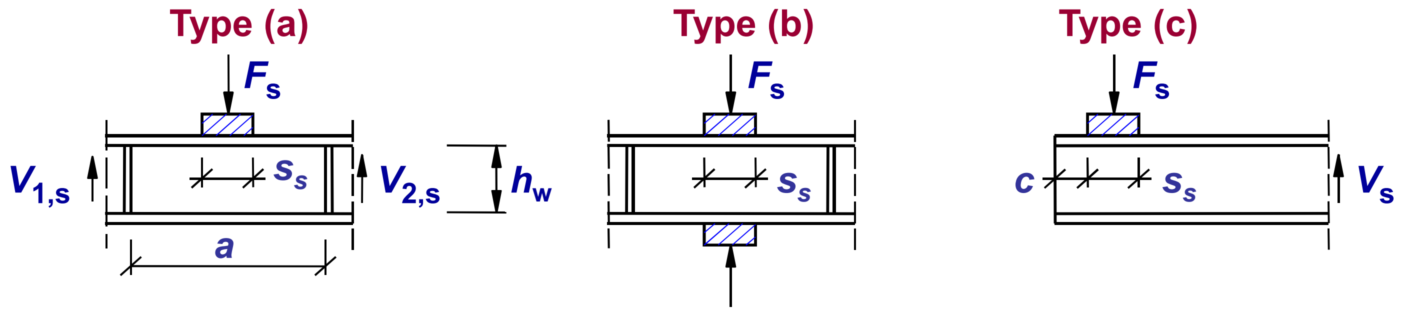
Notes

Buckling under local loading

3. Buckling under local loading

3 types of loading are distinguished:

- **a) through the flange ,**
- **b) through the flange and transferred directly to the other one,**
- **c) through the flange adjacent to an unstiffened end.**



Local design resistance:

$$F_{Rd} = L_{eff}\, t_w \frac{f_y}{\gamma_{M1}}$$

effective length of web $L_{eff} = \chi_F \ell_y$

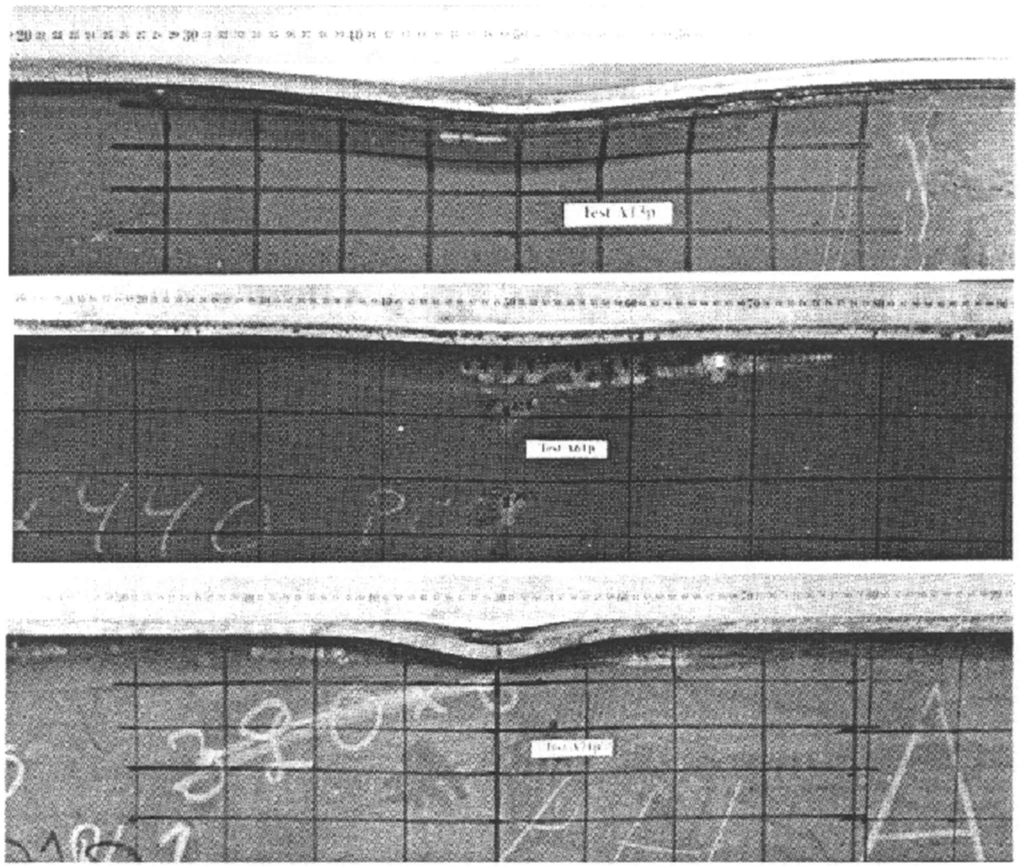
reduction factor due to local buckling (governed by critical stress)

effective loaded length (governed by s_s)

[In detail see Eurocode, or course: Stability of plates]

Buckling under local loading

Example of local web buckling:



Interaction *N* + *M* + *F*

Verification for local buckling:

$$\eta_2 = \frac{F_{Ed}}{F_{Rd}} = \frac{F_{Ed}}{L_{eff} t_w \dfrac{f_y}{\gamma_{M1}}} \leq 1{,}0$$

Interaction *N* + *M* + *F*:

$$\eta_2 + 0{,}8\, \eta_1 \leq 1{,}4$$

i.e.:

$$\frac{F_{Ed}}{L_{eff} t_w \dfrac{f_y}{\gamma_{M1}}} + 0{,}8 \left(\frac{N_{Ed}}{\dfrac{f_y\, A_{eff}}{\gamma_{M0}}} + \frac{M_{Ed} + N_{Ed}\, e_N}{\dfrac{f_y\, W_{eff}}{\gamma_{M0}}} \right) \leq 1{,}4$$

Assessment

- Ideal and actual plate – differences.
- Eurocode approaches concerning buckling effects.
- Verification of class 4 sections.
- Design resistance to shear.
- Behaviour of webs under shear.
- Types of local loading.
- Verification for local loading.

Notes to users of the lecture

- This session requires about 90 minutes of lecturing.
- Within the lecturing, buckling of plates under direct, shear and local loading is described. The lecture starts with linear theory of buckling and resulting critical stress, followed with non-linear theory of buckling of actual imperfect plate and its resistance. The buckling resistances under direct stress, shear and local loading in accordance with Eurocode are commented.
- Further readings on the relevant documents from website of www.access-steel.com and relevant standards of national standard institutions are strongly recommended.
- Keywords for the lecture:

  buckling of plates, ideal plate, real plate, buckling due to direct stresses, buckling under shear, local buckling, interaction formulas for buckling.

Notes for lecturers

- Subject: Buckling of plates.
- Lecture duration: 90 minutes.
- Keywords: buckling of plates, ideal plate, real plate, buckling due to direct stresses, buckling under shear, local buckling, interaction formulas for buckling.
- Aspects to be discussed: Ideal plate, critical stress, real plate, reduction factor. Behaviour of plates under shear loading. Behaviour of plates under local loading. Eurocode approach.
- After the lecturing, determination of effective cross section parameters (class 4 effective parameters) should be practised.
- Further reading: relevant documents www.access-steel.com and relevant standards of national standard institutions are strongly recommended.
- Preparation for tutorial exercise: see examples prepared for the course.