Design of Steel-to-Concrete Joints

This Design manual summarises the reached knowledge in the RFCS Project RFSR-CT-2007-00051 New Market Chances for Steel Structures by Innovative Fastening Solutions between Steel and Concrete, INFASO. The material was prepared in cooperation with researchers from Institute of Structural Design and Institute of Construction Materials, Universität Stuttgart, Department of Steel and Timber Structures, Czech Technical University in Prague, and practitioners from Gabinete de Informática e Projecto Assistivo Computador Ltda., Combra, Goldbeck West GmbH, Bielefeld, stahl-verbindungbau GmbH, Dreieich and European Convention for Constructional Steelwork, Bruxelles, one targeting on fastening technique modelling and others focusing on steel joint design.

The models in the text are based on component method and enable the design of steel to concrete joints in vertical position, e.g. beam to column or to wall connections, and horizontal ones, base plates. The behaviour of components in terms of resistance, stiffness, and deformation capacity is summed up for components in concrete and steel parts: header studs, stirrups, concrete in compression, concrete panel in shear, steel reinforcement, steel plate in bending, threaded studs, embedded plate in tension, beam web and flange in compression and steel contact plate. In the Chapters 5 and 6 are described the possibility of assembly of components behaviour into the whole joint behaviour for resistance and stiffness separately. The presented assembly enables the interaction of normal forces, bending moments and shear forces acting in the joint. The global analyses in Chapter 7 is taken into account the joint behaviour. The connection design is sensitive to tolerances, which are recapitulated for beam to column connections and base plates in Chapter 8. The worked examples in Chapter 9 demonstrates the application of theory to design of pinned and moment resistant base plates, pinned and moment resistance beam to column connections and the use of predicted values into the global analyses.
Design of steel-to-concrete joints, Design manual I

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The present document and others related to the research project INFASOR RFSR-CT-2007-00051 New Market Chances for Steel Structures by Innovative Fastening Solutions between Steel and Concrete and the successive dissemination project RFS2-CT-2012-00022 Valorisation of Knowledge for Innovative Fastening Solution between Steel and Concrete, which have been co-funded by the Research Fund for Coal and Steel (RFCS) of the European Community.


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Printing by European Convention for Constructional Steelwork
February 2014
178 pages, 138 figures, 32 tables
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Symbols

Lower case

a factor considering the shoulder width, length
b length
c minimum edge distance, effective width, length
c_{cr,N} critical edge distance, c_{cr,N} = 1.5 \ h_{ef}
c_w drag coefficient
d diameter
b diameter of the bolt
d_b diameter of the head of headed stud
d_s diameter of the shaft of headed stud
d_{sre} diameter of the stirrup
d_{knom} nominal diameter of the anchor shaft
d_w diameter of the washer
e_{xy} length between the bolt axis and the edge of the plate
e eccentricity
f_{bd} design bond strength according to EN1992-1-1:2004
f_{cd} design strength of concrete
f_k characteristic strength of concrete
f_{ck,cube} characteristic square strength of concrete
f_u strength of structural steel
f_{ub} strength of the bolt
f_{ak} characteristic strength of steel
f_y nominal yield strength of steel
f_{ya} average yield strength
f_{yb} nominal value of yield strength of the bolt
f_{yd} design yield strength of steel
f_{y,sre} design yield strength of the stirrups
f_{yk} characteristic yield strength of steel
h height
h_{ef} effective embedment depth according to product specifications
k coefficient depending on the type of forming

k_1 factor for concrete cone strength in case of headed studs
k_2 factor for the headed studs for component P
k_A factor considering the cross-section
k_a form factor at porous edge sections
k_b stiffness of the bolt
k_{b,re} bond stiffness due to supplementary reinforcement, stirrups
k_{C1} stiffness due to the displacement of the anchorage in case of concrete cone failure with supplementary reinforcement, combination C1
k_{C2} stiffness due to the displacement of the head, due to the pressure under the head on the concrete, and steel elongation, combination C2
k_{c,de} stiffness of the descending branch for component CC
k_{c,soft} stiffness of the concrete cone in the softening branch
k_i concentration factor
k_p stiffness coefficient of the plate
k_{p,de} stiffness of the descending branch for component P
k_s stiffness of the anchor shaft for component S
k_{s,re} steel stiffness due to supplementary reinforcement, stirrups
k_v empirical value depending on the type of anchor
l_1 anchorage length
l_{ep} elongated length
l_{eff} effective length of T-stub, defined in accordance with EN1993-1-8:2006
l_{v,eff} effective length of shear area
m distance between threaded and headed studs
m_p\ell plastic moment resistance per unit, defined as m_p\ell = \frac{0.25 \ t^2 \ f_y}{\gamma_{Mo}}
n location of the prying force, number
n_{re} total number of legs of stirrups
p internal pressure
r radius of the fillet of a rolled profile
s actual spacing of anchors
critical spacing for anchors $S_{cr,N}$
thickness $t$
thickness of the T-stub, flange $t_f$
thickness of T-stub, column $t_w$
thickness of the anchor plate $t_{p1}$
thickness of the base plate $t_{p2}$
fictive effective width $w_{fic}$
distance between the anchor and the crack on the concrete surface assuming a crack propagation from the stirrup of the supplementary reinforcement to the concrete surface with an angle of 35$^\circ$ $x$
distance of tension/compressed part $z$

**Upper case**

$A$ cross section area
$A_{c0}$ loaded area
$A_{c1}$ maximum spread area
$A_{c,N}$ actual projected area of concrete cone of the anchorage at the concrete surface, limited by overlapping concrete cones of adjacent anchors ($s < s_{cr,N}$), as well as by edges of the concrete member ($c < c_{cr,N}$)
$A_{0,N}$ reference area of the concrete cone of an individual anchor with large spacing and edge distance projected on the concrete surface
$A_{eff}$ effective area
$A_{h}$ area on the head of the headed stud
$A_{net}$ net cross section area
$A_{s}$ tensile stress area in the bolt
$A_{nom}$ nominal cross section area of all shafts
$A_{re}$ nominal cross section area of all legs of the stirrups
$B_{t,Rd}$ design tension resistance of a single bolt-plate assembly
$B_{c,Rd} = 0.9 \cdot f_{ub} \cdot A_s / \gamma_{Mb}$
$D$ diameter of column
$E$ modulus of elasticity the steel $E = 210\,000\,\text{MPa}$
$F$ force or load
$F_{c,Rd}$ resistance of compressed part

$F_d$ design load
$F_k$ characteristic load
$F_{memb}$ axial force
$F_{t,Ed}$ external tensile force
$F_{t,Rd}$ external ultimate resistance
$F_{T,Rd}$ resistance of tension part
$I$ moment of inertia
$I_t$ torsion constant
$K$ general stiffness
$L$ length
$L_b$ length of anchor bolt
$L_{cr}$ buckling length
$L_D$ the elongation length of the bolt, which may be taken as the total grip length (thickness of material plus washers) plus half the sum of the height of the bolt head and the height of the nut
$L_h$ length of the anchor shaft
$I_{p, bp}$ equivalent moment of inertia
$M_{c,Rd}$ bending moment capacity
$M_{j,Rd}$ design moment resistance of a joint
$M_{N,Rd}$ interaction resistance bending with compression
$M_{pl,Rd}$ plastic moment resistance defined as $M_{pl,Rd} = l_{efy} \cdot m_{pl}$
$M_{t,Rd}$ torsion capacity
$N_{act}$ actual load on the anchor
$N_{b,Rd}$ design buckling resistance
$N_{cr}$ critical buckling load
$N_{Ed}$ tension/compression load
$N_{ETA}$ tension load for which the displacements are derived in the product specifications
$N_{pl,Rd}$ design capacity in tension/compression
$N_{Rd}$ design capacity
$N_{Rd,b, re}$ design tension resistance for bond failure of stirrups
$N_{Rd,c3}$ design failure load or the combined model
$N_{Rd,c}$ design tension resistance for concrete cone failure of headed stud
Greek symbols

\(\alpha\)  factor according to EN1992:2006 for hook effect and large concrete cover
\(\alpha_c\)  factor of component concrete break out in tension
\(\alpha_p\)  factor for the component head pressing
\(\alpha_s\)  factor of component stirrups
\(\alpha_j\)  material coefficient
\(\gamma_F\)  partial safety factor for actions
\(\gamma_M\)  material safety factor
\(\gamma_{Mb}\)  partial safety factor for bolts \(\gamma_{Mb} = 1.25\)
\(\gamma_{Mc}\)  partial safety factor for concrete \(\gamma_{Mc} = 1.5\)
\(\gamma_{Ms}\)  partial safety factor for steel \(\gamma_{Ms} = 1.15\)
\(\gamma_{MV}\)  partial safety factor for shear resistance of studs \(\gamma_{MV} = 1.25\)
\(\gamma_{Mw}\)  partial safety factor for welds \(\gamma_{Mw} = 1.25\)
\(\gamma_{Mo}\)  partial safety factor for resistance of Class 1, 2 or 3 cross-sections \(\gamma_{Mo} = 1.0\)
\(\gamma_{M1}\)  partial safety factor for resistance of a member to buckling \(\gamma_{M1} = 1.0\)
\(\gamma_{M2}\)  partial safety factor resistance of net section at bolt holes \(\gamma_{M2} = 1.25\)
\(\delta\)  deformation, displacement
\(\delta_{act}\)  displacement corresponding to \(N_{act}\)
\(\delta_c\)  displacement corresponding to \(N_{act}\) for concrete cone
\(\delta_f\)  corresponding displacement at failure load \(N_{Rd,are}\) or \(N_{Rd,bre}\)
\(\delta_{N,ETA}\)  displacement given in the product specifications for a corresponding tension load
\(\delta_{Rd,bre}\)  deformation corresponding to design resistance for bond failure of stirrups
\(\delta_{Rd,c}\)  deformation corresponding to design resistance for concrete cone failure
\(\delta_{Rd,p}\)  deformation corresponding to design resistance for pull out failure
\(\delta_{Rd,s}\)  deformation corresponding to \(N_{rd}\)
\(\delta_{Rd,s}\)  deformation corresponding to design resistance for steel failure
\(\delta_{Rd,are}\)  deformation corresponding to design resistance for steel failure of stirrups

\(N_{Rd,cs}\)  design failure load for the concrete strut
\(N_{Rd,p}\)  design tension resistance for pull out failure of headed stud
\(N_{Rd,re}\)  design failure load for the supplementary reinforcement
\(N_{Rd,s}\)  design tension resistance for failure of headed stud
\(N_{Rd,s,re}\)  design tension resistance for steel failure of stirrups
\(N_{Rd,c}\)  characteristic resistance of a single anchor without edge and spacing effects
\(N_u\)  ultimate resistance
\(N_y\)  yielding resistance
\(Q\)  prying force
\(R_d\)  design capacity
\(R_k\)  characteristic resistance
\(S_i\)  elastic stiffness
\(S_{ini}\)  initial stiffness
\(V_{ETA}\)  shear load for which the displacements are derived in the product specifications
\(V_{pl,Rd}\)  shear capacity
\(V_{Rd}\)  design failure load for the anchor under shear
\(V_{Rd,c}\)  design shear resistance for concrete cone failure
\(V_{Rd,cp}\)  design shear resistance for concrete pryout
\(V_{Rd,p}\)  design shear resistance for pullout
\(V_{Rd,s}\)  design shear resistance for steel failure
\(W_e\)  external work
\(W_{eff}\)  section modulus of effective area
\(W_{el}\)  elastic section modulus
\(W_i\)  internal work
\(W_{pl}\)  plastic section modulus
δ_{RD,sy} deformation corresponding to design yield resistance for steel failure
δ_u elongation
δ_{V,ETA} displacement given in the product specifications for a corresponding shear load
δ_{u,eta} strain limit for the stirrups due to bond
ε_{su,re} ultimate design strain limit for steel
ε_{u,re} strain limit for the stirrups under tension
ε_{u,re} strain limit for the stirrups under tension
ε_u ultimate strain
θ angle
λ slenderness of member
μ coefficient of friction
ν Poisson’s ratio, ν = 0.30
σ stress
ψ \text{N} factor accounting for geometric effects in anchor group, ψ_{AN} = A_{c,N}/A_{c,N}^0
ψ_{rc,N} factor accounting for negative effect of closely spaced reinforcement in the concrete member on strength of anchors with h_{ef} < 100 mm
ψ_{k,N} factor accounting for the influence of edges of the concrete member on the distribution of stresses in the concrete ψ_{k,N} = 0.7 + 0.3 · c/c_{er,N} ≤ 1.0
ψ_{supp} support factor considering the confinement of the stirrups ψ_{supp} = 2.5 - x/h_{ef} ≥ 1.0
Φ rotation

Subscripts
A area
act actual
b bolt, bond
bd design bond
c column, concrete
cb concrete block
ck characteristic concrete
cp concrete pry out
cs concrete strut
cr critical
d design
e external
eff effective
ETA European technical approval
g grout
h head
i internal
k characteristic
lim limit
Mc material concrete
Ms material steel
N tension
nom nominal
po pullout
p plate
pl plastic
Rd resistance design
Rk characteristic resistance
re failure
rec reinforcement
Sd internal design
s shaft of anchor, stud
soft softening
supp support
T tension part
t tension
tot total
p plate
p1 anchor plate
p2 base plate
u ultimate
uk characteristic ultimate
V shear
w column web
x, y directions
y yield
yd design yield
yk characteristic yield
1 INTRODUCTION

The mixed building technology allows to utilise the best performance of all structural materials available such as steel, concrete, timber and glass. Therefore the building are nowadays seldom designed from only one structural material. Engineers of steel structures in practice are often faced with the question of economical design of steel to concrete joints, because some structural elements, such as foundations, stair cases and fire protection walls, are optimal of concrete. A gap in knowledge between the design of fastenings in concrete and steel design was abridged by standardized joint solutions developed in the INFASO project, which profit from the advantage of steel as a very flexible and applicable material and allow an intelligent connection between steel and concrete building elements. The requirements for such joint solutions are easy fabrication, quick erection, applicability in existing structures, high loading capacity and sufficient deformation capacity. One joint solution is the use of anchor plates with welded headed studs or other fasteners such as post-installed anchors. Thereby a steel beam can be connected by butt straps, cams or a beam end plate connected by threaded bolts on the steel plate encased in concrete. Examples of typical joint solutions for simple steel-to-concrete joints, column bases and composite joints are shown in Fig. 1.1.

The Design Manual I gives an overview of the existing design rules and introduces components developed. To present the use of the developed design rules, worked examples are given within the Design Manual I. More detailed information about the background documents, the experiments and development of the new design rules might be found in the final report, (Kuhlman et al 2013) and in Design manual II. This manual is focused to more complex worked examples, the application of a software tool for design, sensitivity study of proposed analytical models and its boundary conditions as well as design tables of optimal solutions.

Chapter 2 gives a general overview about the component method and presents the existing models for steel-to-concrete joints. It also includes a short summary of the joint models and components developed in the project. In Chapter 3 and Chapter 4 the concrete and the steel components for the modelling of steel-to-concrete joints are described in more detail. The components already described in the codes as well as components of the newly derived models are introduced. Values for stiffness and resistance are presented. In Chapter 5 the single components are assembled to evaluate the overall joint resistance. Chapter 6 shows how the joint stiffness can be derived due to the stiffness’s of the single components. For the global analysis of a structure the joint behaviour/stiffness may have an influence. The effects of the joint modelling on the global analysis are explained in Chapter 7. The tolerances for steel-to-concrete joints and their effect on the construction are discussed in Chapter 8. In the Chapter 9 worked examples for the whole range of the steel-to-concrete joints are prepared. This examples are demonstrating the possibilities of the new design rules and allow an easy
access for the engineers in practice. The references are given to this Design manual I (DM I) and to Eurocodes (EN199x-1-x). Chapter 10 summarises the offered opportunity for innovations.

Chapters 1 and 2 were prepared by U. Kuhlman and J. Ruopp, Chapter 3 by J. Hofmann and A. Sharma, Chapters 4, 5 and 6 by F. Wald, Bečková Š. and Schwarz I., Chapter 7 by da Silva L. Simoes, H. Gervásio and J. Henriques and F. Gentili and Chapter 8 by M. Krimpmann. The worked examples 9.1 to 9.3 were set by Š. Bečková and I. Schwarz, 9.4 by Š. Bečková, I. Schwarz and M. Krimpmann, 9.5 by J. Ruopp, 9.6 and 9.7 by J. Henriques and F. Gentili, with help of the headed studs design models by A. Sharma.
2 COMPONENT METHOD FOR STEEL TO CONCRETE JOINTS

2.1 Design method

In the past decades, the component method has been set as a unified approach for the efficient analysis of steel and composite joints, see (Da Silva 2008). The basic principle of the component method consists of determining the complex non-linear joint response through the subdivision into basic joint components. The joint can be regarded as a set of individual basic components that contribute to its structural behaviour by means of resistance, stiffness and deformation capacity. The component method allows designers to take options more efficiently because the contribution of each component to the joint behaviour may be optimized according to the limiting components. Thus, one of the main advantages of the component method is that the analysis of an individual component can be done independently of the type of joint. In a second calculation step the single components are assembled by the designers according to the joint configuration.

Joint components may be divided by the type of loading. Accordingly, three groups of components are usually identified: components for tension, compression and shear. Additionally, a second division may be done according to their location: panel zone or connecting zone. In Fig. 2.1 these two definitions are illustrated based on a double sided composite joint.

![Fig. 2.1 Division of joint into groups and zones](image)

In practice these components are modelled by translational springs with non-linear force-deformation response that are exposed to internal forces. The joint may then be represented by a spring model as illustrated in Fig. 2.2.
The component method is given by EN1993-1-8:2006 and EN1994-1-1:2010 for the analysis of steel and composite joints. The application of the method requires following steps:

1. Identification of the basic joint components
2. Characterization of the structural properties of the basic joint components
3. Assembly of the component properties

In the referred codes, a list of basic joint components is provided for the most common joint configurations. Basic joint components are then characterized in terms of strength, stiffness and deformation capacity allowing to obtain the $F-\delta$ curve, see Fig. 2.3, reproducing its behaviour. Finally, through the assembly procedure the joint properties are determined. The joint behaviour may be later reproduced by an $M-\Phi$ curve, see Fig. 2.4, in the structural analysis.

### 2.2 Classification of joints

#### 2.2.1 Global analyses

The classification of the joints is prepared to examine the extent to which the stiffness or strength have to be considered in the calculation according the design accuracy. In total there are three different calculation methods which require different joint properties. These calculation methods and the joint properties are compared within Tab. 2.1.
**Elastic method**

If the elastic calculation method is applied, only the joint stiffness $S_j$ is considered. $S_j$ is implemented in the structural calculation as spring element or one-dimensional beam element in order to determine the internal forces. If the bending moment does not exceed $2/3$ of the moment resistance of the joint the initial stiffness $S_{j,ini}$ can be used to describe the elastic behaviour. For calculations, where the plastic moment capacity is reached, the joint stiffness can be calculated with the secant stiffness $S_{j,ini}/\mu$. The joints are classified for this method by taking into consideration the rotational stiffness.

**Rigid plastic method**

In the second calculation method the elastic behaviour of the joint is neglected. Internal forces of the structural calculation are calculated from 1st order plastic hinge theory only satisfying equilibrium conditions. Within this method only the plastic moment capacity is considered, but the joints must have sufficient deformation capacity to allow full plastic redistribution. In this case the joints are classified by the resistance.

**Elastic plastic method**

If the third method is applied the overall moment-rotation-relationship of the joint has to be considered. This relationship is used within the joint modelling of the structural calculation. For simplification a bilinear approach of the moment rotation curve may be used. Typically the reduced secant stiffness is applied. If the elastic plastic method is used, the joint has to be classified by stiffness and strength.

The advantages of this method are shown in the following example. In Fig. 2.5 a steel frame with horizontal and vertical loading is shown. Instead of modelling the column bases as a pinned joint as it is common in practice, the column bases may be classified as semi-rigid and modelled with a rotational spring. Thereby the column bases may stabilise the structure and reduce the bending moment in the steel-to-steel beam to column joints. So a classification of the column bases as semi-rigid instead of pinned makes the steel structure more safe and economical.
2.2.2 Stiffness

The first part of this chapter deals with the classification of beam to column/wall and beam to beam joints, the second part with the classification of column bases. Depending on its initial rotational stiffness $S_{\text{ini}}$ a joint may be classified as pinned, rigid or semi-rigid. Normally pinned joints can transfer axial and shear force. Rotation of the joint does not cause significant bending moments. If a joint cannot be classified as normally pinned or rigid it is classified as semi-rigid. Rigid joints have a rotational stiffness which legitimise to treat the joint as rigid in the global analysis.

**Joints classified according to the connecting beams**

Rigid joints, in Fig. 2.6 zone 1, is classified as rigid if

$$S_{\text{ini}} \geq K_b E I_b / L_b$$  \hspace{1cm} (2.1)

If a bracing system reduces the horizontal displacement more than 80 %, then $K_b = 8$. For other frames provided that in every storey the following equation (2.2) is valid, then $K_b = 25$.

$$\frac{K_b}{K_c} \geq 0.1$$  \hspace{1cm} (2.2)
Semi-rigid joints, in Fig. 2.6 zone 2, are all joints which are not classified as pinned or rigid. For frames where Eq. 2.3 applies the joints should be classified as semi rigid and not as rigid.

\[ \frac{K_h}{K_c} < 0.1 \]  \hspace{1cm} (2.3)

Nominally pinned joints, in Fig. 2.6 zone 3, are expecting to have a limited bending stiffness compared to the bending stiffness of the connected beam.

\[ S_{j,ini} \leq 0.5 \cdot E \cdot \frac{l_b}{L_b} \]  \hspace{1cm} (2.4)

where
- \( K_h \) is mean value of \( \frac{l_b}{L_b} \) for all the beams at the top of that storey
- \( K_c \) is mean value of \( \frac{l_c}{L_c} \) for all columns of that storey
- \( l_b \) is the second moment of area of beam
- \( l_c \) is the second moment of area of column
- \( L_b \) is the span of beam
- \( L_c \) is the storey height of a column

Column bases classified according to the connecting column

Column bases are classified as rigid if the following conditions are satisfied. There are two possible cases which have to be considered. If there is an additional bracing in a frame and the additional bracing reduces the horizontal movement at least by 80%, then the column base affects the accuracy of the column design, which depends on the column relative slenderness. This column base might be assumed as rigid according to EN1993-1-8:2006 cl. 5.2a, if

\[ \lambda_0 \leq 0.5 \]  \hspace{1cm} (2.5)

for

\[ 0.5 < \lambda_0 < 3.93 \]  \hspace{1cm} \( S_{j,ini} \geq 7 \left( 2 \lambda_0 - 1 \right)E l_c/L_c \]  \hspace{1cm} (2.6)

and for

\[ \lambda_0 \geq 3.93 \]  \hspace{1cm} \( S_{j,ini} \geq 48 E l_c/L_c \]  \hspace{1cm} (2.7)

where
- \( \lambda_0 \) is the relative slenderness of a column in which both ends are assumed as pinned.

For all other constructions, the cases where the storey’s sway is not prevented, the column base might be classified according to cl. 5.2d in EN1993-1-8:2006 as rigid if

\[ S_{j,ini} \geq 30 E l_c/L_c \]  \hspace{1cm} (2.8)

2.2.3 Strength

A joint is classified for strength as pinned, full-strength or partial-strength, see Tab. 2.1 and Fig. 2.7. The classification by strength may be found in EN1993-1-8:2006 cl. 5.2.3. Nominally pinned joint should have a design moment resistance less than 25% of the design moment resistance, which would be required for a full-strength joint. They must have sufficient rotational capacity. A Partial-strength joint is a joint, which cannot be classified as pinned or full-strength.
The design moment resistance of a full-strength joint is bigger than the design moment resistance of the beam or column connected to it.

![Fig. 2.7 Classification due to resistance](image)

If the design resistance of the beam $M_{b,pl,Rd}$ is smaller than the design resistance of the column $M_{c,pl,Rd}$, $M_{b,pl,Rd}$ is replaced for connections at the top of a column by $M_{c,pl,Rd}$, see Fig. 2.7. If the design resistance of the beam $M_{b,pl,Rd}$ is smaller than the double design resistance of the column $M_{c,pl,Rd}$ than in the figure above $M_{b,pl,Rd}$ is replaced for connections within the column height by $2M_{c,pl,Rd}$.

### 2.2.4 Deformation capacity

In EN1993-1-8:2006 an explicit classification for deformation or rotational capacity of the joint is not implemented. The complexity on classification according to deformation capacity is in the lack of knowledge of the upper values of material properties by designers, which do not allow a safe prediction of the failing component. In EN 1993-1-8 cl 6.4 design rules for the rotation capacity are given based on best engineering practice. If the system is calculated with a plastic global analysis a sufficient rotation capacity is needed. No investigation of the rotation capacity of the joint is necessary, if the moment resistance of the joint $M_{j,Rd}$ is at least 20% bigger than the plastic moment resistance $M_{pl,Rd}$ of the connected beam, see (2.9). Then the plastic hinge appears in the beam and the rotational capacity has to be satisfied by the beam section.

$$M_{j,Rd} \geq 1.2 \cdot M_{pl,Rd} \quad (2.9)$$

If the moment resistance of the joint is not 1.2 times the plastic moment resistance of the connected beam and a plastic hinge is assumed in the joint, minimum rotational capacities for bolted and welded joints have to be checked.

**Bolted joints**

The rules for bolted joints may be found in EN1993-1-8:2006 cl 6.4.2. A bolted joint is assumed to have a sufficient rotation capacity if following conditions can be applied:

If the failure load $M_{j,Rd}$ is determined by the resistance of the column web panel and for this panel $d/t_w \leq 69 \varepsilon$

where

- $d$ is the nominal bolt diameter and
- $t_w$ is the thickness of the web
If the thickness of the flange of the column or the beam end plate is sufficiently thin to satisfy the following formula.

\[
t \leq 0.36 \sqrt{\frac{f_{ub}}{f_y}}
\]

(2.10)

where

- \(f_{ub}\) is ultimate strength of the bolts
- \(f_y\) is yield strength of the flange or the end plate

**Welded joints**

The rules for welded joints may also be found in EN1993-1-8:2006 cl 6.4. For a welded beam to column connection the rotation capacity \(\phi_{cd}\) may be calculated with the following equation. In this case the web has to be stiffened in the compression area but not in the tension area and the moment resistance is not determined by the resistance of the column web panel.

\[
\phi_{cd} = 0.025 \frac{h_c}{h_b}
\]

(2.11)

where

- \(h_c\) is the depth of the column
- \(h_b\) is the depth of the beam

For a welded beam to column connection where the compression area and the tension area in the column are not stiffened, the rotation capacity may be assumed to be at least 0.015 rad.

### 2.3 Steel-to-concrete joints

#### 2.3.1 Available models

Design models for steel-to-concrete joints are currently available in the three standard documents:

- EN1993-1-8:2006 includes values for stiffness and resistance for all steel components and values for stiffness and resistance for concrete components in compression. There are no rules for concrete components in tension or shear.
- EN1994-1-1:2010 enhancement of the rules from EN 1993-1-8 on composite joints such as the connection of composite girder to steel columns.
- CEN/TS 1992-4-1:2009 summarises values for the design resistance of fasteners in concrete. But no values for stiffness and ductility are available.

#### 2.3.2 Steel and composite structures

Design rules in the Eurocode are given for different joint configurations. The model for the column bases is described in the EN1993-1-8:2006 and the model for the composite joint in EN1994-1-1:2010.

**Column bases with base plates**

The analytical prediction model for column base with base plate is described in the EN1993-1-8:2006. With these design rules column bases loaded by axial force and bending moments are calculated. The model is only including concrete components for the compression forces.
For the tension force only steel components are considered. The design resistance of column bases with steel base plates is described in EN1993-1-8:2006, cl 6.2.8. First according to the eccentricity of the axial force $e$ and the geometry of the column base one of the four loading types is chosen, and the lever arm $z$ is calculated. For this see Tab. 2.2. Then the loading of the tension and the compression components are calculated. The failure load is determined by the weakest activated component. These components are for:

**Tension**

- Base plate in bending under tension cl 6.2.6.11 in EN1993-1-8
- Anchor bolt in tension cl 6.2.6.12 in EN1993-1-8
- Column web in tension cl 6.2.6.8 in EN1993-1-8

**Compression**

- Base plate in bending under compression cl 6.2.6.10 in EN1993-1-8
- Concrete in compression cl 6.2.6.9 in EN1993-1-8
- Column web and flange in compression cl 6.2.6.7 in EN1993-1-8

**Shear**

- Anchor bolts in shear cl 6.2.2.6 to 6.2.2.9 in EN1993-1-8

According to procedure in EN1993-1-8:2006 cl 6.3.4 one of the four cases of the loading and geometry is chosen, see Tab.2.2. Then the rotational stiffness is calculated. One complexity creates change of the loading type depending on the loading cases. From this different rotational stiffness values for different combinations of bending moment and axial forces are resulting. The design of the embedded column base according to Eurocodes was developed by (Pertold et al, 2000) based on set of tests and finite element modelling. This model is prepared to approve resistance to combine base plate with embedding.

**Composite joints**

The composite joint is described in the Section 8 in EN1994-1-1:2010. The composite joint may be used for the connection of composite beams to steel columns. The design rules are an enhancement of the rules according to EN1993-1-8:2006 and new components are added. These additional components are:

- Longitudinal steel reinforcement in tension cl. 8.4.2.1 EN1994-1-1:2010
- Steel contact plate in compression cl 8.4.2.2 EN1994-1-1:2010
- Column web in transverse compression cl 8.4.3 EN1994-1-1:2010
- Reinforced components cl 8.4.4 EN1994-1-1:2010
- Column web panel in shear cl 8.4.4.1 EN1994-1-1:2010
- Column web in compression cl 8.4.4.2 EN1994-1-1:2010

For all other components EN1993-1-8:2006 is applied.
Tab. 2.2 The loading situations for the definition of the lever arm

<table>
<thead>
<tr>
<th>Number</th>
<th>Description of loading</th>
<th>Sketch</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Left side in tension, Right side in compression</td>
<td><img src="image1" alt="Sketch 1" /></td>
<td>[ z = z_{T,l} + z_{C,r} ] Bending moment is dominating</td>
</tr>
<tr>
<td>2</td>
<td>Left side in tension, Right side in tension</td>
<td><img src="image2" alt="Sketch 2" /></td>
<td>[ z = z_{T,l} + z_{T,r} ] Tensile force is dominating</td>
</tr>
<tr>
<td>3</td>
<td>Left side in compression, Right side in tension</td>
<td><img src="image3" alt="Sketch 3" /></td>
<td>[ z = z_{C,l} + z_{T,r} ] Bending moment is dominating</td>
</tr>
<tr>
<td>4</td>
<td>Left side in compression, Right side in compression</td>
<td><img src="image4" alt="Sketch 4" /></td>
<td>[ z = z_{C,l} + z_{C,r} ] Compression force is dominating</td>
</tr>
</tbody>
</table>

Fig. 2.8 Composite joint
### Tab. 2.3 Failure modes observed for anchors in concrete

<table>
<thead>
<tr>
<th>Loading</th>
<th>Failure modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tension</td>
<td><img src="image1" alt="Steel failure" /> <img src="image2" alt="Concrete cone failure" /> <img src="image3" alt="Pull-out / Pull-through" /></td>
</tr>
<tr>
<td></td>
<td><img src="image4" alt="Splitting failure" /> <img src="image5" alt="Local Blow-out failure" /> <img src="image6" alt="Steel failure" /></td>
</tr>
<tr>
<td>Shear</td>
<td><img src="image7" alt="Steel failure" /> <img src="image8" alt="Concrete edge failure" /> <img src="image9" alt="Pull-out failure" /></td>
</tr>
<tr>
<td></td>
<td><img src="image10" alt="Pry-out failure" /></td>
</tr>
</tbody>
</table>

#### 2.3.3 Concrete structures

In CEN/TS1992-4-1:2009 the design of fastenings in concrete is given. In these rules the failure modes of the fasteners and the concrete are described in a detailed way. For tension and shear loading various failure modes exist. Failure modes are given according to CEN/TS 1992-4-1:2009, see Tab. 2.3. All possible failure modes are determined. The smallest resistance defines the design resistance of the joint. The design rules for the resistance include different types of geometries. Also edge effects, concrete with and without cracks and different kinds of fasteners are considered. However for stiffness no design rules are given and the use of additional stirrups is covered in a very conservative way.

#### 2.3.4 Components for joints with anchor plate

**Headed studs in tension / Headed studs with stirrups in tension**

Load-displacement-curves of test specimens have shown, that in cases were additional reinforcement is used, also other components besides the reinforcement have a contribution on the overall load bearing capacity of the fixture. If, for instance, the reinforcement starts to yield, compression struts may develop and a small concrete cone failure can be the decisive component. With the design model the interaction of the concrete cone and the stirrups is considered. This allows the increase of the design resistance and the determination of the stiffness of the two combined components concrete cone and stirrups in tension in cases, where both of them are interacting. In Fig. 2.9 a headed stud with additional reinforcement and the assembly of single components is shown.
Embedded plate in tension

Ductile behaviour and a larger rotation capacity of column bases can be initiated with a thin anchor plate in combination with a base plate welded to the end of the column. In Fig. 2.10 three different kinds of geometries of embedded plates are shown, see Kuhlman et al, 2013.

The headed studs are welded on the bottom side of the base plate to connect the thin plate to the concrete. The column base plate is connected to the anchor plate by the threaded bolts. If the threaded bolts and the headed studs are in one line like, see Fig. 2.10, the anchor plate has no influence on the behaviour of the joint. If the threaded bolts and the headed studs are not in one line the anchor plate is activated. The model of the embedded plate represents an additional failure mode for the T-stub in tension. If the T-stub reaches its limit state, the thin base plate may still increase its capacity due to the membrane effect. The component embedded plate in tension shows a ductile behaviour as large deformations occur before failure. A detailed explanation of this component is given in Chapter 7.

The Tab. 2.4 summarises the components, which are used to model the simple and rigid steel beam to concrete column/wall joints and column bases using anchor plates.

<table>
<thead>
<tr>
<th>Component</th>
<th>Headed stud in tension</th>
<th>Concrete breakout in tension</th>
<th>Stirrups in tension</th>
<th>Pull-out failure of the headed stud</th>
<th>Headed stud in shear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure</td>
<td>3.1.1</td>
<td>3.1.2</td>
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<td>3.1.5</td>
<td>3.1.6</td>
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<tr>
<td>Chapter</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Component</th>
<th>Friction</th>
<th>Concrete in compression</th>
<th>Concrete panel in shear</th>
<th>Longitudinal steel</th>
<th>Slip of the composite beam</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure</td>
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<td></td>
</tr>
<tr>
<td><img src="image1.png" alt="Image" /></td>
<td>Threaded studs in tension/shear</td>
<td>3.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image2.png" alt="Image" /></td>
<td>Punching of the anchor plate</td>
<td>4.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image3.png" alt="Image" /></td>
<td>Anchor plate in bending and tension</td>
<td>4.6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image4.png" alt="Image" /></td>
<td>Colum/beam flange and web in compression</td>
<td>4.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image5.png" alt="Image" /></td>
<td>Steel contact plate</td>
<td>4.4</td>
<td></td>
<td></td>
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</tbody>
</table>

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<tr>
<td><img src="image5.png" alt="Image" /></td>
<td>Steel contact plate</td>
<td>4.4</td>
</tr>
</tbody>
</table>
3 COMPONENTS IN CONCRETE

3.1 Component model for headed studs

For components embedded in concrete the displacement behaviour and therefore the F-δ-curve is influenced by the concrete properties itself and the interaction between the anchorage and the concrete. The influence of concrete on the behaviour of anchorages in tension have to be considered. The scatter in concrete is much larger than that observed for the material steel, see (Pallarés and Hajjar, 2009).

For design, a material safety factor for concrete according to EN1992-1-1:2004 of \( \gamma_M = 1.5 \) is used. The characteristic values for the resistances are derived by assuming a normal distribution and a probability of 90 % for the 5 % fractal that corresponds to the characteristic value. The given displacements and stiffness’s are mean values and can scatter with coefficient of variation up to 50 %.

The complete F-δ-curve for the design of a headed stud in tension is described by a rheological model using and combining different components for the headed stud. The individual components for anchorages with supplementary reinforcement are:

- Component S  Steel failure of the headed stud \( (\delta_{Rd,s} / N_{Rd,s}) \)
- Component CC Concrete cone failure \( (\delta_{Rd,c} / N_{Rd,c}) \)
- Component RS Steel failure of the stirrups \( (\delta_{Rd,ss}\, / \, N_{Rd,ss}) \)
- Component RB Bond failure of the stirrups \( (\delta_{Rdb,ss}\, / \, N_{Rdb,ss}) \)
- Component P Pull out failure of the headed stud \( (\delta_{Rd,p} / N_{Rd,p}) \)

The combination is given in Fig. 3.1.

![Components](image)

a) with supplementary reinforcement b) without supplementary reinforcement

Fig. 3.1 Spring models for the different components of anchorages embedded in concrete

3.1.1 Headed studs in tension, component S

If a headed stud is loaded in tension, the load is first transferred from the loading point at the base plate to the bearing areas of the headed stud. Therefore the shaft will elongate up to the design yielding strength \( f_{yd} = f_{yk}/\gamma_M \). For design the behaviour is assumed as linear elastic up to the yielding load of the headed stud. The corresponding elongation due to the introduced stress is calculated with the equation using the Hooke’s law. The elongation corresponding to the yield load is given by
\[
\delta_{Rd,sy} = \frac{N_{Rd,s}}{A_{s,nom} L_h} \frac{L_h}{E_s} [\text{mm}]
\]

where
- \(L_h\) is length of the anchor shaft [mm]
- \(N_{Rd,s}\) is design tension resistance of the headed stud [N]
- \(E_s\) is elastic modulus of the steel, \(E_s = 210\,000\,\text{N/mm}^2\) [N/mm²]
- \(A_{s,nom}\) is nominal cross section area of all shafts

\[
A_{s,nom} = \frac{\pi d_{s,nom}^2}{4} [\text{mm}^2]
\]

where
- \(d_{s,nom}\) is nominal diameter of the shaft [mm]

The design load at steel yielding failure is calculated as given below

\[
N_{Rd,s} = A_{s,nom} \frac{f_{uk}}{\gamma_{Ms}} = n \pi \left( \frac{d_{s,nom}^2}{4} \right) \frac{f_{uk}}{\gamma_{Ms}} [\text{N}]
\]

where
- \(f_{uk}\) is characteristic ultimate strength of the shaft material of the headed stud [N/mm²]
- \(n\) is number of headed studs in tension [-]
- \(\gamma_{Ms}\) is partial safety factor for steel [-]

Exceeding the design steel yielding strength \(f_{yd}\), the elongation will strongly increase without a significant increase in load up to a design strain limit \(\varepsilon_{su}\). For the design, this increase of strength is neglected on the safe side and the stiffness is assumed to be zero, \(k_s = 0\,\text{N/mm}\). Depending on the product the failure shall be assumed at the yielding point. In general, fasteners as headed studs are deemed to have an elongation capacity of at least \(\varepsilon_{su} = 0.8\,\%\). This limit shall be used to determine the response of the fasteners unless it is proven by means of tests that they have a higher elongation capacity.

Therefore the stiffness \(k_s\) is described as given below depending on the displacement or load

\[
k_{s1} = \frac{A_{s,nom} E_s}{L_h} \text{ for } N_{act} < N_{Rd,sy} [\text{N/mm}]
\]

\[
k_{s2} = 0 \text{ for } \delta \geq \delta_{Rd,sy} \leq \varepsilon_{su} \text{ and } N_{act} = N_{Rd,sy} [\text{N/mm}]
\]

where
- \(\delta_{Rd,sy}\) is displacement at yielding of the shaft, see Eq. (3.1) [mm]
- \(\varepsilon_{su}\) is maximum elongation capacity of the shaft, 0.8 % [-]

### 3.1.2 Headed studs in tension, component CC

The component concrete breakout in tension is described using the design load \(N_{Rd,c}\) for concrete cone failure and the displacement in the softening branch after failure. Up to the design load the component can’t be assumed as absolutely rigid without any displacement. The displacement corresponding to design load is given by
The design load at concrete cone failure is calculated as

\[ N_{Rd,c} = N_{Rk,c}^0 \Psi_{AN} \Psi_{s,N} \frac{\Psi_{ef,N}}{\gamma_{Mc}} [N] \]  

(3.7)

where

- \( N_{Rk,c}^0 \) is characteristic resistance of a single anchor without edge and spacing effects
- \( k_1 \) is basic factor 8.9 for cracked concrete and 12.7 for non-cracked concrete [-]
- \( h_{ef} \) is embedment depth given according to the product specifications [mm]
- \( f_{ck} \) is characteristic concrete strength according to EN206-1:2000 [N/mm²]
- \( \Psi_{A,N} \) is factor accounting for the geometric effects of spacing and edge distance [-]

\[ \Psi_{A,N} = \frac{A_{c,N}^0}{A_{c,N}} [-] \]  

(3.9)

where

- \( \Psi_{s,N} \) is factor accounting for the influence of edges of the concrete member on the distribution of stresses in the concrete

\[ \Psi_{s,N} = 0.7 + 0.3 \frac{c}{c_{cr,N}} \leq 1 [-] \]  

(3.10)

where

- \( \Psi_{ef,N} \) is factor accounting for the negative effect of closely spaced reinforcement in the concrete member on the strength of anchors with an embedment depth \( h_{ef} < 100 \text{ mm} \)
  
  - 0.5 + \( h_{ef} / 200 \) for \( s < 150 \text{ mm} \) (for any diameter) [-]
  - or \( s < 100 \text{ mm} \) (for \( d_s \leq 10 \text{ mm} \))
  - for \( s \geq 150 \text{ mm} \) (for any diameter) [-]

- \( \gamma_{Mc} \) is 1.5 for concrete [-]
- \( A_{c,N}^0 \) is reference area of the concrete cone of an individual anchor with large spacing and edge distance projected on the concrete surface [mm²]. The concrete cone is idealized as a pyramid with a height equal to \( h_{ef} \) and a base length equal to \( s_{cr,N} \) with

\[ s_{cr,N} = 3.0 h_{ef} [\text{mm}] \]  

(3.11)

\[ c_{cr,N} = 0.5 s_{cr,N} = 1.5 h_{ef} [\text{mm}] \]  

(3.12)

where

- \( A_{c,N} \) is actual projected area of concrete cone of the anchorage at the concrete surface, limited by overlapping concrete cones of adjacent anchors, \( s < s_{cr,N} \), as well as by edges of the concrete member, \( c < c_{cr,N} \). It may be deduced from the idealized failure cones of single anchors [mm²]

To avoid a local blow out failure the edge distance shall be larger than 0.5 \( h_{ef} \). Due to sudden and brittle failure, the initial stiffness for concrete cone is considered as infinity, i.e. till the actual load, \( N_{act} \) is less than or equal to the design tension resistance for concrete cone, the
displacement $\delta_c$ is zero. Once the design load is exceeded, the displacement increases with decreasing load, descending branch. Thus, the load-displacement behaviour in case of concrete cone breakout is idealized as shown in Fig. 3.2.

Fig. 3.2 Idealized load-displacement relationship for concrete cone breakout in tension

The stiffness of the descending branch $k_{c,de}$ for the design is described with the following function

$$k_{c,de} = \alpha_c \sqrt{f_{ck}} h_{ef} \cdot \psi_{AN} \cdot \psi_{s,N} \cdot \psi_{re,N} \text{ [N/mm]} \quad (3.13)$$

where
- $\alpha_c$ is factor of component concrete breakout in tension, currently $\alpha_c = -537$
- $h_{ef}$ is embedment depth of the anchorage [mm]
- $f_{ck}$ is characteristic concrete compressive strength [N/mm²]
- $A_{c,N}$ is projected surface of the concrete cone [mm²]
- $A_{c,N}^0$ projected surface of the concrete cone of a single anchorage [mm²]

The displacement $\delta_c$ as a function of the acting load $N_{act}$ is described using the design resistance and the stiffness of the descending branch.

For ascending part

$$N_{act} \leq N_{Rd,c} \text{ and } \delta_c = 0 \quad (3.14)$$

For descending branch

$$\delta_c > 0 \text{ mm and } \delta_c = \frac{N_{act} - N_{Rd,c}}{k_{c,de}} \quad (3.15)$$

3.1.3 Stirrups in tension, component RS

The component stirrups in tension was developed based on empirical studies. Therefore the tests results were evaluated to determine the displacement of the stirrups depending on the load $N_{act}$ acting on the stirrup. The displacement is determined like given in the following equation

$$\delta_{Rd,s,re} = \frac{2 N_{Rd,s,re}^2}{\alpha_s f_{ck} d_{s,re} n_{re}^2} \text{ [mm]} \quad (3.16)$$

where
\( \alpha_s \) is factor of the component stirrups, currently \( \alpha_s = 12 \times 100 [-] \)

\( N_{Rd,s,re} \) is design tension resistance of the stirrups for tension failure [N]

\( d_{s,re} \) is nominal diameter of thereinforcement leg [mm]

\( f_{ck} \) is characteristic concrete compressive strength [N/mm²]

\( n_{re} \) is total number of legs of stirrups [-]

The design load for yielding of the stirrups is determined as given

\[
N_{Rd,s,re} = A_{s,ref,yd,le} = n_{re} \pi \left( \frac{d_{s,le}^2}{4} \right) f_{yd,le} [N]
\] (3.17)

where

\( A_{s,ref} \) is nominal cross section area of all legs of the stirrups [mm²]

\( d_{s,le} \) is nominal diameter of the stirrups [mm]

\( f_{yd} \) is design yield strength of the shaft material of the headed stud [N/mm²]

\( n_{re} \) is total number of legs of stirrups [-]

Exceeding the design steel yielding strength \( f_{yd,le} \) the elongation will increase with no significant increase of the load up to a strain limit \( \varepsilon_{su,le} \) of the stirrups. For the design this increase of strength is neglected on the safe side. In general reinforcement steel stirrups shall have an elongation capacity of at least \( \varepsilon_{su,le} = 2.5 \% \). So the design strain limit \( \varepsilon_{su,le} \) is assumed to be 2.5 \%. The displacement as a function of the acting load is determined as

\[
k_{s,rel} = \frac{\sqrt{n_{re}^2 \alpha_s f_{ck} d_{s,le}^4}}{\sqrt{2} \delta} \quad \text{for } \delta < \delta_{Rd,s,re} [N/mm]
\] (3.18)

\[
k_{s,rez} = 0 \quad \text{for } \delta \geq \delta_{Rd,s,re} \leq \varepsilon_{su,le} [N/mm]
\] (3.19)

### 3.1.4 Stirrups in tension - bond failure, component RB

The displacement of the concrete component stirrups in tension is determined under the assumption that bond failure of the stirrups will occur. This displacement is calculated with equation (3.19) as

\[
\delta_{Rd,b,le} = \frac{2 N_{Rd,b,le}^2}{\alpha_s f_{ck} d_{s,le}^4 n_{re}^2} [mm]
\] (3.20)

where

\( \alpha_s \) is factor of the component stirrups, currently \( \alpha_s = 12 \times 100 [-] \)

\( N_{Rd,b,le} \) is design tension resistance of the stirrups for bond failure [N]

\( d_{s,le} \) is nominal diameter of the stirrups [mm]

\( f_{ck} \) is characteristic concrete compressive strength [N/mm²]

The design anchorage capacity of the stirrups according CEN/TS-model [5] is determined the design tension resistance of the stirrups for bond failure

\[
N_{Rd,b,le} = \sum n_{s,le} \left( \frac{l_1 \pi d_{s,le} f_{bd}}{\alpha} \right) [N]
\] (3.21)

where

\( n_{s,le} \) is number of legs [-]
$l_1$ is anchorage length [mm]

$d_{s,re}$ is nominal diameter of the stirrups [mm]

$f_{bd}$ is design bond strength according to EN1992-1-1:2004 [N/mm$^2$]

$\alpha$ is factor according to EN1992-1-1:2004 for hook effect and large concrete cover, currently $0.7 \cdot 0.7 = 0.49$ [-]

$$k_{b,rel} = \frac{\sqrt{n_{re}^2 \alpha_s f_{ck} d^2_{s,re}}}{\sqrt{2} \delta} \quad \text{for} \quad \delta < \delta_{Rd,b,re} \quad \text{[N/mm]}$$

$$k_{b,rel} = 0 \quad \text{for} \quad \delta \geq \delta_{Rd,b,re} \leq \varepsilon_{su,re} \quad \text{[N/mm]}$$

### 3.1.5 Headed studs in tension, component P

The pull out failure of the headed studs will take place if the local stresses at the head are larger than the local design resistance. Up to this level the displacement of the headed stud will increase due to the increasing pressure under the head.

$$\delta_{Rd,p,1} = k_p \cdot \left( \frac{N_{Rd,c}}{A_h \cdot f_{ck} \cdot n} \right)^2 \quad \text{[mm]}$$

$$\delta_{Rd,p,2} = 2 \ k_p \cdot \left( \min \left( \frac{N_{Rd,p;N_{Rd,re}}}{A_h \cdot f_{ck} \cdot n} \right) \right)^2 - \delta_{Rd,p,1} \quad \text{[mm]}$$

$$k_p = \alpha_p \cdot \frac{k_a \cdot k_A}{k_2} \quad \text{[-]}$$

where

- $A_h$ is area on the head of the headed stud [mm$^2$]

$$A_h = \frac{\pi}{4} \cdot (d_h^2 - d_s^2)$$

$$k_a = \sqrt{5/a} \geq 1$$

where

- $a_p$ is factor considering the shoulder width [mm]

$$a_p = 0.5 \cdot (d_h - d_s)$$

where

- $k_A$ is factor considering the cross section depending on factor $k_a$ [-]

$$k_A = 0.5 \cdot \sqrt{d^2_s + m \cdot (d^2_h - d^2_s)} - 0.5 \cdot d_h$$

where

- $n$ is number of the headed studs [-]
α_p  is factor of the component head pressing, currently is α_p = 0.25 [-]
k_2  is factor for the headed studs in non-cracked concrete, currently 600 [-]
m  is factor for the headed studs in cracked concrete, currently 300 [-]
d_h  is diameter of the head [mm]
d_s  is diameter of the shaft [mm]
N_{Rd,p}  is design load at failure in cases of pull out

\[ N_{Rd,p} = n_{uk} A_h / \gamma_{Mc} \]  (3.31)

where

p_{uk}  is characteristic ultimate bearing pressure at the headed of stud [N/mm²]
N_{Rd,c}  is design load for concrete cone failure without supplementary reinforcement

\[ N_{Rd,c} = N_{Rk,c}^0 \psi_{A,N} \psi_{s,N} \psi_{re,N} / \gamma_{Mc} \]  [N]  (3.32)

where

N_{Rk,c}  is design load at failure of the supplementary reinforcement minimum value of

\[ N_{Rd,s,\text{re}} = A_{s,\text{re}} f_{yd,\text{re}} = n \pi d_{s,\text{re}}^2 f_{yd,\text{re}} \text{ and } N_{Rd,b,\text{re}} = \sum n_{s,\text{re}} \frac{l_{s,\text{re}} \pi d_{s,\text{re}} f_{bd}}{\alpha} \]  [N]  (3.33)

The stiffness as a function of the displacement is determined as

\[ k_{p,1} = \frac{(A_h f_{ck} n)^2}{\delta_{act} k_p} \]  [N/mm]  (3.34)

\[ k_{p,2} = \frac{(A_h f_{ck} n)^2 (\delta + \delta_{Rd,p1})}{2 \delta_{act} k_p} \]  [N/mm]  (3.35)

\[ k_{p,3} = \min(N_{Rd,p}, N_{Rd,\text{re}}) / \delta + k_{p,pp} [1 - \delta_{Rd,p,2} / \delta] \]  [N/mm]  (3.36)

The stiffness \( k_{p,de} \) depends on the failure modes. If the supplementary reinforcement fails by yielding (\( N_{Rd,s,\text{re}} < N_{Rd,b,\text{re}} \) and \( N_{Rd,s,\text{re}} < N_{Rd,p} \)) the design stiffness \( k_{p,de} \) is assumed as \( 10^4 \) N/mm², negative due to descending branch.

In all other cases (e.g. \( N_{Rd,s,\text{re}} > N_{Rd,b,\text{re}} \) or \( N_{Rd,s,\text{re}} > N_{Rd,p} \)) \( k_{p,de} \) shall be assumed as infinite due to brittle failure. The stiffness in case of pull out failure is calculated using the minimum value of the stiffness’s calculated with equation (3.34) to (3.36).

\[ k_{p,de} = \min(k_{p,1}; k_{p,2}; k_{p,3}) \]  [N/mm]  (3.37)

3.1.6 Headed studs in shear, component V

The load-displacement behaviour mainly depends on the pressure to the concrete near the surface of the concrete member. Due to concrete crushing at the surface of the concrete member, the displacement under shear loading varies very large with a coefficient of variation about 40 % to 50 %. However a semi-empirical calculation shows that the displacement at failure mainly depends on the acting loading, the diameter of the anchors and the embedment
depth. Therefore the displacement under shear loading for a given load level is calculated, see (Hofmann 2005), using the following equation only as an estimation

$$ \delta_{Rd,v} = k_v \sqrt{\frac{V_{Rd}}{d}} h_{ef}^{0.5} \text{[mm]} $$  \hspace{1cm} (3.38)

where

- $k_v$: empirical value depending on the type of anchor [-], for headed studs $k_v = 2$ to $4$
- $V_{Rd}$: design failure load as the minimum of the design failure loads calculated for the different failure modes ($V_{Rd,s}$, $V_{Rd,cp}$, $V_{Rd,c}$, $V_{Rd,p}$) given according to the technical product specification CEN/TS 1992-4-1 or (FIB Bulletin 58, 2011)

The displacement at ultimate load up three times larger than the displacement at the design load level due to the assumption, that the concrete near the surface is not fully crushed at design load level.

### 3.2 Combination of components

To come up with the total stiffness of the connection with headed studs anchored in concrete with or without supplementary reinforcement, the stiffness’s must be combined. The combination depends on whether the components are acting in parallel, equal displacements, or in serial, equal load. Three combinations are given, see (Hofmann, 2005):

- **Combination C1**
  Concrete cone failure with or without supplementary reinforcement, $k_{s,re} = 0$ and $k_{h,re} = 0$

- **Combination C2**
  Displacement due to steel elongation and head pressure, pull out

- **Combination C3**
  Total connection of headed studs anchored in concrete with supplementary reinforcement

![Fig. 3.3 Combinations of different single components for an anchorage with supplementary reinforcement](image)

**3.2.1 Combination of concrete cone and stirrups, C1 = CC + RS/RB**

If both components are summarized, the load is calculated using the sum of the loads at the same displacement $\delta$ due to the combination of the components using a parallel connection.
from the rheological view. Two ranges must be considered. The first range is up to the load level at concrete failure \( N_{Rd,c} \) the second up to a load level of failure of the stirrups \( N_{Rd,s,re} \) or \( N_{Rd,b,re} \).

\[
k_{C,1.1} = k_{c1} + k_{s,re} = \infty \text{ for } N_{act} \leq N_{Rd,c} \quad [N/mm] (3.39)
\]

This leads to the following equation

\[
k_{C,1.1} = \frac{\sqrt{n_{f,0}^2 \alpha_s f_{ck} d_{s,0}^4}}{\sqrt{2} \delta} \quad \text{for } N_{act} \leq N_{Rd,c} \quad [N/mm] (3.40)
\]

In the second range the load is transferred to the stirrups and the stiffness decreases. The stiffness is calculated if \( N_{act} \) is larger than \( N_{Rd,c} \) with the following equation

\[
k_{C,1.2} = k_{c2} + k_{s,re} \quad \text{for } N_{act} > N_{Rd,c} \quad [N/mm] (3.41)
\]

This leads to a relative complex equation

\[
k_{C,1.2} = \frac{N_{Rd,c}}{\delta} + k_{c,de} = k_{c,de} + \frac{\sqrt{n_{f,0}^2 \alpha_s f_{ck} d_{s,0}^4}}{\sqrt{2} \delta} \quad \text{for } N_{act} < N_{Rd,s,re} < N_{Rd,b,re} \quad [N/mm] (3.42)
\]

If the load exceeds the ultimate load given by \( N_{Rd,s,re} \) or \( N_{Rd,b,re} \) the stiffness of the stirrups are negligible. Therefore the following equation applies:

\[
k_{C,1.3} = k_{c} + k_{s,re} = 0 \quad \text{for } N_{act} = N_{Rd,s,re} \geq N_{Rd,b,re} \quad [N/mm] (3.43)
\]

### 3.2.2 Combination of steel and pullout, \( C2 = S + P \)

If both components are summarized the load is calculated using the sum of the displacements at the same load \( N_{act} \) due to the combination of the components using a serial connection from the rheological view. This is done by summing up the stiffness’s as given below

\[
k_{C,2} = \left( \frac{1}{k_s} + \frac{1}{k_p} \right)^{-1} \quad [N/mm] (3.44)
\]

This loads to the following equation

\[
k_{C,2} = \left( \frac{L_h}{A_{s,nom} E_s} + \frac{1}{k_p} \right)^{-1} = \left( \frac{L_h}{A_{s,nom} E_s} + \frac{1}{\min(k_{p1}; k_{p2}; k_{p3})} \right)^{-1} \quad [N/mm] (3.45)
\]

where

\( k_p \) is the minimum stiffness in case of pullout failure as the minimum of \( k_{p1}, k_{p2} \) and \( k_{p3} \).
### 3.2.3 Combination of all components, $C3 = CC + RS/RB + P + S$

To model the whole load-displacement curve of a headed stud embedded in concrete with a supplementary reinforcement the following components are combined:

- concrete and stirrups in tension, components $CC$ and $RB/RS$, as combination $C1$,
- shaft of headed stud in tension, component $S$,
- pull-out failure of the headed stud component $P$ as Combination 2.

The combinations $C1$ and $C2$ is added by building the sum of displacements. This is due to the serial function of both components. That means that these components are loaded with the same load but the response concerning the displacement is different. The combination of the components using a serial connection leads to the following stiffness of the whole anchorage in tension:

$$\frac{1}{k_{C3}} = \frac{1}{k_{C1}} + \frac{1}{k_{C2}} \text{[N/mm]} \quad (3.46)$$

where

- $k_{C1}$ is the stiffness due to the displacement of the anchorage in case of concrete cone failure with supplementary reinforcement, see combination $C1$ [N/mm], if no supplementary reinforcement is provided $k_{C1}$ is equal to $k_c$
- $k_{C2}$ is the stiffness due to the displacement of the head, due to the pressure under the head on the concrete, and steel elongation, see combination $C2$ [N/mm]

### 3.2.4 Design failure load

In principle two failure modes are possible to determine the design failure load $N_{Rd,C3}$ for the combined model. These modes are failure of

- the concrete strut $N_{Rd,cs}$,
- the supplementary reinforcement $N_{Rd,re}$.

The design failure load in cases of concrete strut failure is calculated using the design load in case of concrete cone failure and an increasing factor to consider the support of the supplementary reinforcement, angle of the concrete strut,

$$N_{Rd,cs} = \psi_{\text{supp}} N_{Rd,c} \text{[N]} \quad (3.47)$$

where

- $N_{Rd,c}$ is design failure load in case of concrete cone failure, see Eq. 3.7 [N]
- $\psi_{\text{support}}$ is support factor considering the confinement of the stirrups

$$2.5 - \frac{x}{h_{ef}} \geq 1 [-] \quad (3.48)$$

where

- $x$ is distance between the anchor and the crack on the concrete surface assuming a crack propagation from the stirrup of the supplementary reinforcement to the concrete surface with an angle of 35° [mm]
**3.2.5 Combination of tension and shear components**

The displacements in tension and shear is calculated by the sum of the displacement vectors.

**3.3 Simplified stiffness's based on technical specifications**

**3.3.1 Headed stud in tension without supplementary reinforcement**

For simplification the displacements and the stiffness of headed studs or anchorages is estimated using technical product specifications. The elongation $\delta_{Rd}$ is estimated up to the design load $N_{Rd}$ using the displacements given in the technical product specification. The displacement is estimated by the following equation

$$\delta_{Rd,N} = \frac{\delta_{N,\text{ETA}}}{N_{\text{ETA}}} N_{Rd}$$

where

- $\delta_{N,\text{ETA}}$ is displacement given in the product specifications for a corresponding load
- $N_{\text{ETA}}$ is tension load for which the displacements are derived in the product specifications
3.3.2 Headed stud in shear

For the design the displacement $\delta_v$ is estimated up to the design load $V_{Rd}$ using the displacements given in the technical product specification. The displacement is estimated using the displacements far from the edge $\delta_{v,ETA}$ for short term and long term loading. The displacement is estimated by the following equation

$$
\delta_{Rd,v} = \frac{\delta_{v,ETA}}{V_{ETA}} V_{Rd}
$$

(3.52)

where
- $\delta_{v,ETA}$ is displacement given in the product specifications for a corresponding load
- $V_{ETA}$ is shear load for which the displacements are derived in the product specifications
- $V_{Rd,c}$ is design shear resistance

The stiffness of the anchorage is calculated with the following equation

$$
k_{Rd,v} = \frac{\delta_{v,ETA}}{V_{ETA}}
$$

(3.53)

where
- $\delta_{v,ETA}$ is displacement given in the product specifications for a corresponding load
- $V_{ETA}$ is shear load for which the displacements are derived in the product specifications

3.3.3 Concrete breakout in tension

The characteristic load corresponding to the concrete cone breakout in tension for a single headed stud without edge influence is given by equation

$$
N_{Rk,c}^0 = k_1 h_{ef}^{1.5} \sqrt{f_{ck}}
$$

(3.54)

where
- $k_1$ is basic factor for concrete cone breakout, which is equal to 8.9 for cracked concrete and 12.7 for non-cracked concrete, for headed studs, [-]
- $h_{ef}$ is effective embedment depth given according to the product specifications [mm] [-]
- $f_{ck}$ is characteristic concrete strength according to EN206-1:2000 [N/mm²]

The design load for concrete cone breakout for a single anchor, $N_{Rd,c}^0$ is obtained by applying partial safety factor of concrete $\gamma_{Mc}$ to the characteristic load as
For concrete, the recommended value of is $\gamma_{Mc} = 1.5$.

For a group of anchors, the design resistance corresponding to concrete cone breakout is given by equation (3.56), which is essentially same as equation (3.7)

\[
N_{Rd,c} = N_{Rk,c}^0 \psi_{AN} \psi_{s,N} \psi_{re,N} / \gamma_{Mc}
\]

where

- $N_{Rk,c}^0$ is characteristic resistance of a single anchor without edge and spacing effects
- $\psi_{AN}$ is factor accounting for the geometric effects of spacing and edge distance given as $\psi_{AN} = \frac{A_{c,N}}{A_{c,N}}$
- $A_{c,N}^0$ is reference area of the concrete cone for a single anchor with large spacing and edge distance projected on the concrete surface [mm$^2$]. The concrete cone is idealized as a pyramid with a height equal to $h_{ef}$ and a base length equal to $s_{cr,N}$ with $s_{cr,N} = 3.0 \ h_{ef}$, thus $A_{c,N}^0 = 9 \ h_{ef}^2$.
- $A_{c,N}^0$ is reference area of the concrete cone of an individual anchor with large spacing and edge distance projected on the concrete surface [mm$^2$]. The concrete cone is idealized as a pyramid with a height equal to $h_{ef}$ and a base length equal to $s_{cr,N}$ with $s_{cr,N} = 3.0 \ h_{ef}$ [mm].
- $A_{c,N}$ is actual projected area of concrete cone of the anchorage at the concrete surface, limited by overlapping concrete cones of adjacent anchors $s < s_{cr,N}$, as well as by edges of the concrete member $c < c_{cr,N}$.
- $c$ is minimum edge distance $c = 1.5 \ h_{ef}$ [mm]
- $c_{cr,N}$ is critical edge distance $c_{cr,N} = 1.5 \ h_{ef}$ [mm]
- $\psi_{re,N}$ is factor accounting for the negative effect of closely spaced reinforcement in the concrete member on the strength of anchors with an embedment depth $h_{ef} < 100$ mm.
- $\gamma_{Mc}$ is 1.5 for concrete [-]

### 3.3.4 Pull out failure of the headed studs

The design load corresponding to the pull out failure of the headed stud, $N_{Rd,p}$ is given by

\[
N_{Rd,p} = \frac{p_{uk} A_h}{\gamma_{Mc}}
\]

where

- $p_{uk}$ is characteristic ultimate bearing pressure at the head of stud [N/mm$^2$]
- $A_h$ is area on the head of the headed stud [mm$^2$]

\[
A_h = \frac{\pi}{4} \cdot (d_h^2 - d_s^2)
\]

- $d_h$ is diameter of the head [mm]
- $d_s$ is diameter of the shaft [mm]
- $\gamma_{Mc}$ is 1.5 for concrete [-]
3.3.5 Interaction of components for concrete and stirrups

In case of headed stud anchored in concrete with supplementary reinforcement, stirrups, the stirrups do not carry any load till the concrete breakout initiates, i.e. till $N_{act}$ is less than or equal to $N_{Rd,c}$. Once, the concrete breakout occurs, the load shared by concrete decreases with increasing displacement as depicted in Fig. 3.4. The load shared by concrete $N_{act,c}$ corresponding to a given displacement $\delta$ is therefore given by equation

$$N_{act,c} = N_{Rd,c} + k_{c,de} \delta$$  
(3.57)

where $k_{c,de}$ is the slope of descending branch of Fig. 3.4, negative value, given by Eq. (3.7). Simultaneously, in case of concrete with supplementary reinforcement, the stirrups start to carry the load. The load carried by the stirrups corresponding to a given displacement $\delta$ is given by equation

$$N_{act,te} = n_{re} d_{se}^2 \sqrt{\frac{\alpha_s f_{ck} \delta}{2}}$$  
(3.58a)

where
- $\alpha_s$ is factor of the component stirrups, currently is $\alpha_s = 12 100 [-]$
- $d_{se, nom}$ is nominal diameter of the stirrups [mm]
- $f_{ck}$ is characteristic concrete compressive strength [N/mm²]
- $n_{re}$ is total number of legs of stirrups [-]

The total load $N_{act}$ carried by concrete cone and stirrups corresponding to any given displacement $\delta$ is therefore given as the sum of the two components:

$$N_{act} = N_{act,c} + N_{act,te} = N_{Rd,c} + k_{c,de} \delta + \min(n_{re} d_{se}^2 \sqrt{\frac{\alpha_s f_{ck} \delta}{2}}; N_{Rd,s,te}; N_{Rd,b,te})$$  
(3.59)

The displacement corresponding to peak load of the system is obtained by differentiating the right hand side of Eq. (3.60) and equating it to zero. If the bond failure or steel failure of stirrups is not reached at an earlier displacement then the design peak load carried by the system $N_{u,c+s}$ is given by

$$N_{u,c+s} = N_{Rd,c} + \frac{3}{8} \frac{n_{re} d_{se}^4 \alpha_s f_{ck}}{k_{c,de}}$$  
(3.60)

where
- $N_{Rd,c}$ is design load at concrete cone failure given by equation (3.7)
- $\alpha_s$ is factor of the component stirrups, currently is $\alpha_s = 12 100 [-]$
- $d_{se, re}$ is Nominal diameter of the stirrups [mm]
- $f_{ck}$ is characteristic concrete compressive strength [N/mm²]
- $n_{re}$ is total number of legs of stirrups [-]
- $k_{c,de}$ is stiffness of descending branch for concrete cone failure, given by eq. (3.13)

In a relatively rare case of all studs loaded in tension, both the legs of the hanger reinforcement are not uniformly loaded and the distribution of forces is difficult to ascertain. Due to this reason and also to avoid the problems with serviceability requirements, it is recommended that in such a case, the contribution of hanger reinforcement is ignored.
3.3.6 Determination of the failure load

The failure load $N_u$ is given by the minimum of the failure load corresponding to each considered failure mode.

3.3.7 Friction

For base plates the friction is defined in EN1993-1-8 cl 6.2.2. For the resistance the resistance values of friction and bolts may be added as long as the bolt holes are not oversized. For the friction between a base plate and the grout underneath the plate the following calculation may be used.

$$F_{f,Rd} = C_{f,d} N_{c,Ed}$$ (3.61)

where

- $C_{f,d}$ is coefficient for friction, for sand-cement mortar $C_{f,d} = 0.2$
- $N_{c,Ed}$ is axial compressive force of the column.

In this design manual the friction is not only applied to compression forces caused by axial forces but also for compression forces generated by bending moments. This principle is applied in EN1993-1-8:2006 for beam to the column end joints with end plates in cl 3.9.2(3).

3.4 Base plate in bending and concrete block in compression

3.4.1 Concrete 3D strength

The components concrete in compression and base plate in bending represent the behaviour of the compressed part of a steel to concrete connection. The resistance of these components depends primarily on the bearing resistance of the concrete block under the flexible base plate, see (Melchers, 1992). The resistance of concrete is influenced by flexibility of base plate. In case of loading by an axial force, the stresses in concrete are not uniformly distributed, they are concentrated around the footprint of the column under the plate according to its thickness, see (Dewolf, Sarisley, 1980). For the design the flexible base plate is replaced by reducing the effective fully rigid plate. The grout layer between the base plate and concrete block influences the resistance and stiffness of the component. That is why this layer is also included into this component, see (Penserini, Colson, 1989). Other important factors which influence the resistance are the concrete strength, the compression area, the location of the plate on the concrete foundation, the size of the concrete block and its reinforcement.

The stiffness behaviour of column base connection subjected to bending moment is influenced mostly by elongation of anchor bolts. The Component concrete in compression is mostly stiffer in comparison to the component anchor bolts in tension. The deformation of concrete block and base plate in compression is important in case of dominant axial compressive force.

The strength of the component $F_{Rd,u}$, expecting the constant distribution of the bearing stresses under the effective area, is given by

$$F_{Rd,u} = A_{c0} f_{jd}$$ (3.62)

The design value of the bearing strength $f_{jd}$ in the joint loaded by concentrated compression, is determined as follows. The concrete resistance is calculated according to cl. 6.7(2) in EN1992-1-1:2004 see Fig. 3.6 is
\[ F_{Rd,u} = A_{c0} f_{cd} \sqrt{\frac{A_{c1}}{A_{c0}}} \leq 3.0 A_{c0} f_{cd} \]  

(3.63)

where

\[ A_{c0} = b_1 d_1 \quad \text{and} \quad A_{c1} = b_2 d_2 \]  

(3.64)

where \( A_{c0} \) is the loaded area and \( A_{c1} \) the maximum spread area. The influence of height of the concrete block to its 3D behaviour is introduced by

\[ h \geq b_2 - b_1 \quad \text{and} \quad h \geq d_2 - d_1 \]

\[ 3 b_1 \geq b_2 \quad \text{and} \quad 3 d_1 \geq d_2 \]  

(3.65)

From this geometrical limitation the following formulation is derived

\[ f_{ld} = \frac{b_1}{b_{eфф}} \frac{f_{Rd,u}}{f_{cd}} = \frac{\beta_1 A_{c0} f_{cd}}{A_{c0}} = \frac{A_{c1}}{A_{c0}} = \beta_1 f_{cd} k_1 \leq 3 A_{c0} f_{cd} = 3.0 f_{cd} \]  

(3.66)

The factor \( \beta_1 \) represents the fact that the resistance under the plate might be lower due to the quality of the grout layer after filling. The value 2/3 is used in the case of the characteristic resistance of the grout layer is at least 0.2 times the characteristic resistance of concrete and thickness of this layer is smaller than 0.2 times the smallest measurement of the base plate. In different cases, it is necessary to check the grout separately. The bearing distribution under 45° is expected in these cases, see (Steenhuis et al, 2008) and Fig. 3.5 Concrete compressive strength for calculation of 3D concentration

Fig. 3. The design area \( A_{c0} \) is conservatively considered as the full area of the plate \( A_p \).
3.4.2 Base plate flexibility

In case of the elastic deformation of the base plate is expected homogenous stress distribution in concrete block is expected under the flexible base plate based on the best engineering practice. The formula for the effective width \( c \) is derived from the equality of elastic bending moment resistance of the base plate and the bending moment acting on the base plate, see (Astaneh et al., 1992). Acting forces are shown in Fig. 3.7.

Elastic bending moment of the base plate per unit length is

\[
M' = \frac{1}{6} t^2 \frac{f_y}{\gamma_{M0}}
\]  

(3.69)

and the bending moment per unit length on the base plate of span \( c \) and loaded by distributed load is

\[
M' = \frac{1}{2} f_j c^2
\]  

(3.70)

where \( f_j \) is concrete bearing strength and from Eq. (3.69) and (3.70) is

\[
c = t \sqrt{\frac{f_y}{3 \cdot f_{jd} \cdot \gamma_{M0}}}
\]  

(3.71)

The flexible base plate, of the area \( A_p \), is replaced by an equivalent rigid plate with area \( A_{eq} \), see Fig. 3.8. Then the resistance of the component, expecting the constant distribution of the bearing stresses under the effective area is given by
\[ F_{\text{Rd,u}} = A_{\text{eq}} \cdot f_{\text{id}} \]  (3.72)

The resistance \( F_{\text{Rd}} \) should be higher than the loading \( F_{\text{Ed}} \)

\[ F_{\text{Ed}} \leq F_{\text{Rd,u}} \]  (3.73)

Fig. 3.8 Effective area under the base plate

### 3.4.3 Component stiffness

The proposed design model for stiffness of the components base plate in bending and concrete in compression is given also in (Steenhuis et al, 2008). The stiffness of the component is influenced by factors: the flexibility of the plate, the Young’s modulus of concrete, and the size of the concrete block. By loading with force, a flexible rectangular plate could be pressed down into concrete block. This flexible deformation is determined by theory of elastic semi-space

\[ \delta_r = \frac{F \alpha a_r}{E_c A_p} \]  (3.74)

where

- \( F \) is acting load
- \( \alpha \) is shape factor of the plate
- \( a_r \) is width of equivalent rigid plate
- \( E_c \) is elastic modulus of concrete
- \( A_p \) is area of the plate

The factor \( \alpha \) depends on the material characteristics. The Tab. 3.1 gives values of this factor dependent on the Poisson's ratio, for concrete is \( \nu \approx 0.15 \). The table shows also the approximate value of factor \( \alpha \), that is \( 0.58 \cdot \sqrt{L/a_r} \).

**Tab. 3.1 Factor \( \alpha \) and its approximation for concrete**

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<thead>
<tr>
<th>( l/a_r )</th>
<th>( \alpha )</th>
<th>Approximation as ( \alpha = 0.58 \cdot \sqrt{L/a_r} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.90</td>
<td>0.85</td>
</tr>
<tr>
<td>1.5</td>
<td>1.10</td>
<td>1.04</td>
</tr>
<tr>
<td>2</td>
<td>1.25</td>
<td>1.20</td>
</tr>
<tr>
<td>3</td>
<td>1.47</td>
<td>1.47</td>
</tr>
<tr>
<td>5</td>
<td>1.76</td>
<td>1.90</td>
</tr>
<tr>
<td>10</td>
<td>2.17</td>
<td>2.69</td>
</tr>
</tbody>
</table>

For steel plate laid on concrete block it is
\[ \delta_r = \frac{0.85 \, F}{E_c \sqrt{I \cdot a_r}} \] (3.75)

where

- \( \sigma_r \) is deformation under the rigid plate
- \( l \) is length of the plate

The model for the elastic stiffness behaviour of component is based on a similar interaction between concrete block and steel plate. The flexible plate is expressed as an equivalent rigid plate based on the same deformation, modelled in Fig. 3.9.

![Fig. 3.9 A flange of flexible T-stub](image)

Independent springs support the flange of a unit width. Then, the deformation of the plate is a sine function.

\[ \delta(x) = \delta \sin \left( \frac{\pi x}{c_{fl}} \right) \] (3.76)

The uniform stress on the plate is rewritten by the fourth differentiate and multiplied by \( E \, l_p' \)

\[ \delta(x) = E \, l_p' \left( \frac{\pi}{c_{fl}} \right)^4 \delta \sin \left( \frac{\pi x}{c_{fl}} \right) = E \frac{t^3}{12} \left( \frac{\pi}{c_{fl}} \right)^4 \delta \sin \left( \frac{\pi x}{c_{fl}} \right) \] (3.77)

where

- \( E \) is elastic modulus of steel
- \( l_p' \) is moment of inertia per unit length of the steel plate \( (l_p' = t^3 / 12) \)
- \( t \) is thickness of the plate

\[ \delta(x) = \sigma(x) \frac{h_{ef}}{E_c} \] (3.78)

where

- \( h_{ef} \) is equivalent concrete height of the portion under the steel plate

Assume that

\[ h_{ef} = \xi c_{fl} \] (3.79)

Factor \( \xi \) expresses the rotation between \( h_{ef} \) and \( c_{fl} \). Hence

\[ \delta(x) = \sigma(x) \xi \frac{c_{fl}}{E_c} \] (3.80)

After substitution and using other expressing it is
\[ c_{fl} = t \sqrt[3]{\frac{(\pi/2)^4}{12} \frac{E}{E_c}} \]  

(3.81)

The flexible length \( c_{fl} \) may be replaced by an equivalent rigid length

\[ c_r = \frac{c_{fl}}{2 / \pi} \]  

(3.82)

The factor \( \xi \) shows the ratio between \( h_{eq} \) and \( c_{fl} \). The value \( a_r \) represents height \( h_{eq} \). Factor \( \alpha \) is approximated to \( 1.4 \cdot a_r = t_w + 2c_r \) and \( t_w = 0.5 \cdot c_r \). Then it is written

\[ h_{eq} = 1.4 \cdot (0.5 + 2) \cdot c_r = 1.4 \cdot 2.5 \cdot c_{fl} \cdot \frac{2}{\pi} = 2.2 \cdot c_{fl} \]  

(3.83)

Hence \( \xi = 2.2 \).

For practical joints is estimated by \( E_c \approx 30\,000 \text{ N/mm}^2 \) and \( E \approx 210\,000 \text{ N/mm}^2 \), what leads to

\[ c_{fl} = t \sqrt[3]{\frac{(\pi/2)^4}{12} \frac{E}{E_c}} = t \sqrt[3]{\frac{(\pi/2)^4}{12} \frac{210000}{30000}} = 1.98 \cdot t \]  

(3.84)

or

\[ c_r = \frac{c_{fl}}{2 / \pi} = 1.98 \cdot \frac{2}{\pi} \cdot t = 1.25 \cdot t \]  

(3.85)

The equivalent width \( a_r \) is in elastic state replace with

\[ a_{eq,el} = t_w + 2.5 \cdot t = 0.5 \cdot c_r + t \]  

(3.86)

or

\[ a_{eq,el} = 0.5 \cdot 1.25 \cdot t + 2.5 \cdot t = 3.125 \cdot t \]  

(3.87)

From the deformation of the component and other necessary values which are described above, the formula to calculate the stiffness coefficient is derived

\[ k_c = \frac{F}{\delta E} = \frac{E_c \sqrt{a_{eq,el} L}}{1.5 \cdot 0.85 \cdot E} = \frac{E_c \sqrt{a_{eq,el} L}}{1.275 \cdot E} = \frac{E_c \cdot \sqrt{t \cdot L}}{0.72 \cdot E} \]  

(3.88)

where

- \( a_{eq,el} \) is equivalent width of the T-stub
- \( L \) is length of the T-stub

### 3.5 Concrete panel

The resistance and deformation of the reinforced concrete wall in the zone adjacent to the joint is hereby represented by a joint link component, see (Huber and Cermeneg, 1998). Due to the nature of this joint, reinforced concrete, the developed model is based on the strut-and-tie method, commonly implemented in the analysis of reinforced concrete joints. The problem is 3D, increasing its complexity, as the tension load is introduced with a larger width than the
compression, which may be assumed concentrated within an equivalent dimension of the anchor plate, equivalent rigid plate as considered in T-stub in compression. Thus, a numerical model considering only the reinforced concrete wall and an elastic response of the material has been tested to identify the flow of principal stresses. These show that compression stresses flow from the hook of the longitudinal reinforcement bar to the anchor plate. In this way the strut-and-tie model (STM) represented in Fig. 10a is idealized. Subsequently, in order to contemplate the evaluation of the deformation of the joint, a diagonal spring is idealized to model the diagonal compression concrete strut, as illustrated in Fig. 10. The ties correspond to the longitudinal steel reinforcement bars. The properties of this diagonal spring are determined for resistance and stiffness.

The resistance is obtained based on the strut and nodes dimension and admissible stresses within these elements. The node at the anchor plate is within a tri-axial state. Therefore, high stresses are attained as confinement effect. In what concerns the strut, because of the 3D nature, stresses tend to spread between nodes. Giving the dimensions of the wall of infinite width, the strut dimensions should not be critical to the joint. Thus, the node at the hook of the bar is assumed to define the capacity of the diagonal spring. The resistance of the spring is then obtained according to the dimensions of this node and to the admissible stresses in the node and in the strut. For the latter, the numerical model indicates the presence of transverse tension stresses which have to be taken into consideration.

The deformation of the diagonal spring is obtained by assuming a non-linear stress-strain relation for the concrete under compression, as defined in (Henriques, 2012). The maximum stress is given by the limiting admissible stress as referred above. Then, deformation is calculated in function of the length of the diagonal strut and the concrete strain.

Tab. 3.2 provides the stresses for nodes and struts according to EN1992-1-1:2004. Node 1 is characterized by the hook longitudinal reinforcement bar. The represented dimension is assumed as defined in CEB-FIP Model Code 1990. In what concerns the width of the node, based on the numerical observations, it is considered to be limited by the distance between the external longitudinal reinforcement bars within the effective width of the slab. The numerical model demonstrates that the longitudinal reinforcement bars are sufficiently close, as no relevant discontinuity in the stress field is observed. Though, this is an issue under further
investigation and depending on the spacing of the reinforcing bars, this assumption may or may not be correct (Henriques, 2013).

<table>
<thead>
<tr>
<th>Tab. 3.2 Stresses in strut-and-tie elements according to EN1992-1-1:2004</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Element</strong></td>
</tr>
<tr>
<td>Node 1</td>
</tr>
<tr>
<td>Node 2</td>
</tr>
<tr>
<td>Strut</td>
</tr>
</tbody>
</table>

Finally, to simplify the assembling of the joint model, the diagonal spring representing the joint link component is converted into a horizontal spring. The properties of the horizontal spring are directly obtained from the diagonal spring determined as a function of the angle of the diagonal spring.

3.6 Longitudinal steel reinforcement in tension

In the composite joint configuration under consideration, the longitudinal reinforcement in tension is the only component able to transfer tension forces introduced by the bending moment to the supporting member e.g., a reinforced concrete wall. This component determines the behaviour of the joint. According to EN1994-1, the longitudinal steel reinforcement may be stressed to its design yield strength. It is assumed that all the reinforcement within the effective width of the concrete flange is used to transfer forces. The resistance capacity of the component may then be determined as in Eq. (3.89). Regarding the deformation of the component, the code provides stiffness coefficients for two composite joint configurations, single and double-sided joints. The stiffness coefficient for single-sided joints may be estimated as in Eq. (3.90). This stiffness coefficient depends essentially on the elongation length of the longitudinal reinforcement contributing to the deformation of the component. Analogous to the code provisions, the dimension \( h \) involved in Eq. (3.90) is assumed as shown in Fig. 3.12.

\[
F_{s,r} = A_{s,r} f_{yr} \quad (3.89)
\]

\[
k_{s,r} = \frac{A_{s,r}}{3.6 h} \quad (3.90)
\]
The tension component of the joint is calculated according to

\[ F_t = -M_{y,Ed}/h_s \]  

(3.91)

### 3.7 Slip of the composite beam

The slip of composite beam does not directly influence the resistance of the joint. However, the level of interaction between concrete slab and steel beam defines the maximum load the longitudinal reinforcement can achieve. Therefore in such joint configuration, where reinforcement is the only tension component, the level of interaction affects the joint resistance. In the EN1994-1-1:2008, the influence of the slip of composite beam is taken into account. The stiffness coefficient of the longitudinal reinforcement, see Eq. (3.92) should be multiplied with the reduction factor \( k_{\text{slip}} \) determined as follows:

\[
k_{\text{slip}} = \frac{1}{1 + \frac{E_s k_{sr}}{k_{SC}}} \]  

(3.92)

\[
k_{SC} = \frac{N k_{sc}}{\theta - (\theta - 1) \frac{h_s}{d_s}} \]  

(3.93)

\[
\theta = \sqrt{\frac{(1 + \xi) N k_{sc} l d_s}{E_s l_a}} \]  

(3.94)

\[
\xi = \frac{E_s l_a}{d_s^2 E_s A_s} \]  

(3.95)

where

- \( h_s \) is the distance between the longitudinal reinforcing bars and the centre of compression of the joint, that may be assumed as the midpoint of the compression flange of the steel beam
- \( d_s \) is the distance between the longitudinal reinforcing bars and the centroid of the steel beam section, see Fig. 13
I_a is the second moment area of the steel beam section

l is the length of the beam in hogging bending adjacent to the joint, in the case of the tested specimens is equal to the beam's length

N is the number of shear connectors distributed over the length l

k_sc is the stiffness of one shear connector

4 STEEL COMPONENTS

4.1 T-stub in tension

The base plate in bending and anchor bolts in tension is modelled by the help of T-stub model based on the beam to column end plate connection model. Though in its behaviour there are some differences. Thickness of the base plate is bigger to transfer compression into the concrete block. The anchor bolts are longer due to thick pad, thick base plate, significant layer of grout and flexible embedding into concrete block. The influence of a pad and a bolt head may be higher.

Due to longer free lengths of bolts, bigger deformations could arise. The anchor bolts, compared to bolts, are expecting to behave ductile. When it is loaded by tension, the base plate is often separated from the concrete surface. This case is shown in (Wilkinson et al, 2009). By bending moment loading different behaviour should be expected. The areas of bolt head and pad change favourably distribution of forces on T-stub. This influence is not so distinctive during calculation of component stiffness. The all differences from end plate connections are involved
in the component method, see EN1993-1-8:2006. The design model of this component for resistance as well for stiffness is given in (Wald et al, 2008).

![Fig. 4.2 Length of anchor bolt](image)

**4.1.1 Model**

When the column base is loaded by bending moment as it is shown in Fig. 4.3, anchor bolts transfer tensile forces. This case of loading leads to elongation of anchor bolts and bending of the base plate. Deformed bolts can cause failure as well as reaching of the yield strength of the base plate. Sometimes failure in this tensile zone is caused by both, see (Di Sarno et al, 2007).

![Fig. 4.3 Tensile zone and equivalent T-stub in case of loading by bending moment](image)

Column with connected base plate taken, as it is shown in Fig. 4.4, into model of T-stub.

![Fig. 4.4 T-stub separated from the concrete block with no prying force](image)

There are two models of deformation of the T-stub of the base plate according to presence of prying. In the case the base plate separated from the concrete foundation, there is no prying force \( Q \), see Fig. 4.4. In other case, the edge of the plate is in contact with concrete block, the bolts are loaded by additional prying force \( Q \). This force is balanced just by the contact force at the edge of the T-stub, see Fig. 4.5.

When there is contact between the base plate and the concrete block, beam theory is used to describe deformed shape of the T-stub.
Fig. 4.5 Beam model of T-stub and prying force $Q$

Deformed shape of the curve is described by differential equation

$$EI \delta'' = -M$$  \hspace{1cm} (4.1)

After writing the above equation for both parts of the beam model 1 and 2, application of suitable boundary conditions, the equations could be solved. The prying force $Q$ is derived just from these solved equations as

$$Q = \frac{F}{2} \cdot \frac{3(m^2 n A - 2 \cdot L_b)}{2 n^2 A (3 m + n) + 3 L_b}$$  \hspace{1cm} (4.2)

When the base plate is in contact with concrete surface, the prying of bolts appears and on the contrary no prying forces occur in the case of separated base plate from the concrete block due to the deformation of long bolts. This boundary, between prying and no prying has to be determined. Providing that $n = 1.25$ m it may be expressed as

$$L_{b,\text{min}} = \frac{8.82 \cdot m^3 A_b}{l_{\text{eff}}^3} < L_b$$  \hspace{1cm} (4.3)

where
- $A_c$ is the area of the bolt
- $L_b$ is equivalent length of anchor bolt
- $l_{\text{eff}}$ is equivalent length of T-stub determined by the help of Yield line method, presented in following part of work

For embedded bolts length $L_b$ is determined according to Fig. 4.2 as

$$L_b = L_{hf} + L_{be}$$  \hspace{1cm} (4.4)

where
- $L_{be}$ is $8 d$ effective bolt length

When the length of bolt $L_b > L_{b,\text{min}}$ there is no prying. Previous formulae is expressed for boundary thickness $t_{\text{lim}}$, see (Wald et al, 2008), of the base plate as

$$t_{\text{lim}} = 2.066 \cdot m^3 \frac{A_s}{\sqrt[3]{l_{\text{eff}} L_b}}$$  \hspace{1cm} (4.5)
If the base plate are loaded by compression force and by bending moment and not by tensile force it is recommended to neglect these prying forces. In other cases it needs to be checked.

### 4.1.2 Resistance

The design resistance of a T-stub of flange in tension of effective length $\ell_{\text{eff}}$ is determined as minimum resistance of three possible plastic collapse mechanisms. For each collapse mechanism there is a failure mode. Following collapse modes, shown in Fig. 4.6, is used for T-stub in contact with the concrete foundation, see in EN1993-1-8:2006.

**Mode 1**

According to this kind of failure the T-stub with thin base plate and high strength anchor bolts is broken. In the base plate plastic hinge mechanism with four hinges is developed.

$$F_{1,Rd} = \frac{4 \ell_{\text{eff}} m_{\text{pl,Rd}}}{m}$$  \hspace{1cm} (4.6)

**Mode 2**

This mode is a transition between failure Mode 1 and 3. At the same time two plastic hinges are developed in the base plate and the limit strength of the anchor bolts is achieved.

$$F_{2,Rd} = \frac{2 \ell_{\text{eff}} m_{\text{pl,Rd}} + \Sigma B_{t,Rd} \cdot n}{m + n}$$  \hspace{1cm} (4.7)

**Mode 3**

Failure mode 3 occurs by the T-stub with thick base plate and weak anchor bolts. The collapse is caused by bolt fracture.

$$F_{3,Rd} = \Sigma B_{t,Rd}$$  \hspace{1cm} (4.8)

The design strength $F_{\text{Rd}}$ of the T-stub is derived as the smallest of these three possible modes:

$$F_{\text{Rd}} = \min(F_{1,Rd}, F_{2,Rd}, F_{3,Rd})$$  \hspace{1cm} (4.9)
Because of the long anchor bolts and thick base plate different failure mode arises compare to an end plate connection. When the T-stub is uplifted from the concrete foundation, there is no prying, new collapse mode is obtained, see Fig. 4.7. This particular failure mode is named Mode 1-2.

![Figure 4.7 T-stub without contact with the concrete foundation, Mode 1-2](image)

**Mode 1-2**

The failure results either from bearing of the anchor bolts in tension or from the yielding of the plate in bending, where a two hinges mechanism develops in the T-stub flange. This failure does not appear in beam to column connection because of the small deformation of the bolts in tension, see (Wald et al, 2008).

\[
F_{1-2,Rd} = \frac{2 \ l_{\text{eff}} \ m_{\text{pl,Rd}}}{m}
\]  

(4.10)

The relationship between Mode 1-2 and modes of T-stub in contact with concrete is shown in Fig. 4.8.

![Figure 4.8 Failure mode 1-2](image)

The boundary between the mode 1-2 and others is given in the same way like the boundary of prying and no prying – according to the limiting bolt length \( L_{b,\text{min}} \).

During the Mode 1-2 large deformations of the base plate can develop. Finally these deformations could lead to contact between the concrete block and the edge of the T-stub (prying forces can arise even in this case). After loading Modes 1 or 2 should be obtained like the first. But for reaching this level of resistance, which is necessary to obtain these modes, very large deformations are required. And so high deformations are not acceptable for design. In conclusion, in cases where no prying forces develop, the design resistance of the T-stub is taken as
\[
F_{Rd} = \min \left( F_{1-2,Rd}, F_{3,Rd} \right)
\]

(4.11)

where

\[
F_{3,Rd} = \Sigma B_{L,Rd}
\]

(4.12)

The equivalent length of T-stub \( l_{\text{eff}} \), which is very important for the resistance determination, is calculated by the help of the yield line method, which is explained in the following part of the work.

**Yield line method**

Although numerical methods, based on extensive use of computers, are potentially capable of solving the most difficult plate problems, yield-line analysis is such an alternative computational technique (Thambiratnam, Paramasivam, 1986). It provides such an alternative design method for plates. This simple method, which uses concepts and techniques familiar to structural engineers, provides realistic upper bounds of collapse loads even for arbitrary shapes and loading conditions. The advantages of the yield-line method are: simplicity and economy, information is provided on the real load-carrying capacity of the slab, the basic principles used are familiar to structural engineers, the method also gives acceptable estimates for the ultimate load-carrying capacity of structural steel plates, and resulting designs are often more economical. On the other hand, the present limitations of the method are: the method fails in vibration analysis and cannot be used in the case of repeated static or dynamic loads (but is applied effectively for suddenly applied one-time loads), and theoretically, the law of superposition is not valid. The yield-line method offers, especially for the practicing engineer, certain advantages over the elastic stress analysis approaches.

**Assumptions**

The correct failure pattern is known, the critical load is obtained either from virtual work or from equilibrium considerations. Both approaches use the following basic assumptions: at impending collapse, yield lines are developed at the location of the maximum moments, the yield lines are straight lines, along the yield lines, constant ultimate moments \( m_u \) are developed, the elastic deformations within the slab segments are negligible in comparison with the rigid body motions, created by the large deformations along the yield lines, from the many possible collapse mechanisms, only one, pertinent to the lowest failure load, is important. In this case the yield-line pattern is optimum, when yield lines are in the optimum position, only ultimate bending moments, but no twisting moments or transverse shear forces are present along the yield lines. The location and orientation of yield lines determine the collapse mechanism. The Fig. 4.9 introduces an example of yield line.

![Fig. 4.9 Possible yield line patterns](image)
The work method

The work method, see (Johansen, 1949), gives an upper-bound solution to the critical load at which the slab, with a certain ultimate resisting moment, fails. A particular configuration is searched, from a family of possible yield-line patterns which gives the lowest value of the ultimate load. The solution is based on the principle of virtual work.

The effective length of T-stub

The effective length \( l_{\text{eff}} \) of a T-stub is influenced by the failure mode of the T-stub. When there are more than one possible failure modes, it means more than one effective length, the calculation is done with the smallest (shortest) length, see EN1993-1-8:2006. The Fig. 4.10 shows, that two groups of yield line patterns can arise circular yield line and non-circular yield line. The main difference between these two types is related to contact between the T-stub and concrete foundation. By the non-circular patterns prying forces are developed. In this work there are taken into account only the modes without the contact of the edge of the base plate to the concrete foundation, it means without prying forces in bolts.

Fig. 4.10 The yield line patterns

As it was written in previous paragraphs, the effective length could be determined by the yield line method. Hence the yield line of the base plate must be designed. The collapse Mode 1 of the plate, which is shown in Fig. 4.11, is expected.

Fig. 4.11 Expected collapse mode

For this collapse mode there are following formulas:

\[
m_{\text{pl,Rd}} = \frac{1}{4} t^2 f_{yd}
\]

\[
\tan \theta = \frac{\delta}{m} \approx \theta
\]

\[
F_{\text{pl}} = \frac{4l_{\text{eff}} m_{\text{pl,Rd}}}{m}
\]

where

\( m_{\text{pl,Rd}} \) is plastic bending moment resistance of the base plate per unit length

\( F_{\text{pl}} \) is force acting in the bolt position
The assumptions to determine the yield line of the base plate are following the yield line is a straight line, this line is perpendicular to a line which pass through the bolt and tangent to the column, or this line is tangent to the column and parallel to the edge of the base plate. With these assumptions are determined. Following calculation procedure of the effective length of the T-stub in plate corner is given in (Wald et al, 2000) and (Heinisuo et al, 2012).

Fig. 4.12 The yield line parameters

\[ \alpha \] represents the angle between the yield line and the edge and \( c \) the minimal distance between the corner of the plate and the yield line. With the previous geometrical relation, the following relations is obtained

\[ \tan \alpha = \frac{x}{y} \]  \hspace{1cm} (4.16)

where

\( x, y \) are coordinates of the bolt, which could vary

For the design of the parameter \( c \), the work method of the yield line theory is used. The internal work is

\[ W_i = \sum_n \left[ \tilde{b}_j, m_{uj}, 1 \right] = m_{pl} \left( \frac{1}{y} x + \frac{1}{x} y \right) \]  \hspace{1cm} (4.17)

The external work is

\[ W_e = P_u \Delta = F_{pl} \Delta \]  \hspace{1cm} (4.18)

where \( \Delta \) represents the deformation of the plate in the bolt position, see Fig. 4.13.

Fig. 4.13 The deformation of the plate represented by value \( \Delta \)

According to previous figure is \( \Delta \) replaced with
\[ \Delta = \frac{d}{c} = \sqrt{x^2 + y^2} \]  \hspace{1cm} (4.19)

After replacement \( \Delta \) in the formula of the external work and putting it into equality with the internal work as

\[ \frac{\sqrt{x^2 + y^2}}{c} F_{pl} = m_{pl} \left( \frac{x}{y} + \frac{y}{x} \right) \]  \hspace{1cm} (4.20)

and then the effective length of the T-stub is

\[ l_{eff} = \frac{c m}{4} \frac{\sqrt{x^2 + y^2}}{c} \]  \hspace{1cm} (4.21)

The ultimate load is given by

\[ F_{pl} = c m_{pl} \frac{\sqrt{x^2 + y^2}}{x y} \]  \hspace{1cm} (4.22)

\[ \frac{\partial F_{pl}}{\partial c} = m_{pl} \frac{\sqrt{x^2 + y^2}}{x y} = cst \]  \hspace{1cm} (4.23)

With the yield line assumption the characteristics of the different possible failure models could be designed.

**The effective length of T-stub**

Two groups of yield line patterns called circular and non-circular yield lines are distinguished in EN1993-1-8:2006. The major difference between circular and non-circular patterns is related to contact between the T-stub and rigid foundation. The contact may occur only for non-circular patterns and prying force will develop only in this case. This is considered in the failure modes as follows:

**Mode 1**

The prying force does not have influence on the failure and development of plastic hinges in the base plate. Therefore, the formula (4.2) applies to both circular and non-circular yield line patterns.

**Mode 2**

First plastic hinge forms at the web of the T-stub. Plastic mechanism is developed in the base plate and its edges come into contact with the concrete foundation. As a result, prying forces develop in the anchor bolts and bolt fracture is observed. Therefore, Mode 2 occurs only for non-circular yield line patterns, which allow development of prying forces.
Mode 3
This mode does not involve any yielding of the plate and applies therefore to any T-stub. In the design procedure, the appropriate effective length of the T-stub should be used for Mode 1

\[ l_{\text{eff}, 1} = \min(l_{\text{eff}, cp}; l_{\text{eff}, np}) \]  

and for Mode 2

\[ l_{\text{eff}, 2} = \min(l_{\text{eff}, np}) \]  

The design resistance of the T-stub is given by the formula (4.8). Tab. 4.1 and Tab. 4.2 indicate the values of \( l_{\text{eff}} \) for typical base plates in cases with and without contact. See Fig. 4.14 for the used symbols.

Tab. 4.1 The effective length \( l_{\text{eff}} \) of a T-stub with bolts inside the flanges (Wald et al, 2008)

<table>
<thead>
<tr>
<th>Prying case</th>
<th>No prying case</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l_1 = 2 \alpha m \cdot (4 m \cdot 1.25 e) )</td>
<td>( l_1 = 2 \alpha m \cdot (4 m + 1.25 e) )</td>
</tr>
<tr>
<td>( l_2 = 2 \pi m )</td>
<td>( l_2 = 4 \pi m )</td>
</tr>
<tr>
<td>( l_{\text{eff}, 1} = \min(l_1; l_2) )</td>
<td>( l_{\text{eff}, 1} = \min(l_1; l_2) )</td>
</tr>
<tr>
<td>( l_{\text{eff}, 2} = l_1 )</td>
<td>( l_{\text{eff}, 2} = l_1 )</td>
</tr>
</tbody>
</table>

Tab. 4.2 Effective length \( l_{\text{eff}} \) for bolts outside the flanges (Wald et al, 2008)

<table>
<thead>
<tr>
<th>Prying case</th>
<th>No prying case</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l_1 = 4 \alpha m_x + 1.25 e_x )</td>
<td>( l_1 = 4 \alpha m_x + 1.25 e_x )</td>
</tr>
<tr>
<td>( l_2 = 2 \pi m_x )</td>
<td>( l_2 = 2 \pi m_x )</td>
</tr>
<tr>
<td>( l_3 = 0.5 b_p )</td>
<td>( l_3 = 0.5 b_p )</td>
</tr>
<tr>
<td>( l_4 = 0.5 w + 2 m_x + 0.625 e_x )</td>
<td>( l_4 = 0.5 w + 2 m_x + 0.625 e_x )</td>
</tr>
<tr>
<td>( l_5 = e + 2 m_x + 0.625 e_x )</td>
<td>( l_5 = e + 2 m_x + 0.625 e_x )</td>
</tr>
<tr>
<td>( l_6 = \pi m_x + 2 e )</td>
<td>( l_6 = 2 \pi m_x + 4 e )</td>
</tr>
<tr>
<td>( l_7 = \pi m_x + w )</td>
<td>( l_7 = 2 (\pi m_x + w) )</td>
</tr>
<tr>
<td>( l_{\text{eff}, 1} = \min(l_1; l_2; l_3; l_4; l_5; l_6; l_7) )</td>
<td>( l_{\text{eff}, 1} = \min(l_1; l_2; l_3; l_4; l_5; l_6; l_7) )</td>
</tr>
<tr>
<td>( l_{\text{eff}, 2} = \min(l_1; l_2; l_3; l_4; l_5) )</td>
<td>( l_{\text{eff}, 2} = \min(l_1; l_2; l_3; l_4; l_5) )</td>
</tr>
</tbody>
</table>
4.1.3 Stiffness

The prediction of the base plate stiffness is based on (Steenhuis et al, 2008). The stiffness of the component analogous to the resistance of the T-stub is influenced by the contact of the base plate and the concrete foundation (Wald et al, 2008). The formula for deformation of the base plate loaded by the force in bolt $F_b$ is

$$\delta_p = \frac{1}{2} \frac{F_b m^3}{3EI} = \frac{2F_b m^3}{E \cdot l_{eff} t^3} = \frac{2F_b}{E \cdot k_p} \quad (4.26)$$

and deformation of the bolt is

$$\delta_p = \frac{F_b l_b}{E_b A_b} = \frac{F_b}{E_b k_b} \quad (4.27)$$

The stiffness of the T-stub is written as

$$k_T = \frac{F_b}{E (\delta_p + \delta_b)} \quad (4.28)$$

In following conditions cases prying force are appearing in the T-stub

$$\frac{A_s}{L_b} \geq \frac{l_{eff,ini} t^3}{8.82 \, m^3} \quad (4.29)$$

Formulas for stiffness coefficient of the base plate and of the bolt are

$$k_p = \frac{l_{eff,ini} t^3}{m^3} = \frac{0.85 \, l_{eff} t^3}{m^3} \quad (4.30)$$

$$k_b = 1.6 \frac{A_s}{L_b} \quad (4.31)$$

In case of no prying, it means when

$$\frac{A_s}{L_b} \leq \frac{l_{eff,ini} t^3}{8.82 \, m^3} \quad (4.32)$$

Formulas are as following:

$$k_p = \frac{F_p}{E \delta_p} = \frac{l_{eff,ini} t^3}{2 \, m^3} = \frac{0.425 \, l_{eff} t^3}{m^3} \quad (4.33)$$

$$k_b = \frac{F_p}{E \delta_p} = 2.0 \frac{A_s}{L_b} \quad (4.34)$$

The stiffness of the component of base plate in bending and bolts in tension is summarised from above simplified predictions as

$$\frac{1}{k_T} = \frac{1}{k_{b,i}} + \frac{1}{k_{p,i}} \quad (4.35)$$
For base plates are used the bolt pads under the bolt nut to help to cover the tolerances. The impact of an area of the bolt pad/nut changes the geometrical characteristics of T-stub. The influence is taken into account by the help of equivalent moment of inertia $I_{p,bp}$ and addition of stiffness $k_w$ to the previous stiffness $k_p$. By practical design this influence is neglected for simplicity, see (Hofmann, 2005), even if it may be significant for resistance.

### 4.2 Threaded stud in tension

The threaded studs are efficient connectors welded by fabricator or on side with high level of automation, see (Metric studs 2009,2013 and Pitrakkos and Tizani, 2013). The tension resistance of a threaded stud may be limited by yielding resistance

$$N_{y,s} = n_a A_s f_{yk} \quad (4.36)$$

ultimate resistance

$$N_{u,s} = n_a A_s f_{uk} \quad (4.37)$$

initial stiffness

$$S_{l,s} = n_a \frac{E A_s}{l_{eff}} \quad (4.38)$$

where

- $n_a$ is the number of threaded studs in a row
- $A_s$ is the area in tension of one threaded stud
- $l_{eff}$ is the effective length of the threaded stud
- $f_{yk}$ is the yield stress of the threaded stud
- $f_{uk}$ is the ultimate stress of the threaded stud

This solution procedure is applied to the headed stud connection the anchor plate to concrete block.

### 4.3 Punching of the anchor plate

The anchor plate under the threaded stud or above the headed stud may reach its load capacity due to shear resistance

$$F_{ap,Rd} = A_{p1,eff} \cdot \frac{f_{yk}}{\gamma_{M0}} \quad (4.39)$$

The stress area $A_{p1,eff}$ is determined from the thickness of the anchor plate $t_{p1}$ and effective length $l_{v1,eff}$ of the sheared area

$$A_{ap,eff} = l_{v1,eff} \cdot t_{p1} \quad (4.40)$$
Due to high bending of the threaded stud under the large deformations of the thin plate is assumed the effective length of shear area as half of the circumference only

\[ l_{v1,\text{eff}} = 2\pi \cdot (a_w + \frac{d_{ts}}{2}) \]  

(4.41)

where

\( a_w \) is throat thickness of weld of threaded stud [mm]

\( d_{ts} \) is diameter of the headed/threaded stud [mm]

This failure is assumed at all places, where a stud loaded by tension force is welded directly to a steel plate. The endless stiffness of this component should be assumed in calculations as no visible significant deformation performs due to punching trough steel plate during loading.

### 4.4 Anchor plate in bending and tension

The anchor plate is designed as a thin steel plate located at the top of concrete block and loaded predominantly in compression and shear. By loading the column base by the bending or tension is the anchor plate exposed to the tensile force from the threaded studs. If the threaded studs are not located directly under the headed studs, which are embedded in concrete, the anchor plate is exposed to bending, see Fig. 2.15. After the plastic hinges of the T-stub are developed, the anchor plate between the plastic hinges is elongates by tensile force.

The resistance of the component, see (Kuhlman et al, 2012), is not restricted to plastic mechanism only. The deformed shape with the elongated anchored plate between the threaded and headed studs is caring the additional load and may be taken into account. The behaviour, till the plastic hinges are developed, is modelled as the based plate in bending with help of T stub model, see Chapter 3.4. The anchor plate in tension resistance is
where

\[ t_{p1} \] is the thickness of the anchor plate

\[ b_{ap,eff} = n_1 \cdot \left( d_1 + 2 \cdot \sqrt{2} \cdot a_w \right) \] is the anchor plate effective width

\[ a_w \] is throat thickness of weld of threaded stud

\[ n_1 \] is the number of threaded studs

\[ d_1 \] is the diameter of threaded stud

As the tensile force is developing in anchor plate the headed and threaded studs are exposed to horizontal force, see in Fig. 4.16. The elastic-plastic deformation at the stage of full plastification of the T stub is evaluated, see in Fig 4.17, by model of beam with four supports and three plastic hinges, see Fig. 4.15. The elongation of the anchor plate allows the uplift of the threaded stud. The model assumes that the supports, i.e. the headed and threaded studs, don't move in the horizontal direction and the headed stud in the vertical direction. E.g. the horizontal force depends linearly to the vertical one, see Fig. 4.18 and Fig. 4.19. The resulting horizontal force from tension in anchor plate is taken into account for evaluation of resistance of the components in shear and for the interaction of shear and tensile resistances.

\[
F_{\text{Lap,Rd}} = A_{\text{ap,1}} \cdot \frac{f_{yk}}{Y_{M0}} = t_{p1} \cdot b_{ap,eff} \cdot \frac{f_{yk}}{Y_{M0}}
\] (4.42)

Fig. 4.16 Plastic hinges and bending moments in the anchor plate

In case of activation of the membrane action in anchor plate is verified the resistance of the related components in tension in vertical direction and in shear in horizontal direction. In the procedure is derived:

- the bending resistance of the anchor plate,
- the tensile resistance of the anchor plate,
- the bending and tensile deformation of the anchor plate.

and further is limited the resistance of the component anchor plate in bending and tension by

- the vertical resistance of the threaded stud (tensile and punching resistance) and the headed studs (tensile resistance, concrete cone failure, stirrups failure, bond failure).
- the horizontal resistance of the threaded stud (shear and bearing resistance) and the headed studs (shear and pry out resistance).
- the interaction in the threaded stud (tension and shear resistances) and the headed studs (tension and shear resistances).

The plastic resistance of the anchor plate is

\[ M_{ap,pl} = \frac{l_{eff,1} \cdot t_{p1}^2}{4} \cdot \frac{f_{yk}}{\gamma_{M0}} \]  

(4.43)

where

- \( t_{p1} \) is thickness of the anchor plate [mm]
- \( l_{eff,1} \) is the effective width of the anchor plate [mm]

The effective width of the anchor plate is minimum of the

\[
l_{eff,1} = \min \left\{ \frac{4 \ m_1 + 1.25 \ e_{a1}}{2 \ \pi \ m_1}, \frac{5 \ n_1 \ d_1 \cdot 0.5}{2 \ m_1 + 0.625 \ e_{a1} + 0.5 \ p_1}, \frac{\pi \ m_1 + 2 \ e_{b1}}{\pi \ m_1 + p_1} \right\} \]

(4.44)

where \( 5 \ h_1 \ d_1 \) is the effective width of the T stub between the headed and threaded studs.

The vertical deformation of the anchor plate under bending may be assumed for a beam with four supports and three plastic hinges as

\[ \delta_T = \frac{1}{E} \int_{I_b} \cdot \frac{1}{6} \cdot b^2 \cdot M_{ap,pl} + \frac{1}{E} \int_{I_c} \cdot \frac{1}{3} \cdot b \cdot c \cdot M_{ap,pl} \]  

(4.45a)

The elastic part of the deformation is

\[ \delta_{T,el} = \frac{2}{3} \cdot \delta_T \]  

(4.45b)

The elastic-plastic part of the deformation, see Fig. 4.17, is

\[ \delta_{T,pl} = 2.22 \ \delta_{T,el} \]  

(4.45c)

The force at the bending resistance of the anchor plate is evaluated from equilibrium of internal forces

\[ N_{pl} \cdot \delta_T \cdot \frac{b_2}{b} + M_{Ed} \cdot \frac{\delta_T}{b} = 2 \cdot M_{ap,pl} \cdot \frac{\delta_T}{a} + 2 \cdot M_{ap,pl} \cdot \frac{\delta_T}{b} \]  

(4.45)

\[ N_{pl} \cdot b_2 + M_{Ed} = 2 \cdot M_{ap,pl} \cdot b \cdot \left( \frac{1}{a} + \frac{1}{b} \right) \]  

(4.46)

for \( M_{Ed} = N_{Rd} \cdot e \)

\[ \text{is } N_{pl} \cdot b_2 + N_{Rd} \cdot e = 2 \cdot M_{ap,pl} \cdot b \cdot \left( \frac{1}{a} + \frac{1}{b} \right) \]  

(4.47)
\[ N_{pl} = 2 \cdot M_{ap,pl} \cdot b \cdot \left( \frac{1}{a} + \frac{1}{b} \right) \frac{1}{(b_2 + e)} \] (4.48)

The vertical resistance of the component anchor plate in tension is limited by the resistance of the components: threaded stud in tension, punching of the threaded stud and tensile resistance of the anchor plate. For the thin anchor plate is decisive the punching of the threaded stud. The deformed length of the anchor plate between the threaded and headed studs at the resistance in punching of the anchor plate under the threaded stud is

\[ a_{ap} = a + \Delta a = a + \frac{a \cdot F_{ap,Rd}}{t_{p1} \cdot b_{ap,eff} \cdot E} \] (4.49)

![Fig. 4.17 Linear relation of acting vertical forces \( F_v \) and vertical deformation \( \delta_v \)]

The component of vertical deformation by the elongation of the anchor plate, see Fig. 4.14, is

\[ \delta_{p,tot} = \delta_{T,pl} + \sqrt{a_{ap}^2 - a^2} \] (4.50)

The component of the horizontal force at the resistance in punching of the anchor plate under the threaded stud, see Fig. 4.18, is

\[ F_{p,Rd,H} = \frac{a}{\delta_{p,tot}} \cdot F_{tp,Rd} \] (4.51)

![Fig. 4.18 Linear relation of vertical \( F_v \) and horizontal forces \( F_H \)]

The horizontal force \( F_{ap,H} \) is limited by shear resistance of the threaded and headed studs \( V_{Rd} \), see in Figs 4.19. The resistance to vertical force is
The interaction of the tensile and shear forces is verified for the threaded and headed studs, see Tab. 3.4 in EN1993-1-8:2006 by

\[
\frac{F_{V,Ed}}{F_{V,Rd}} + \frac{F_{T,Ed}}{1.4 \cdot F_{T,Rd}} \leq 1
\]  

(4.53)

The interaction of tensile and shear forces is verified for the headed stud anchoring to concrete, see Chapter 3.2.5 by

\[
\left(\frac{F_{V,Ed}}{F_{V,Rd}}\right)^\frac{3}{2} + \left(\frac{F_{T,Ed}}{F_{T,Rd}}\right)^\frac{3}{2} \leq 1
\]  

(4.54)

## 4.5 Column/beam flange and web in compression

The resistance of the column flange and web in compression may be expected as for the beam flange, see Chapter 6.2.6.7 in EN1993-1-8:2006. In this model the column/beam web has its full plastic resistance on the lever arm of column/beam flanges

\[
F_{C,1,Rd} = \frac{M_{c,Rd}}{(h - t_f)}
\]  

(4.55)

in EN1993-1-8:2006 Eq. (4.1), where

\[M_{c,Rd}\] is the design moment resistance of the beam cross-section, see EN1993-1-1:2004

\[h\] is the depth of the connected column/beam

\[t_f\] is the column/beam flange thickness

If the height of the column/beam including the haunch exceeds 600 mm the contribution of the beam web to the design compression resistance should be limited to 20%. If a beam is reinforced with haunches the proposal for design is in cl 6.2.6.7(2). The stiffness of this component in compression is expected to be negligible.
4.6 Steel contact plate

The resistance of the steel contact plate in joint may be taken as its full plastic resistance

\[ F_{cp} = f_{y, cp} A_{cp} \]  

(4.56)

where

- \( f_{y, cp} \) is the yield strength of the steel contact plate
- \( A_{cp} \) is the effective area of the contact plate under compression

A height or breadth of the contact plate exceeds the corresponding dimension of the compression flange of the steel section, the effective dimension should be determined assuming dispersion at 45° through the contact plate. It should be assumed that the effective area of the contact plate in compression may be stressed to its design yield strength \( f_{yd} \), see EN1994-1-1:2010. The stiffness of the component the steel contact plate is negligible.

4.7 Anchor bolts in shear

In most cases the shear force is transmitted via friction between the base plate and the grout. The friction capacity depends on the compressive normal force between the base plate and the grout and the friction coefficient, see Chapter 3.3.7. At increasing horizontal displacement the shear force increases till it reaches the friction capacity. At that point the friction resistance stays constant with increasing displacements, while the load transfer through the anchor bolts increases further. Because the grout does not have sufficient strength to resist the bearing stresses between the bolts and the grout, considerable bending of the anchor bolts may occur, as is indicated in Fig. 4.20, see (Bouwman et al, 1989). The tests shows the bending deformation of the anchor bolts, the crumbling of the grout and the final cracking of the concrete. Based on the work (DeWolf and Sarisley, 1980) and (Nakashima, 1998) and of tests (Bouwman et al, 1989) the analytical model for shear resistance of anchor bolts was derived in EN1993-1-8 cl 6.2.2, see (Gresnight et al, 2008). Also, the preload in the anchor bolts contributes to the friction resistance. However, because of its uncertainty, e.g. relaxation and interaction with the column normal force, it was decided to neglect this action in current standard.

Fig. 4.20 Test specimen loaded by shear force and tensile force

The design shear resistance \( F_{v,Rd} \) may be derived as follows

\[ F_{v,Rd} = F_{f,Rd} + n F_{vb,Rd} \]  

(4.57)
where

- \( F_{f,Rd} \) is the design friction resistance between base plate and grout layer
  \[ F_{f,Rd} = C_{f,d} N_{c,Ed,v,Rd} \] (4.58)

- \( C_{f,d} \) is the coefficient of friction between base plate and grout layer. The following values may be used for sand-cement mortar \( C_{f,d} = 0.20 \), see Chapter 3.3.7.

- \( N_{c,Sd} \) is the design value of the normal compressive force in the column. If the normal force in the column is a tensile force \( F_{f,Rd} = 0 \)

- \( n \) is the number of anchor bolts in the base plate

- \( F_{vb,Rd} \) is the smallest of \( F_{1,vb,Rd} \) and \( F_{2,vb,Rd} \)

- \( F_{1,vb,Rd} \) is the shear resistance of the anchor bolt and
  \[ F_{2,vb,Rd} = \frac{\alpha_{bc} f_{ub} A_s}{\gamma_M} \] (4.59)

- \( A_s \) is the tensile stress area of the bolt or of the anchor bolt

- \( \alpha_{bc} \) is a coefficient depending on the yield strength \( f_{yb} \) the anchor bolt
  \[ \alpha_{bc} = 0.44 - 0.0003 f_{yb} \] (4.60)

- \( f_{yb} \) is the nominal yield strength the anchor bolt
  where \( 235 \text{ N/mm}^2 \leq f_{yb} \leq 640 \text{ N/mm}^2 \)

- \( \gamma_2 \) is the partial safety factor for anchor bolt
5 ASSEMBLY FOR RESISTANCE

5.1 Column base

5.1.1 Column base with base plate

The calculation of the column base resistance, based on the plastic force equilibrium on the base plate and applied in EN1993-1-8:2006, is described in (Wald et al, 2008). Based on the combination of acting load, see Fig. 5.1, three patterns may be distinguished:

Pattern 1 without tension in anchor bolts occurs due to high normal force loading. The collapse of concrete appears before developing stresses in the tension part.

Pattern 2 with tension in one anchor bolt row arises when the base plate is loaded by small normal force compared to the ultimate bearing capacity of concrete. During collapse the concrete bearing stress is not reached. The breaking down occurs because of yielding of the bolts or because of plastic mechanism in the base plate.

Pattern 3 with tension in both rows of anchor bolts occurs when the base plate is loaded by tensile normal force. The stiffness is guided by yielding of the bolts or because of plastic mechanism in the base plate. This pattern occurs often in base plates designed for tensile force only and may lead to contact of baseplate to the concrete block.

The connection is loaded by axial force $N_{Ed}$ and bending moment $M_{Ed}$, see Fig. 5.1. The position of the neutral axis is calculated according to the resistance of the tension part $F_{T,Rd}$. Then the bending resistance $M_{Rd}$ is determined assuming a plastic distribution of the internal forces, see (Dewolf, Sarisley, 1980). For simplicity of the model, only the effective area is taken into account. The effective area $A_{eff}$ under the base plate, which is taken as an active part of equivalent rigid plate, is calculated from an equivalent T-stub, with an effective width $c$, see Chapter 3.4.2. The compression force is assumed to act at the centre of the compressed part. The tensile force is located at the anchor bolts or in the middle when there are more rows or bolts, see (Thambiratnam, Paramasivam, 1986). Like for another cross sections of the composite structures there should be a closer look at the resistance for the ultimate limit state ULS and to the elastic behaviour under the serviceability limit state SLS. In the ultimate limit state the failure load of the system is important. Under service loads is checked the elastic behaviour and that the concrete cone will not fail. This would lead to cracks and with the time to a corrosion of the reinforcement of the concrete wall and finally to a failure of the construction.

![Diagram](image_url)

Fig. 5.1 The force equilibrium of the base plate a) no tension in anchor bolts, b) one row of the anchor bolts in tension, c) two rows of the anchor bolts in tension
Fig. 5.2 Force equilibrium for the column base, one row of the anchor bolts in tension

The equilibrium of forces is calculated according to Fig. 5.2 as follows:

\[ N_{Ed} = F_{c,Rd} + F_{t,Rd} \] (5.1)

\[ M_{Rd} = F_{c,Rd} \cdot z_c + F_{t,Rd} \cdot z_t \] (5.2)

where

\[ F_{c,rd} = A_{eff} \cdot f_{jd} \] (5.3)

\[ e = \frac{M_{Rd}}{N_{Ed}} \geq z_c \] (5.4)

formulas for tension and compressed part is derived

\[ \frac{M_{Rd}}{z} - \frac{N_{Ed} \cdot z_c}{z} \leq F_{c,Lrd} \] (5.5)

\[ \frac{M_{Rd}}{z} + \frac{N_{Ed} \cdot z_{c1}}{z} \leq F_{c,Rd} \] (5.6)

Then, the column base moment resistance \( M_{Rd} \) under a constant normal force \( N_{Ed} \) is expressed as follow:

with tension force in the anchor bolts

\[ M_{Rd} = \min \left\{ F_{t,Rd} \cdot z + N_{Ed} \cdot z_c, F_{c,Rd} \cdot z - N_{Ed} \cdot z_t \right\} \] (5.7)

without tension force, both parts are compressed
The procedure is derived for open section of I/H cross section. For rectangular hollow section RHS may be taken directly taking into account two webs. For circular/elliptical hollow sections CHS/EHS may be modified, see Fig. 5.2 and (Horová, 2011). Using sector coordinates depends the effective area \( A_{\text{eff}} = 2 \theta r c \) on the angle \( \theta \). The lever arm and the resistance of the component in compression is

\[
z_c = r \cdot \cos \frac{\theta}{2}
\]

\[
F_{c,Rd} = F_{c1,Rd} = \pi \cdot r \cdot c
\]

The resistance of the base plate connection under different loading is illustrated in M-N interaction diagram. In Fig. 5.3a there is an example of this diagram with its important points.

![Important points of interaction diagram](image)

**Fig. 5.3a An example of M-N interaction diagram for the base plate connection**

### 5.1.2 Column base with anchor plate

The bending resistance of the base plate with anchor plate is assembled from the tensile/compression resistances of its component. The additional components to the column bases without the anchor plate is the anchor plate in bending and in tension. The procedure for evaluation of the resistance is the same in all connections loaded by bending moment and normal force.

First the resistance of the components in tension is evaluated: the base plate, the threaded studs, the anchor plate and the headed studs. The activated area in contact under the base and anchor plate is calculated from the equilibrium of internal forces for the tensile part resistance. From the known size of the contact area is calculated the lever arm and the
bending resistance of the column base for particular acting normal force by the same procedure like for column base with the base plate only without the anchor plate.

During design of the base plate with the anchor plate is the elastic-plastic stage at serviceability limit verified separately, similar to the composite steel and concrete beam design. If the headed and threaded studs are not over each another the resistance of the base plate is influenced by the resistance of the component the anchor plate in tension and related components like punching of treated studs. The elastic-plastic resistance at Serviceability limit state is calculated based on the bending resistance of the anchor plate only. Moment rotational diagram at Fig. 5.4b sums up the behaviour of column base which is influenced by the elastic bending of the anchor plate (1), its elastic-plastic bending (2) and its tension (3).

![Moment rotational diagram of column base with anchor plate](image)

**Fig. 5.3b Moment rotational diagram of column base with anchor plate**

### 5.2 Simple steel to concrete joint

This joint typically represents a connection of a steel structure to a concrete wall. The anchor plate is loaded by shear load $V_{Ed}$ and a bending moment $M_{y,Ed}$. The developed model assumes a stiff anchor plate and deformations due to the anchor plate are neglected. The connection between the girder and the anchor plate may be regarded as pinned, rigid or semi-rigid. For most structures the connection between the beam and the anchor plate may be assumed as pinned. In this case of a simple connection the anchor plate is only loaded by shear load and a corresponding bending moment caused by the eccentricity of the shear load. The connection between the girder and the anchor plate may be realised with butt straps or cams or any other simple connection, see Fig. 5.4.
If the connection between the girder and the anchor plate cannot be assumed as pinned, there might be larger bending moments in the joint. In this Chapter the system described is a pinned connection between the beam and the anchor plate with an eccentricity $e_v$. However if there is a bending moment derived in the global analysis, the eccentricity $e_v$ may be assumed no longer a purely geometrical value anymore but is calculated by

$$e_v = \frac{M_{y,Ed}}{V_{Ed}}$$  \hspace{1cm} (5.11)

The developed component model describes the structural behaviour of the simple joints. The joints are consisting of an anchor plate with headed studs with and without additional reinforcement in cracked as well as non-cracked concrete. To prove a sufficient resistance for the ultimate limit state, the following steps have to be done:

- evaluation of the tension force caused by the shear load,
- verification of the geometry of the tension zone,
- evaluation of the tension resistance,
- evaluation of the shear resistance,
- verification of interaction conditions.

In the following the mechanical joint model for the simple joints is described. Due to the eccentricity of the applied shear load a moment is acting on the anchor plate. This moment causes forces, which are shown in Fig. 5.5. The anchor row on the non-loaded side of the anchor plate is in tension. This anchor row represents the tension component of the joint $N_{Ed,2}$ and forms a vertical equilibrium with the compression force $C_{Ed}$ under the anchor plate on the loaded side. The shear forces are carried by the headed studs, $V_{Ed,1}$ and $V_{Ed,2}$, and the friction between steel and concrete $V_f$.

The tension component of the joint, which is represented by the headed studs in tension or headed studs with stirrups in tension, in the case of using additional reinforcement, is described in Chapter 3. If no additional reinforcement is used, the following failure modes may occur: steel failure of the shaft, pull-out failure of the headed stud due to the high compression of the stud head on the concrete and concrete cone failure of the anchorage. When using additional reinforcement however, the stirrups contribute to the deformation and the resistance of the tension component. Besides the steel failure and the pull-out failure of the headed studs, a concrete failure due to yielding of the stirrups, an anchorage failure of the stirrups and a smaller
concrete cone failure may appear. A detailed description of these components is found in Chapter 3.

![Diagram of Forces at the Anchor Plate](image)

**Fig. 5.5 Forces at the anchor plate caused by the shear force $V_{Ed}$ and its eccentricity $e_V$**

For the compression zone a rectangular stress block is assumed under the loaded side of the plate. The stresses in the concrete are limited according to EN1993-1-8 cl 6.2.5. The design bearing strength of the concrete is $f_{jd}$. When there is no grout and the anchor plate has a common geometry, $f_{jd}$ may be assumed as $f_{jd} = 3f_{cd}$. The stress area $A_c$ is given by the width of the anchor plate $b$ and the length of the compression zone $x_c$ perpendicular to the load, resulting from the equilibrium with the assumed tension force in the studs on the non-loaded side $N_{Ed,2}$. As the anchor plate is regarded as stiff, the compression zone starts at the edge of the plate. The stiffness of this component is assumed according to Chapter 3.

**Equilibrium**

$$\sum N: C_{Ed} = N_{Ed,2} \quad (5.12)$$

**Compression force**

$$C_{Ed} = f_{jd} \cdot x_c \cdot b$$

for most cases $f_{jd} = 3f_{cd} \quad (5.13)$

The position of the shear load $V_{Ed,1}$ and $V_{Ed,2}$ has been derived according to the stress distribution given by the results of numerical calculations. There it is seen that the resulting shear force is placed with a distance of about $d$ in average from the anchor plate, when $d$ is the diameter of the headed stud. As a simplification of the mechanical joint model it is assumed that the shear forces of both anchor rows appear in the same line, see Fig. 5.6. In case of a high tension in the first row of studs only small additional shear forces $V_{Ed,2}$ is applied the 2nd stud row. The position of the friction force $V_f$ is assumed between the concrete surface and the anchor plate.
Forming the moment equilibrium according to (5.14), the size of the tension and the compression component of the joint is calculated. The rotational equilibrium is calculated for the point in the middle of the compression zone in one line with $V_{Ed,1}$ and $V_{Ed,2}$. The shear force is turning clockwise with a lever arm of $e_v + d + t_p$. The tension force $N_{Ed,2}$ is turning counter clockwise with a lever arm of $z$. The friction force is turning counter clockwise with a lever arm of $d$. The tension component carried by the second stud row $N_{Ed,2}$ is calculated with the following formula.

$$V_{Ed} \cdot (e_v + d + t_p) = N_{Ed,2} \cdot z + V_f \cdot d \quad (5.12)$$

$$N_{Ed,2} = \frac{V_{Ed} \cdot (e_v + d + t_p) - V_f \cdot d}{z} \quad (5.13)$$

If the pinned joint is loaded by diagonal pull, additional normal forces have to be considered in the moment equation, see Eq. (5.16). This equation requires, that the normal force does not lead to an uplift of the anchor plate. In this case both anchor rows would be subjected to tension forces and no shear resistance due to friction forces is carried by the pinned joint.

$$V_{Ed} \cdot (e_v + d + t_p) + N_{Ed} \cdot \left( z - \frac{s_2}{2} \right) = N_{Ed,2} \cdot z + V_f \cdot d \quad (5.14)$$

As already described above, the assumed tension load in the headed studs on the non-loaded side and the compression component form a vertical equilibrium. This approach requires an iterative process, as the area of the compression zone is dependent on the assumption for the tension load in the studs on the non-loaded side. But the shear resistance of the joint is not only limited by the acting moment. Therefore as a last step the resistance of the shear components have to be verified. The joint shear resistance is defined by the sum of the shear resistance of the studs and the friction between the concrete surface and the anchor plate, see Fig. 5.7 The resistance due to friction $V_f$ is defined by the coefficient $\mu$ for friction between steel and concrete. In cl 6.2.2 of EN1993-1-8:2006 a friction coefficient of $\mu = 0.2$ is proposed. The stiffness is assumed as infinite, as the displacement is zero if the shear force is smaller than $V_f$. 
After subtracting the component of the friction force, the rest of the applied shear load has to be carried by the headed studs. The total shear resistance depends on two possible failure modes due to shear: Steel failure of the headed studs as well as Concrete cone failure respectively pry-out failure. Also the distribution of the shear load among the anchor rows depends on the failure mode. Furthermore interaction between tension and shear loads in the stud row on the non-loaded side of the anchor plate has to be considered resulting in a reduced resistance of these studs. In case of a steel failure of the headed studs, it is assumed that at ultimate limit state the front anchor row is subjected to 100% of its shear resistance, as there are acting no tensional forces. The remaining part of the shear load is carried by the back row of anchors, depending on the interaction conditions. In contrast when verifying the anchorage for concrete failure, the shear load is distributed half to the front and half to the back row of anchors. Thereby the interaction condition for concrete failure has to be considered. The following interaction conditions are used:

$$n_N^{3/2} + n_V^{3/2} \leq 1$$  \hspace{2cm} (5.15)

$$n_N^2 + n_V^2 \leq 1$$  \hspace{2cm} (5.16)

where

- $n_N$ is the minimum value for $N_{Ed,i}/N_{Rd,i}$
- $n_V$ is the minimum value for $V_{Ed,i}/V_{Rd,i}$

### Additional verifications required

In the preceding description not all verifications are covered. Additional checks, which are not described in this manual have to be done:

- Verification of the steel components connected to the anchor plate.
- Calculation of the anchor plate. The calculated tension and compression forces causes bending moments in the anchor plate. The anchor plate must be able to carry these bending moments. The anchor plate has to be stiff and therefore in the plate no yielding is allowed.
- Additional checks for the reinforcement in the concrete wall to prevent local failure of the concrete due to the compression force with have to be done, see EN1992-1:2004.
- The concrete wall must be able to carry the loads transferred by the anchor plate.

The verification of the design resistance of the joint is described in the Table 5.1 in a stepwise manner.
Tab. 5.1 Verification of the design resistance of the joint

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
<th>Formula</th>
</tr>
</thead>
</table>
| 1    | Evaluation of the tension force caused by the shear load | z is depending on $x_c$  
$N_{Ed,2} = \frac{V_{Ed} \cdot (e_v + d + t_p) - V_f \cdot d}{z}$ |
|      | Estimation of $x_c$ and calculation of the tension component $N_{Ed,2}$ | |
| 2    | Verification of compression height. |  
$\sum N: C_{Ed} = N_{Ed,2}$ \quad $x_c = \frac{C_{Ed}}{b \cdot f_{id}}$ \nIf $x$ is estimated too small go back to Step 1 and try again.\nFor most cases $f_{id} = 3 f_{cd}$ |
| 3    | Evaluation of the tension resistance | Without Stirrups  
$N_{Rd,u} = \min \left\{ \frac{N_{Rd,u,s}}{N_{Rd,p}}, N_{Rd,u,c} \right\}$  
With Stirrups  
$N_{Rd,u} = \min \left\{ \frac{N_{Rd,u,s}}{N_{Rd,p}}, N_{Rd,c}, N_{Rd,re,1}, N_{Rd,re,2} \right\}$ |
| 4    | Calculation of the shear resistance |  
$V_{Rd,s} = 0.7 \cdot N_{Rd,u,s}$  
$V_{Rd,cp} = k \min [N_{Rd,cs}, N_{Rd,re,1}, N_{Rd,cs,2}, N_{Rd,u,group}]$ |
| 5    | Verification of interaction conditions | Possible failure modes  
Steel failure of the headed studs  
$V_{Ed,2} = V_{Ed} - V_{Rd,s} - V_f$  
$V_{Ed,2} = \frac{V_{Ed} - V_f}{2}$  
$\left( \frac{N_{Ed,2}}{N_{Rd,u,s}} \right)^2 + \left( \frac{V_{Ed,2}}{V_{Rd,s}} \right)^2 \leq 1$  
Concrete failure  
$N_{Rd,u}$ is not including $N_{Rd,u,s}$  
$\left( \frac{N_{Ed,2}}{N_{Rd,u}} \right)^{3/2} + \left( \frac{V_{Ed,2}}{V_{Rd,cp}} \right)^{3/2} \leq 1$  
$N_{Rd,u}$ is not including $N_{Rd,u,s}$ |
|      | Are both interaction equations OK? | YES \nDesign calculation finished | NO \nThe load carrying capacity of the joint is not sufficient. \nThe joint has to be improved. |
5.3 Moment resistant steel to concrete joint

A representative spring and rigid link model was idealized for the behaviour of composite beam to reinforced concrete wall joint, subjected to hogging bending moment, which is illustrated in Fig. 5.8. The joint components are:

- longitudinal steel reinforcement in the slab, at Fig. component 1
- slip of the composite beam, component 2;
- beam web and flange, component 3;
- steel contact plate, component 4;
- components activated in the anchor plate connection, components 5 to 10 and 13 to 15;
- the joint link, component 11.

In order to obtain the joint properties, the assembly of the components and of the joint models is described in the present section. For the joint under hogging bending moment, the assembly procedure was based on the mechanical model depicted in Fig. 5.8b. The determination of the joint properties to bending moment may be performed using a direct composition of deformations. The longitudinal steel reinforcement bar in slab, the slip of the composite beam, and the anchor plate components consider the models described in section 3. These models enable a good approximation to the real behaviour of the components, see (Henriques, 2008). The models may be described and composed also based on its stiffness coefficients as used in EN1993-1-8:2006.

The mechanical model represented in Fig. 5.9 presents only one row of components in tension and another in compression. This implies that the assembly procedure is much simpler, as no distribution of load is required amongst rows, as in steel/composite joint with two or more tension rows. Thus, the first step is the assembly of the components per row. Equivalent springs are defined per row, as represented in Fig. 5.9. The equivalent component/spring should perform as the group of components/springs it represents. The determination of its properties takes into consideration the relative position of the components: acting in series or in parallel. In the present case, either for the compression row either for the tension row, all joint components are acting in series. Thus, the determination of the properties of equivalent components/springs was performed as expressed in (5.17) for resistance $F_{eq,t}$ and $F_{eq,c}$, see (Henriques, 2008).
where the indexes $i$ to $n$ represent all components to consider either in tension either in compression, depending on the row under consideration.

Then, because only one tension row and one compression row was considered, the determination of the joint properties, $M_j, \Phi_j$, becomes relatively easy. In order to determine the joint rotation, it is important to define the lever arm $h_r$. According to the joint configuration, it was assumed that the lever arm is the distance between the centroid of the longitudinal steel reinforcement bar and the mid thickness of bottom flange of the steel beam. The centroid of steel contact plate is assumed to be aligned with this reference point of the steel beam. Accordingly, the joint properties are obtained as follows:

\[
F_{eq} = \min\{F_{i\text{t}}, F_{n\text{c}}\}
\]

(5.17)

where, $F_{eq,\text{t}}$ and $F_{eq,\text{c}}$ are the equivalent resistance of the tension and compression rows, respectively, determined using Eq. (5.17).

\[
F_{eq} = \min\{F_{eq,\text{t}}, F_{eq,\text{c}}, F_{jL}\} h_r
\]

(5.18)

where, $F_{eq,\text{t}}$ and $F_{eq,\text{c}}$ are the equivalent resistance of the tension and compression rows, respectively, determined using Eq. (5.17).
6 ASSEMBLY FOR STIFFNESS

6.1 Column base

6.1.1 Column base with base plate

The calculation of stiffness of the base plate, given in (Wald et al, 2008), is compatible with beam to column stiffness calculation. The difference between these two procedures is in the fact that by the base plate joint the normal force has to be introduced, see (Ermopoulos, Stamatopoulos, 1996). In Fig. 6.1 there is the stiffness model which shows a way of loading, compression area under the flange, allocating of forces under the base plate, and a position of the neutral axes.

![Stiffness model of the base plate connection of the column base](image)

By the calculation of the stiffness the effective area is only taken into account. The position of compression force $F_{c,Rd}$ is located at the centre of compression area. The tensile force $F_{t,Rd}$ is located at the anchor bolts. The rotational bending stiffness of the base plate is usually determined during proportional loading with constant eccentricity

$$e = \frac{M_{Ed}}{N_{Ed}} = \text{const.} \quad (6.1)$$

According to the eccentricity three possible basic collapse modes can arise with activation of anchor bolts, see (Wald et al, 2008). For large eccentricity with tension in one row of anchor bolts Pattern 1, see Fig. 6.2a, without tension in row of anchor bolts, small eccentricity, Pattern 2 in Fig. 6.2b, and with tension in both row of anchor bolts Pattern 3.

**Pattern 1** with tension in one bolt row of anchor bolts arises when the base plate is loaded by small normal force compared to the ultimate bearing capacity of concrete. During collapse the concrete bearing stress is not reached. The breaking down occurs because of yielding of the bolts or because of plastic mechanism in the base plate.

**Pattern 2** without tension in anchor bolts grows up during high normal force loading. The collapse of concrete appears before developing stresses in the tension part.
Pattern 3 with tension in one bolt row of anchor bolts arises when both bolt row of anchor bolts may be activated and column base is exposed to tension force in not so common, and the theorems may be derived similarly.

Deformations $\delta_t$ and $\delta_c$ of components depend on the stiffness of tension part $k_t$ and the stiffness of the compression part $k_c$.

$$\delta_{t,l} = \frac{M_{Ed}}{E k_t} - \frac{N_{Ed} z_t}{E z k_t}$$

$$\delta_{c,r} = \frac{M_{Ed}}{E k_c} - \frac{N_{Ed} z_t}{E z k_c}$$  \hspace{1cm} (6.2)

$$\delta_{t,l} = \frac{M_{Ed}}{E k_t} - \frac{N_{Ed} z_t}{E z k_t}$$

The rotation of the base plate could be determined from formulas above

$$\phi = \frac{\delta_{t,l} + \delta_{c,r}}{z} = \frac{1}{E z^2} \left( \frac{M_{Ed} - N_{Ed} z_c}{k_c} + \frac{M_{Ed} + N_{Ed} z_t}{k_t} \right)$$  \hspace{1cm} (6.3)

From the rotation the initial stiffness is derived

$$S_{j,ini} = \frac{E z^2}{\frac{1}{k_c} + \frac{1}{k_t} \sum^1_k}$$  \hspace{1cm} (6.4)

Nonlinear part of the moment-rotation curve is given by coefficient $\mu$, which express the ratio between the rotational stiffness in respect to the bending moment, see (Weynand et al, 1996) and EN1993-1-8:2006

$$\mu = \frac{S_{j,ini}}{S_j} = \left( \kappa \frac{M_{Ed}}{M_{Ed}} \right)^\xi \geq 1$$  \hspace{1cm} (6.5)

where

- $\kappa$ is coefficient introducing the beginning of non-linear part of curve, $\kappa = 1.5$
- $\xi$ is shape parameter of the curve, $\xi = 2.7$

The rotation stiffness is calculated as
For above described components, the stiffness coefficients, showed in Fig. 6.3, is revised from bolt in tension $k_b$, base plate in bending $k_p$, and concrete in compression $k_c$.

\[
S_j = \frac{E z^2}{\mu \sum k^{-1}}
\]  

(6.7)

As it is evident in Fig. 6.3, the stiffness of the tension part $k_t$ consists of the stiffness of the base plate $k_p$ and the stiffness of bolts $k_b$. With these parameters, $S_j$, $\mu$, and $M_{rd}$, we obtain the moment rotation curve, which is the best way how to describe behaviour of the base plate connection, see Fig. 6.4.

The procedure for evaluation of stiffness is derived for open section of I/H cross section. For rectangular hollow section RHS may be taken directly taking into account two webs. For circular/elliptical hollow sections CHS/EHS may be modified, see (Horová, 2011).

6.1.2 Column base with anchor plate

The bending stiffness of the base plate with anchor plate is assembled from the deformation stiffness's of its components, e.g. in the tensile part the base plate, the threaded studs, the anchor plate, and the headed studs and in the compressed part the concrete block in compression and base plate plus anchor plate in bending. The additional components are the anchor plate and treated studs. The deformation springs representing the individual components and its lever arms are summarized in Fig. 6.5. The effective stiffness coefficient, see Chapter 6.3 in EN1993-1-8:2005, is applied to transfer all deformational springs into the position of the threatened stud.
6.2 Simple steel-to-concrete joint

The stiffness of the concrete components are not yet considered in the CEN/TS 1992-4-2 to calculate the deformation behaviour of the Simple joint. In the following the stiffness that have been developed within the INFASO project were applied to the Simple joint and from this the rotational stiffness of the joint is developed. A detailed description of this components may be found in Chapter 3. Thereby the rotational behaviour of the joint caused by the shear load \( V_{Ed} \) is calculated. It is assumed that in the case of a Simple joint the rotation does not influence the global analysis or the bending resistance of the joint to a high extend, see Fig. 6.6 and 6.7. The Simple joint is primarily a shear force connection and the rotation or the rotational stiffness of the joint is not relevant.

If the connection between the girder and the anchor plate cannot be assumed as pinned, there might be larger bending moments in the joint. In the following Chapters the system described is a simple connection between the beam and the anchor plate with an eccentricity \( e_y \). However if there is a real bending moment derived in the global analysis, the eccentricity \( e_y \) may be assumed no longer a purely geometrical value anymore but is calculated by
\[ e_v = \frac{M_{r,Ed}}{V_{Ed}} \]. In this case it is very important to determine the rotational stiffness of the joint because the rotational stiffness may influence the load distribution in the global analysis and the size of the bending moment of the joint, see Fig. 6.8. In order to model the rotational behaviour of the joint, at minimum two components are necessary, a tension component and a compression component. The tension component is represented by the headed stud in tension, see Chapter 3, and the compression component by the component concrete in compression. With these two components and the lever arm \( z \) and the rotational behaviour of the joint may be modelled.

\[ \varphi = \frac{\delta_T + \delta_C}{z} \]  \hspace{1cm} (6.8)

Fig. 6.8 Forces at the anchor plate caused by the shear force \( V_{Ed} \) and its eccentricity \( e_v \)

The shear load \( V_{Ed} \) causes a tension force \( N_{Ed,2} \) in the headed stud on the non-loaded side of the anchor plate. In equilibrium with the tension force there is a compression force \( C_{Ed} \). For the equilibrium of moments and forces also see Chapter 3.

This forces are leading to a deformation \( \delta_T \) caused by the tension force on the non-loaded side of the anchor plate and a deformation \( \delta_C \) caused by the compression force on the loaded side of the anchor plate, see Fig. 6.. With these two deformation values and the lever arm \( z \) the rotation of the stiff anchor plate may be calculated according to the following formula
In the following an overview over the tension and over the compression component is given.

The tension component

The tension component is described in detail in Chapter 3. For these components two alternatives exist, one with additional stirrups and one without, see Fig. 6.10. For every alternative a model including several springs has been developed.

Headed studs in tension

\[
\begin{align*}
N &= 0 \text{ to } N = N_{u,c} \\
\delta_1 &= \delta_{p1} + \delta_h
\end{align*}
\]

(6.9)

\[
\begin{align*}
N &= N_{u,c} \text{ to } N = 0 \\
\delta_2 &= \delta_{p1} + \left(\frac{N - N_{u,c}}{k_c}\right)
\end{align*}
\]

(6.10)

Headed studs with stirrups in tension

\[
\begin{align*}
N &= 0 \text{ to } N = N_{u,c} \\
\delta_1 &= \delta_{p1} + \delta_h
\end{align*}
\]

(6.11)
\[ N = N_{u,c} \text{ to } N = N_u \quad \text{and} \quad \delta_2 = \delta_{p2} + \delta_h + (\delta_c + \delta_s) \quad (6.12) \]

\[ N = N_u \text{ to } N = 0 \quad \text{and} \quad \delta_3 = \delta_2(N_u) + \frac{N - N_u}{k_c} + \frac{N_u - N}{10000} \quad (6.13) \]

In both cases it is necessary to ensure that neither yielding nor pull-out failure of the headed studs is the decisive failure mode. The load-displacement behaviour after of these failure modes are not considered in the equations above.

**The compression component**

For the compression force the spring stiffness may be calculated as follows:

\[ K_c = \frac{E_c \cdot \sqrt{A_{eff}}}{1.275} \quad (6.14) \]

The formula is taken from EN1993-1-8. The influence of the concrete stiffness is not very large on the rotational behaviour.

**Determination of the lever arm z**

Due to the equilibrium for each value of the shear load \( V_{Ed} \), a corresponding tension force \( N_{Ed,2} \) and the compression force \( C_{Ed} \) have to be calculated. As every value of \( V_{Ed} \) corresponds to a different compression force \( C_{Ed} \), there is also a different height of the compression area \( x_c \) and another corresponding lever arm \( z \). For example if a small \( V_{Ed} \) causes a small \( N_{Ed,2} \) and \( C_{Ed} \), the height of the compression zone \( x_c \) is small and the lever arm \( z \) is relatively large. If the shear load is increased, the size of the compression force rises and the height of the compression area \( x_c \) also grows, whereas the lever arm \( z \) decreases.

The changing of the lever arm \( z \) is easily taken into account in a computer based calculation. For a calculation without computer a constant lever arm should be assumed. For the most structures the best solution to determine the lever arm is to calculate the maximum resistance of the tension load of the headed studs. Based on this value the maximum compression force and the minimum \( z \) may be calculated. Only if the anchor plate is extreme small and the tension resistance is extremely large the lever arm should be determined in a different way.

**The rotational stiffness**

Not only the rotation caused by the shear load, but also the rotational stiffness of the joint is calculated. With the help of the rotational stiffness it is possible to model the joint in the global analysis assuming his realistic behaviour. The initial rotational stiffness \( S_{j,\text{ini}} \) may be calculated according to EN1993-1-8. The following equation may be found in EN1993-1-8:2006, cl 6.3.1

\[ S_{j,\text{ini}} = \frac{z^2}{\left(\frac{1}{K_T} + \frac{1}{K_c}\right)} \quad (6.15) \]

where

- \( K_T \) is the stiffness of the tension component
- \( K_c \) is the stiffness of the compression component

If no ductile behaviour is expected, the initial stiffness \( S_{j,\text{ini}} \) is assumed up to the maximum load. In the case of ductility the stiffness \( S_j \) of the joint is changed according to the utilization level of the joint. Therefore the behaviour of the joint is represented by a moment-rotation curve with a trilinear shape, see equation 6.17. The determination of the associated factor \( \mu \) is taken from
EN1993-1-8. It has to be mentioned that in this case large cracks that are undesirable might occur.  

\[ S_j = S_{j,ini}/\mu \]  

(6.16)

6.3 Moment resistant steel to concrete joint

For the joint under hogging bending moment, the assembly procedure was based on the mechanical model represented in Fig. 5.8a. The determination of the joint properties to bending moment is performed using two different approaches: the direct deformation superposition and model based on composition of stiffness coefficients by spring procedure.

The mechanical model represented in Fig. 5.8b presents only one row of components in tension and another in compression. The determination of the properties of equivalent components/springs was performed as expressed in (6.17), for deformation \( \Delta_{eq,t} \) and \( \Delta_{eq,c} \).

\[ \Delta_{eq} = \sum_{i=1}^{n} \Delta_i \]  

(6.17)

where, the index \( i \) to \( n \) represent all components to consider either in tension either in compression, depending on the row under consideration. In order to determine the joint rotation, it is important to define the lever arm \( h_r \). Accordingly, the joint properties are obtained as follows

\[ \phi_j = \frac{\Delta_{eq,t} + \Delta_{eq,c} + \Delta_{IL}}{h_r} \]  

(6.18)

where

\( \Delta_{eq,t} \) and \( \Delta_{eq,c} \) are the equivalent deformation of the tension and compression rows, respectively, determined using (6.17).

7 GLOBAL ANALYSIS INCLUDING JOINT BEHAVIOUR

7.1 Structural analysis

The analysis of structures regarding the steel and composite joints modelling has been conventionally based on the concept of rigid, infinite rotational stiffness, or pinned, no rotational stiffness. However, it is well recognized that the real behaviour is often intermediate between these extreme situations, see (Jaspart, 2002). In these cases, the joints are designated as semi-rigid. In such joints, partial relative rotation between connected members is assumed, contrarily to the traditional concept of no or free rotation.

Consequently, the behaviour of the joint has a non-negligible influence on the structural analysis, see (Jaspart, 1997); and (Maquoi, Chabrolin, 1998) affecting: distribution of internal forces and deformations. In terms of resistance, the influence of the joint properties is obvious, as the structural capacity is limited if the joint is not fully capable of transmitting the internal forces, namely the bending moments. In such cases, the joint rotation capacity also becomes critical, defining the type of failure and the possibility to redistribute the internal forces. Thus, joints are keys parts of the structure, playing an important role in the behaviour of the structure. In what regards to the reinforced concrete joints, the structural analysis remains in the classical concept of rigid or pinned joints EN1992-1-1:2004. This is understandable due to the nature of the joints. In what concerns the steel-to-concrete joints, the joint behaviour is similar to steel
joints. In this way, the effect of the steel-to-concrete joint on the structural behaviour should be considered as in steel structures.

With the component method (Jaspart, 1997), the real behaviour of the steel/composite joints may be efficiently evaluated and characterized in terms of rotational stiffness, bending moment resistance and rotation capacity. Subsequently, their behaviour is introduced in the structural analysis. This allows integrating the joint design with the structural design. Such type of analysis is recommended by the codes, EN1993-1-8:2006 and EN1994-1-1:2010, and should follow the subsequent steps:

- Characterization of the joint properties in terms of rotational stiffness, bending moment resistance and rotation capacity,
- Classification of the joint,
- Joint modelling on the structural model,
- Joint idealization.

The joint classification as already been introduced in section 2.2 and consists in determining the boundaries for the conventional type of joint modelling regarding the stiffness, see Fig. 2.6, and the resistance, see Fig. 2.7. The classification of the joint determines the type of joint modelling that should be adopted for the structural analysis. For stiffness classification, the stiffness of the connected beam is used to define the boundaries. In terms of resistance, the classification is set according to the minimum capacity of the connected members. In terms of rotation capacity, the information available is quite limited. In the code EN1993-1-8:2006 only a qualitative classification is given which consists in the following: i) ductile joints (suitable for plastic analysis) – ductile components govern the behaviour of the joint; ii) semi-ductile joints components with limited deformation capacity govern the joint response; iii) and brittle joints (do not allow redistribution of internal forces) - brittle components control the joint response.

<table>
<thead>
<tr>
<th>Stiffness</th>
<th>Semi-rigid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rigid/Semi-rigid</td>
<td>8 E I b/L b</td>
</tr>
<tr>
<td>Semi-rigid/Pinned</td>
<td>0.5 E I b/L b</td>
</tr>
</tbody>
</table>

Tab. 7.1 Criteria to define the boundaries for classification of beam-to-column steel and composite joints

- **Stiffness**
  - Rigid/Semi-rigid: $8 \frac{E I_b}{L_b}$
  - Semi-rigid/Pinned: $0.5 \frac{E I_b}{L_b}$

- **Resistance**
  - Full-strength/Partial-strength
    - Top of column: $\min\{M_{c,pl,ru}; M_{b,pl,ru}\}$
    - Within column height: $\min\{2M_{c,pl,ru}; M_{b,pl,ru}\}$
  - Partial-strength/Pinned
    - 25% of Full-strength/Partial-strength

In the structural analysis, according to the stiffness and strength classification, three types of joint modelling are possible, as listed in Tab. 7.2. In the case of continuous joint, the full rotation continuity is guaranteed between the connected members. In the case of simple joint, all rotational continuity is prevented between the connected members.

Otherwise, the joint is semi-continuous. In relation to the physical representation of the joint in the structural model, different approaches may be used, as illustrated in Tab. 7.2. In Fig. 7.1a the actual behaviour of the joint is modelled: L-springs $S_{r,b}$ representing the connecting zone and S-springs $S_{r,s}$ representing the panel zone. The infinite rigid stubs assure that the flexibility of the joint will not be taken into consideration more than once. In Fig. 7.1b is presented a model to be used in the software which does not support flexural springs. Stubs with adequate bending stiffness $E I$ and resistance $M$, maintaining the clear separation between bending and shear influences are used to replace rotational springs. Finally, the concentrated model is represented in Fig. 7.1c. In this model, L-springs and S-springs are assembled into one single spring and displaced to the column axis $S_c$. The overall joint behaviour is then represented by a single rotational spring, two in the case of double sided joints. This simplified modelling solution is prescribed by EN1993-1-8:2006. The simplifications adopted are compensated in
the joint transformation. The joint transformation takes into account the shear force acting in the column, and the combination of the shear panel and connections in the joint spring at the beam-to-column axis intersection point, see (Huber et al, 1998).

Tab. 7.2 Criteria to define the boundaries for classification of beam-to-column steel and composite joints EN1993-1-8:2006

<table>
<thead>
<tr>
<th>Joint modelling</th>
<th>Joint Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuous</td>
<td>Full-strength and Rigid</td>
</tr>
<tr>
<td>Semi-continuous</td>
<td>Full-strength and Semi-rigid</td>
</tr>
<tr>
<td></td>
<td>Partial-strength and Rigid</td>
</tr>
<tr>
<td>Simple</td>
<td>Partial-strength and Semi-rigid</td>
</tr>
<tr>
<td></td>
<td>Pinned and Pinned</td>
</tr>
</tbody>
</table>

The joint idealization consists in defining the type of flexural curve which will be attributed to the flexural spring representing the joint. The behaviour of the joints is typically nonlinear; however, its implementation in the flexural spring is not practical for everyday design. In this way, the behaviour of the joint may be simplified as schemed in Fig. 7.2. The selection of the appropriate curve depends on the type of analysis to perform: elastic, elastic-plastic, rigid-plastic. Accordingly the following behaviours may be assumed: i) linear elastic, Fig. 7.2a only requires rotational stiffness; ii) bi-linear or tri-linear elastic-plastic, Fig. 7.2b requires rotational stiffness, resistance and deformation capacity; iii) rigid plastic, Fig. 7.2c requires resistance and rotation capacity. In the case of semi-rigid joint, the joint rotational stiffness to be consider depends on the expected load on the joint, thus the following is considered: i) the acting bending moment is smaller than 2/3 of the joint bending moment resistance $M_{j,Rd}$ and the joint initial rotational stiffness $S_{j,ini}$ may be used; ii) in the other cases, the joint secant rotational stiffness $S_j$ should be used. The latter is obtained dividing the joint initial stiffness $S_{j,ini}$ by the
stiffness modification coefficient $\eta$. The codes EN1993-1-8:2006 and EN1994-1-1:2010 provide the stiffness modification coefficient to consider according to the type of connection.

The stiffness of a joint influences the deformability of the structure, which is reflected in the check for SLS. The influence of non-linear behaviour of joints in terms of ULS is more difficult to assess as it requires a non-linear analysis. The following example illustrates in a simplified way, the influence of joints in the behaviour of the structure. Considering the beam represented in Fig. 7.3, under a linear uniform load $q$ and assuming rigid joints at both ends of the beam leads to the bending moment $M_{j,\infty}$ at both supports, and to the bending moment diagram represented by the dashed line. On the other hand, assuming at both ends of the beam a rotational stiffness of the joints $S_j$, then the bending diagram represented by the continuous line is obtained. This represents a bending moment re-distribution of $\Delta M$ that varies from 0 to $qL^2/12$. This re-distribution is also reflected in the vertical deflection of the beam, which may vary from $qL^4/(384EI)$ to $5qL^4/(384EI)$. 

---

**Fig. 7.2a Linear elastic M-Φ curve idealized for the joint behaviour**

**Fig. 7.2b Bi-linear and tri-linear elastic-plastic M-Φ curve idealized for the joint behaviour**

**Fig. 7.2c Rigid plastic M-Φ curve idealized for the joint behaviour**
7.2 Examples on the influence of joint behaviour

7.2.1 Reference building structures

In order to illustrate the influence of joint behaviour in the global analysis of structures, an example is provided in the following paragraphs. For complete details of the analysis see (Henriques, 2013). The building structures selected for the analysis considered two types of occupancy: office and car park. For the first type, the building structure erected in Cardington and subject to fire tests was chosen, see (Bravery 1993) and (Moore 1995). The building was designed to represent a typical multi-storey building for offices. For the car park building, the structure used in a recent RFCS project regarding Robustness in car park structures subject to a localized fire, see (Demonceau et al, 2012), was selected. Though the main characteristics of the reference building structures are used, modifications were performed whenever required to adapt the structures. Furthermore the performed calculations only considered the analysis of plane sub-structures which were extracted from the complete building structures. As higher variation of the structural system was found in the office building, two sub-structures were selected to represent this type of building while for the car park only one sub-structure was considered. The main characteristics and the adopted modifications of the referred building structures are summarized in the following paragraphs, see (Kuhlmann et al, 2012) and (Maoqui, 1998).

The office building structure

The main geometrical and mechanical properties of the office building are summarized in Tab. 3, together with the adopted modifications. The floor layout is illustrated in Fig. 7.4.
Tab. 7.3 The main properties and performed modifications of the reference structure representing the office building type

<table>
<thead>
<tr>
<th>Reference Structure</th>
<th>Modifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nº of floors and height: 1 x 4.34 m + 7 x 4.14 m</td>
<td>No modifications</td>
</tr>
<tr>
<td>Nº of spans and length in longitudinal direction: 5 x 9 m</td>
<td></td>
</tr>
<tr>
<td>Nº of spans in transversal direction: 2 x 6 m+1 x 9 m</td>
<td></td>
</tr>
<tr>
<td>Columns: British steel profiles, grade S355, cross-section variation along height</td>
<td>All British steel profiles were replaced by common European steel profiles</td>
</tr>
<tr>
<td>Beams: composite, British steel profiles + composite slab; grade S355 and grade S275; Lightweight concrete</td>
<td>with equivalent mechanical properties. Bracing systems were replaced by shear</td>
</tr>
<tr>
<td>Bracing system: cross bracing flat steel</td>
<td>walls in order to introduce in the structural system, steel-to-concrete</td>
</tr>
<tr>
<td>Beam-to-column joints: simple joints</td>
<td>joints.</td>
</tr>
<tr>
<td>Column bases: continuous</td>
<td></td>
</tr>
</tbody>
</table>

The type of joint between horizontal members and vertical members was one of the key parameters of the study. The joint modelling was varied from continuous to simple. Column bases were assumed as simple joints.

Fig. 7.4 Floor layout of the reference structure representing the office building type

The car park building structure

This type of building represents the standard configuration of a car park structure in Europe. The main geometrical and mechanical properties of this type of building are summarized in Tab. 7.4. In this case, only a few modifications were required. Fig. 7.5 illustrates the floor layout.
Tab. 7.4 The main properties and performed modifications for the car park building type

<table>
<thead>
<tr>
<th>Reference structure</th>
<th>Modifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nº of floors and height: 8 x 3 m</td>
<td>No modifications</td>
</tr>
<tr>
<td>Nº of spans and length in longitudinal direction: 6 x 10 m</td>
<td></td>
</tr>
<tr>
<td>Nº of spans in transversal direction: 10 x 16 m</td>
<td></td>
</tr>
<tr>
<td>Columns: steel profiles, grade S460, cross-section variation along height</td>
<td>Dimensions given to the concrete core</td>
</tr>
<tr>
<td>Beams: composite (steel profiles + composite slab); grade S355; normal weight concrete</td>
<td></td>
</tr>
<tr>
<td>Bracing system: concrete core (assumed but not defined)</td>
<td></td>
</tr>
<tr>
<td>Beam-to-column joints: semi-continuous joints</td>
<td>No modifications</td>
</tr>
<tr>
<td>Column bases: simple joints</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 7.5 Structural layout of the car park building type

7.2.2 Design

The structural calculations performed considered an elastic-plastic analysis. In all members and joints, except RC walls, plastic deformations were admissible. For sake of simplicity, the wall behaviour was always assumed elastic without limitation of capacity. However, it was considered that the steel-to-concrete joint includes the part of the wall surrounding the joint. Therefore, partially, hypothetic localized failure of the wall was considered. In terms of loading, two types of combinations were considered: i) Service Limit State; and ii) Ultimate Limit State.

In relation to the calculations, the strategy consisted in performing several numerical simulations where the beam-to-column and beam-to-wall joint properties were varied within the boundaries for joint classification. In addition, two cases considered the extreme situations of all joints either continuous or simple joints. For the other cases, the steel joints and steel-to-concrete joints are semi-continuous. In all calculations, the column bases joints were assumed simple. Tab. 7.5 lists the numerical simulations performed and identifies the joint properties considered in each case. Although the focus was on steel-to-concrete joints, steel joints were also considered to be semi-continuous so that the structural system was consistent.
The different cases presented in Tab. 7.5 considered the combination of different values of joint initial rotational stiffness and resistance capacity. In terms of rotation capacity, it was assumed that unlimited rotation capacity was available. A total of 10 cases were considered for each load combination.

Tab. 7.5 Definition of the cases for each load combination and each sub-structure

<table>
<thead>
<tr>
<th>Case</th>
<th>Initial Rotational Stiffness</th>
<th>Bending Moment Resistance</th>
<th>Col. bases</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Steel-to-concrete joint</td>
<td>Steel joint</td>
<td>Col. bases</td>
</tr>
<tr>
<td>1</td>
<td>R</td>
<td>R</td>
<td>P</td>
</tr>
<tr>
<td>2</td>
<td>R</td>
<td>SR: 0.5 (R/SR+SR/P)</td>
<td>P</td>
</tr>
<tr>
<td>3</td>
<td>SR: 2/3 (R/SR+SR/P)</td>
<td>SR: 0.5 (R/SR+SR/P)</td>
<td>P</td>
</tr>
<tr>
<td>4</td>
<td>SR: 1/3 (R/SR+SR/P)</td>
<td>SR: 0.5 (R/SR+SR/P)</td>
<td>P</td>
</tr>
<tr>
<td>5</td>
<td>SR: 2/3 (R/SR+SR/P)</td>
<td>SR: 0.5 (R/SR+SR/P)</td>
<td>P</td>
</tr>
<tr>
<td>6</td>
<td>SR: 1/3 (R/SR+SR/P)</td>
<td>SR: 0.5 (R/SR+SR/P)</td>
<td>P</td>
</tr>
<tr>
<td>7</td>
<td>SR: 2/3 (R/SR+SR/P)</td>
<td>SR: 0.5 (R/SR+SR/P)</td>
<td>P</td>
</tr>
<tr>
<td>8</td>
<td>SR: 1/3 (R/SR+SR/P)</td>
<td>SR: 0.5 (R/SR+SR/P)</td>
<td>P</td>
</tr>
<tr>
<td>9</td>
<td>P</td>
<td>SR: 0.5 (R/SR+SR/P)</td>
<td>P</td>
</tr>
<tr>
<td>10</td>
<td>P</td>
<td>P</td>
<td>P</td>
</tr>
</tbody>
</table>

R-Rigid; SR-Semi-rigid; P-Pinned; FS-Full-strength; PS-Partial-strength

7.2.3 Structural model

Geometric and mechanical properties of members

The three sub-structures selected for the structural calculations are illustrated in Fig. 7.6. The members’ geometric dimensions and material properties are given in Tab. 7.6. For the bare steel cross-sections, the material behaviour was considered elastic-perfectly-plastic.
## Tab. 7.6 Sub-structures members’ geometric and material properties

<table>
<thead>
<tr>
<th>Sub-structure</th>
<th>Members</th>
<th>Geometric</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Columns: AL-1 and 4, AL-2</td>
<td>Bottom to 2nd floor: HEB320, 2nd floor to Top: HEB260</td>
<td>S355, S355</td>
</tr>
<tr>
<td></td>
<td>Beams*: IPE360+Composite slab (h&lt;sub&gt;slab&lt;/sub&gt; = 130mm)</td>
<td>#Φ6/200mm</td>
<td>S355, LC35/38</td>
</tr>
<tr>
<td></td>
<td>Walls</td>
<td>tw = 300mm, vertical reinforcement Φ20/30cm horizontal Φ10/30cm</td>
<td>C30/37, S500</td>
</tr>
<tr>
<td>II</td>
<td>Columns</td>
<td>Bottom to 2nd floor: HEB 340, 2nd floor to Top: HEB 320</td>
<td>S355, S355</td>
</tr>
<tr>
<td></td>
<td>Beams*: IPE360+Composite slab (h&lt;sub&gt;slab&lt;/sub&gt; = 130mm)</td>
<td>#Φ6/200mm</td>
<td>S355, LC35/38</td>
</tr>
<tr>
<td></td>
<td>Walls</td>
<td>tw = 300 mm, vertical reinforcement Φ20/300 mm horizontal Φ10/300mm</td>
<td>C30/37, S500</td>
</tr>
<tr>
<td>III</td>
<td>Columns</td>
<td>Bottom to 2nd floor: HEB 550, 2nd floor to 4th floor: HEB 400, 4th floor to 6th floor: HEB 300, 6th floor to 8th floor: HEB 220</td>
<td>S460, S460, S460, S460</td>
</tr>
<tr>
<td></td>
<td>Beams*: IPE450+Composite slab (h&lt;sub&gt;slab&lt;/sub&gt; = 120 mm)</td>
<td>#Φ8/200 mm</td>
<td>S355, C25/30</td>
</tr>
<tr>
<td></td>
<td>Walls</td>
<td>tw = 400 mm, # Φ20/200 mm</td>
<td>C30/37, S500</td>
</tr>
</tbody>
</table>

![Fig. 7.6a Geometry of sub-structure I, office building alignment A](image-url)
In order to simplify the structural modelling, the composite beams cross-section was replaced by equivalent rectangular cross-sections, see Table 7.7. Because of the different behaviour of the composite section under sagging and hogging bending moments, the equivalent beams cross-section (EqCS) varies within its length, as identified in Fig. 7.7. In terms of material properties, equivalent yield strength was also determined so that the equivalent cross-section attained a maximum bending moment equal to the resistance of the real composite cross-section.

Fig. 7.6b Geometry of sub-structure II, Office building alignment 3

Fig. 7.6c Geometry of sub-structure III, car park building alignment 2
### Tab. 7.7 Properties of the equivalent cross-sections replacing the real composite cross-sections

#### Sub-structure I

<table>
<thead>
<tr>
<th>Eq. CS-1</th>
<th>Eq. CS-2</th>
<th>Eq. CS-3</th>
<th>Eq. CS-4</th>
<th>Eq. CS-5</th>
</tr>
</thead>
<tbody>
<tr>
<td>I=1.59x10⁸mm⁴</td>
<td>I=3.885x10⁸mm⁴</td>
<td>I=1.63x10⁸mm⁴</td>
<td>I=5.4975x10⁸mm⁴</td>
<td>I=1.58x10⁸mm⁴</td>
</tr>
<tr>
<td>A=7034.56mm²</td>
<td>A=14512.67mm²</td>
<td>A=7087.57mm²</td>
<td>A=12633.20mm²</td>
<td>A=7024.62mm²</td>
</tr>
<tr>
<td>Equivalent rectangular cross-section dimension</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>h=520.08mm</td>
<td>b=13.53mm</td>
<td>h=525.23mm</td>
<td>b=13.49mm</td>
<td>h=580.67mm</td>
</tr>
<tr>
<td>Yield strength ($f_y$) of the equivalent rectangular cross-section to obtain the maximum bending moment ($M_{bh,max}$) of the composite beam cross-section</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_{bh,max}$ =351.41kN.m</td>
<td>$M_{bh,max}$ =605.00kN.m</td>
<td>$M_{bh,max}$ =358.94kN.m</td>
<td>$M_{bh,max}$ =565.00kN.m</td>
<td>$M_{bh,max}$ =349.98kN.m</td>
</tr>
<tr>
<td>$f_y$=576.30N/mm²</td>
<td>$f_y$=441.31N/mm²</td>
<td>$f_y$=578.52N/mm²</td>
<td>$f_y$=462.12N/mm²</td>
<td>$f_y$=575.88N/mm²</td>
</tr>
</tbody>
</table>

#### Sub-structure II

<table>
<thead>
<tr>
<th>Eq. CS-1</th>
<th>Eq. CS-2</th>
<th>Eq. CS-3</th>
<th>Eq. CS-4</th>
<th>Eq. CS-5</th>
</tr>
</thead>
<tbody>
<tr>
<td>I=1.14x10⁸mm⁴</td>
<td>I=2.74x10⁸mm⁴</td>
<td>I=1.20x10⁸mm⁴</td>
<td>I=3.38x10⁸mm⁴</td>
<td>I=1.23x10⁸mm⁴</td>
</tr>
<tr>
<td>A=6012.32mm²</td>
<td>A=11207.20mm²</td>
<td>A=6101.78mm²</td>
<td>A=16431.90mm²</td>
<td>A=6141.54mm²</td>
</tr>
<tr>
<td>Equivalent rectangular cross-section dimension</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>h=476.37mm</td>
<td>b=12.62mm</td>
<td>h=486.39mm</td>
<td>b=12.54mm</td>
<td>h=496.74mm</td>
</tr>
<tr>
<td>Yield strength ($f_y$) of the equivalent rectangular cross-section to obtain the $M_{max}$ of the composite cross-section</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_{max}$=274.86kN.m</td>
<td>$M_{max}$=470kN.m</td>
<td>$M_{max}$=286.85kN.m</td>
<td>$M_{max}$=631kN.m</td>
<td>$M_{max}$=292.05kN.m</td>
</tr>
<tr>
<td>$f_y$=575.81N/mm²</td>
<td>$f_y$=464.75N/mm²</td>
<td>$f_y$=579.90N/mm²</td>
<td>$f_y$=463.83N/mm²</td>
<td>$f_y$=581.62N/mm²</td>
</tr>
</tbody>
</table>

#### Sub-structure III

<table>
<thead>
<tr>
<th>Eq. CS-1</th>
<th>Eq. CS-2</th>
<th>Eq. CS-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>I=6.72x10⁸mm⁴</td>
<td>I=1.42x10⁹mm⁴</td>
<td>I=7.23x10⁸mm⁴</td>
</tr>
<tr>
<td>A=13192.32mm²</td>
<td>A=27012.63mm²</td>
<td>A=13600.91mm²</td>
</tr>
<tr>
<td>Equivalent rectangular cross-section dimension</td>
<td></td>
<td></td>
</tr>
<tr>
<td>h=781.66mm</td>
<td>h=794.22mm</td>
<td>h=798.44mm</td>
</tr>
<tr>
<td>b=16.88mm</td>
<td>b=34.01mm</td>
<td>b=17.00mm</td>
</tr>
<tr>
<td>Yield strength ($f_y$) of the equivalent rectangular cross-section to obtain the $M_{max}$ of the composite cross-section</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_{max}$=988.86kN.m</td>
<td>$M_{max}$=1338.00kN.m</td>
<td>$M_{max}$=1057.61kN.m</td>
</tr>
<tr>
<td>$f_y$=575.37N/mm²</td>
<td>$f_y$=374.20N/mm²</td>
<td>$f_y$=584.00N/mm²</td>
</tr>
</tbody>
</table>

---

**Fig. 7.7a** Identification of the equivalent cross-sections of the beams in sub-structure I
Joint properties

The boundaries values for classification of the joint in terms of rotational stiffness and resistance are listed in Tab. 7.8 for the three sub-structures. The joints were included in the structural models using concentrated flexural springs. For the partial-strength joints, a tri-linear behaviour was assigned, Fig. 7.8. The initial joint rotational stiffness is considered up to $2/3 M_{j,Rd}$, and then the joint rotation at $M_{j,Rd}$ is determined using the secant joint rotational stiffness. The latter is determined using a stiffness modification coefficient $\eta$ equal to 2.

<table>
<thead>
<tr>
<th>Sub-structure I</th>
<th>Joints</th>
<th>Rotational Stiffness</th>
<th>Bending Moment Resistance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>R-SR [kNm/rad]</td>
<td>SR-P [kNm/rad]</td>
</tr>
<tr>
<td>AL-1-right</td>
<td>AL-2-right</td>
<td>108780.0</td>
<td>108780.0</td>
</tr>
<tr>
<td>AL-2-left</td>
<td>AL-2-right</td>
<td>205340.0</td>
<td>3710.0</td>
</tr>
<tr>
<td>AL-2-right</td>
<td>AL-3-left</td>
<td>108780.0</td>
<td>2782.5</td>
</tr>
<tr>
<td>AL-3-right</td>
<td>AL-4-left</td>
<td>108780.0</td>
<td>2782.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sub-structure II</th>
<th>Joints</th>
<th>Rotational Stiffness</th>
<th>Bending Moment Resistance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>R-SR [kNm/rad]</td>
<td>SR-P [kNm/rad]</td>
</tr>
<tr>
<td>AL-A-right</td>
<td>AL-B-left</td>
<td>102293.3</td>
<td>2660.0</td>
</tr>
<tr>
<td>AL-B-right</td>
<td>AL-C-left</td>
<td>94640.0</td>
<td>2100.0</td>
</tr>
<tr>
<td>AL-C-left to</td>
<td>AL-D-right</td>
<td>94640.0</td>
<td>2100.0</td>
</tr>
<tr>
<td>AL-D-right</td>
<td>AL-E-left</td>
<td>94640.0</td>
<td>2100.0</td>
</tr>
<tr>
<td>AL-E-right</td>
<td>AL-F-left</td>
<td>102293.3</td>
<td>2660.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sub-structure III</th>
<th>Joints</th>
<th>Rotational Stiffness</th>
<th>Bending Moment Resistance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>R-SR [kNm/rad]</td>
<td>SR-P [kNm/rad]</td>
</tr>
<tr>
<td>AL-A-right</td>
<td>AL-B-right</td>
<td>238560.0</td>
<td>7056.0</td>
</tr>
<tr>
<td>AL-B-left</td>
<td>AL-B-right</td>
<td>238560.0</td>
<td>7056.0</td>
</tr>
</tbody>
</table>
Fig. 7.8 Partial strength joint mechanical behaviour

**Loading conditions**

The loading considered in each sub-structure was determined for each load combination and varies with the structural conception and building occupancy. The loads and load combinations were defined according to EN1990:2002 and EN1991-1-1:2002. Note that for Sub-structure I and II, the wind action was also considered while for Sub-structure 3 no lateral action was assumed, this action was not quantified in (Demonceau et al, 2012) and it was considered that the stiffness of the wall will absorb it. In the office building structure, the slab works in the transverse direction, therefore the beams in the Sub-structure II are loaded with uniform distributed loads. For the other two sub-structures, the represented beams are the main beams so the loads are transmitted as concentrated loads, at the intersection of the secondary beams. In all cases the self-weight is considered.

**Sub-structures finite element models**

The structural calculations were performed in the finite element software (Abaqus 6.11, 2011). In Tab. 7.9 are listed the types of elements used to reproduce each component of the structural system (members and joints): i) beam elements for beams and columns, ii) shell/plate elements for the RC walls, and iii) spring elements to connect the structural members, in the different degrees of freedom.

<table>
<thead>
<tr>
<th>Structural Model Component</th>
<th>Type of finite element</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beams and Columns</td>
<td>Beam element</td>
<td>2-node linear beam element B31</td>
</tr>
<tr>
<td>Shear Walls</td>
<td>Shell element</td>
<td>4-node shell element S4R (general-purpose) with reduce integration and hourglass control</td>
</tr>
<tr>
<td>Beam-to-column and Beam-to-Wall Joints</td>
<td>Spring element</td>
<td>Non-linear spring element with single degree of freedom</td>
</tr>
</tbody>
</table>

The concentrated joint modelling was selected, where a flexural spring was used to represent the connection at each side of the column. As the parametric study was performed varying the properties of this flexural spring, it was assumed that this spring was already integrating the deformation of the column web panel and was already affected by the transformation parameter $\beta$, so that an iterative calculation was avoid. As the main goal is to analyse the influence of the joint and to obtain some structural requirements to the steel-to-concrete joints, the joint springs are located at the axis of the columns, and the eccentricity associated to the height of this member is neglected. In what concerns the other degrees of freedom, namely axial and shear direction of the beam, translation springs are used to connect the members.
In this way, in each connection, between structural members, three springs are used, one for each relevant degree of freedom.

The use of the above described types of elements was previously validated against academic problems (Simoes da Silva et al, 2010). Simultaneously, the calibration of the required mesh refinement was performed. Tab. 7.10 summarizes the mesh refinement to consider in the different members of the structural models simulated and discussed in the next section.

<table>
<thead>
<tr>
<th>Member</th>
<th>Number of elements or mesh size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beams</td>
<td>40</td>
</tr>
<tr>
<td>Columns</td>
<td>10</td>
</tr>
<tr>
<td>Shear walls</td>
<td>400 mm x 400 mm</td>
</tr>
</tbody>
</table>

The performed numerical calculations are two dimensional; therefore, no out-of-plane deformations occur. Both material and geometrical non-linearities are taken into account. Furthermore, the analysis neglects any possible in-plane buckling phenomena. The structural capacity is in this way only limited by the maximum resistance of the members and joints cross-sections. Finally, in what concerns to the simulation of the column loss, the procedure consisted in replacing the support of the relevant column by the reactions forces obtained in a previous calculation during the application of the real loading, and then to reduce them to zero.

7.2.4 Analysis and discussion for Service Limit State

The structure under service limit state (SLS) has to guarantee the comfort of the users. If in terms of loading this limit state is less demanding, in terms of deformation requirements it is often the most limiting state, and therefore, design guiding. For this load condition, the analysis of the steel-to-concrete joint properties is performed using the two following parameters: beams deflection and structure lateral deformation. For the latter, only Substructures I and II are considered, as no horizontal load (namely wind load) was considered in the analysis of Sub-structure III.

Fig. 7.10 illustrates how the beams deflection was considered. The maximum values obtained for each case are listed in Table 7.11, in a beam connected to a RC member, columns in grey, and in a beam only supported by steel columns. According to the Portuguese National Annex to EN1993-1-1:2006 the limit value \( \delta_{\text{max}} = \frac{L}{300} \) was calculated and is included in the table. It is observed that in Sub-structures I and II, the values are distant from this limit, even if the beams deformation achieves 20 mm in the sub-structure II with simple joints, the value is still 33% below the limit. The beam deformations in sub-structure III are closer to the limit value but still, this value is not exceeded for any of the cases. In Fig. 7.11 are represented the beams deformations for the cases corresponding to the maximum and minimum deflections, for the beams implementing steel-to-concrete joints. These is seen as the envelope of the beams deformation, as these cases consider the two extreme situations in what respects the joint properties: i) continuous (Rigid + Full Strength); and ii) simple (Pinned). Using the beam deformation mode corresponding to the maximum beam deflection, the deformation corresponding to the code limit was extrapolated and is also included in the figure. The figure illustrates the above observations, confirming Substructure III closer to the limit.
Fig. 7.9  Representation of the considered beams deflection

Tab. 7.11 Maximum beams deformation under service limit state [mm]

<table>
<thead>
<tr>
<th>Case</th>
<th>Sub-structure I</th>
<th>Sub-structure II</th>
<th>Sub-structure III</th>
<th>Joint Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Beam 1-2</td>
<td>Beam 3-4</td>
<td>Beam C-D</td>
<td>Beam A-B</td>
</tr>
<tr>
<td>1</td>
<td>2.6</td>
<td>3.0</td>
<td>5.5</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>3.3</td>
<td>3.2</td>
<td>7.8</td>
<td>0.3</td>
</tr>
<tr>
<td>3</td>
<td>3.3</td>
<td>3.5</td>
<td>7.8</td>
<td>0.4</td>
</tr>
<tr>
<td>4</td>
<td>3.3</td>
<td>3.6</td>
<td>7.8</td>
<td>0.4</td>
</tr>
<tr>
<td>5</td>
<td>3.3</td>
<td>3.5</td>
<td>7.8</td>
<td>0.4</td>
</tr>
<tr>
<td>6</td>
<td>3.3</td>
<td>3.6</td>
<td>7.8</td>
<td>0.4</td>
</tr>
<tr>
<td>7</td>
<td>3.3</td>
<td>3.5</td>
<td>7.8</td>
<td>0.4</td>
</tr>
<tr>
<td>8</td>
<td>3.3</td>
<td>3.6</td>
<td>7.8</td>
<td>0.4</td>
</tr>
<tr>
<td>9</td>
<td>3.2</td>
<td>4.6</td>
<td>7.8</td>
<td>0.6</td>
</tr>
<tr>
<td>10</td>
<td>6.1</td>
<td>6.1</td>
<td>20.5</td>
<td>1.5</td>
</tr>
</tbody>
</table>

δ_{max} [mm] 20 20 30 15 33.3 33.3

R-Rigid; P-Pinned; FS-Full-strength

Fig. 7.10  Beam deformations envelop and limit according to PNA to EN1993-1-1:2006 supported by a steel-to-concrete joint

a) Sub-structure I b) Sub-structure II c) Sub-structure III
Besides the beams deformation, the lateral stiffness of the sub-structures is also affected by the joint properties. In Tab. 7.12 are listed the maximum top floor displacements obtained for each case and for Sub-structures I and II. The design limit \( d_{h,\text{top,limit}} \) according to Portuguese National Annex to EN1993-1-1:2006 is also included. As for the beams deflections, it is observed that the observed values are distant from the code limit. Note that as long as the joints are continuous or semi-continuous, the top floor displacement suffers small variations. This is due to the dominant contribution of the RC wall to the lateral stiffness of the sub-structures. In Fig. 7.11 are represented the sub-structures lateral displacement envelopes and the code limit. In Sub-structure II, because two RC walls contribute to the lateral stiffness of the sub-structure, the variation between minimum and maximum is quite reduced.

<table>
<thead>
<tr>
<th>Case</th>
<th>Sub-structure I</th>
<th>Sub-structure II</th>
<th>Joint Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>26.7</td>
<td>13.5</td>
<td>R</td>
</tr>
<tr>
<td>2</td>
<td>27.6</td>
<td>14.0</td>
<td>FS</td>
</tr>
<tr>
<td>3</td>
<td>28.3</td>
<td>14.1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>28.6</td>
<td>14.2</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>28.3</td>
<td>14.1</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>28.6</td>
<td>14.2</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>28.3</td>
<td>14.1</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>28.6</td>
<td>14.2</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>31.4</td>
<td>14.8</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>36.0</td>
<td>16.2</td>
<td></td>
</tr>
</tbody>
</table>

\( d_{h,\text{top,limit}} \) [mm] 94.3 94.3 P P

R-Rigid; P-Pinned; FS-Full-strength

In what concerns the steel-to-concrete joints, under service limit state, the bending moment developed in the joints and the required joint rotation are represented in Fig. 7.12. In Fig. 7.13 the ratio between the bending moment developed in the joints and the joint or beam bending moment capacity is represented. For none of the cases, the joints under SLS attained the maximum bending moment resistance of the joint. As for the deformations, Sub-structure III is the most demanding to the joints. In case 7, almost 70% of the joint bending moment capacity is activated. Because the assumed joint resistance is lower, in case 7 and 8 the percentage of bending moment activated is higher. In Fig. 7.13 is shown the maximum joint rotations observed for each sub-structure and for each case. For the cases where the joints are modelled as pinned, the joint rotation required is naturally higher, but never greater than 11 mrad. In the other cases, the joint rotation is quite low, below 3.2 mrad, which is expectable as not plastic deformation of the joints occurs.
7.2.5 Analysis and discussion for Ultimate Limit State

At Ultimate Limit State (ULS), joints should perform so that the structural integrity is not lost. This requires to the joints either resistance either deformation capacity, allowing the redistribution of internal forces within the structure. In order to quantify such structural demands to the steel-to-concrete joints, calculations considering the load combinations of this limit state are performed. In Fig. 7.14 are summarized the maximum loads obtained on these joints $M_j$, $N_j$, $V_j$. In all cases, hogging bending moment and the axial compression are reported. Though, it should be referred that axial tension is observed in bottom floors of the sub-structures; however, in average, the maximum value does not exceed 10 kN.
<table>
<thead>
<tr>
<th>Joint Location</th>
<th>Sub-structure I</th>
<th>Sub-structure II</th>
<th>Sub-structure III</th>
<th>Joint Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case</td>
<td>$M_i$ [kNm]</td>
<td>$N_j$ [kN]</td>
<td>$V_j$ [kN]</td>
<td>$M_i$ [kNm]</td>
</tr>
<tr>
<td>1</td>
<td>169.0</td>
<td>68.5</td>
<td>181.1</td>
<td>64.7</td>
</tr>
<tr>
<td>2</td>
<td>170.0</td>
<td>61.7</td>
<td>183.3</td>
<td>65</td>
</tr>
<tr>
<td>3</td>
<td>151.2</td>
<td>62.3</td>
<td>178.3</td>
<td>54.2</td>
</tr>
<tr>
<td>4</td>
<td>136.2</td>
<td>62.8</td>
<td>174.3</td>
<td>46.2</td>
</tr>
<tr>
<td>5</td>
<td>151.2</td>
<td>62.3</td>
<td>178.3</td>
<td>54.2</td>
</tr>
<tr>
<td>6</td>
<td>136.3</td>
<td>62.8</td>
<td>174.3</td>
<td>46.2</td>
</tr>
<tr>
<td>7</td>
<td>138.0</td>
<td>62.1</td>
<td>174.8</td>
<td>54.8</td>
</tr>
<tr>
<td>8</td>
<td>121.7</td>
<td>62.4</td>
<td>170.5</td>
<td>46.6</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>65.9</td>
<td>138.9</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>43.3</td>
<td>134.0</td>
<td>0</td>
</tr>
</tbody>
</table>

AL-Alignment; L – Left hand side; R - right hand side; R – Rigid; P – Pinned; FS – Full Strength

Fig. 7.14 shows the ratio between acting bending moment and the bending moment capacity of the steel-to-concrete joints or of the beams, in the case of full strength joints. As expected, for this limit state the ratio increases in comparison to the service limit state though, in none of the cases the full capacity of joints is activated. The higher ratios are observed in Sub-structures I and III, for the cases with lower bending moment resistance.

In Fig. 7.15 are plotted the maximum joint rotations observed in the different calculations. The maximum required joint rotation is approximately 20 mrad for the case studies where the steel-to-concrete joints are modelled as simple joints.
8 TOLERANCES

8.1 Standardized tolerances

The European standard EN1090-2:2008 describes the geometric tolerances in Chapter 11. Limits of geometric deviations contained therein are independent from the execution classes and they are divided into two types.

Essential tolerances called those which are necessary for the mechanical resistance and stability of the completed structure.

Functional tolerances have decisive influence on other criteria such as fit-up and appearance. The permitted limits of deviations are defined for two tolerance classes in generally. Stricter limits apply to class 2. Is not a class set, tolerance class 1 applies.

The essential tolerances, as well as the functional tolerances are normative.

With regard to the connections of steel structures in concrete components, essential tolerances are limited in Chapter 11.2.3.2 for foundation bolts and other supports and in Chapter 11.2.3.3 for column bases. There, with regard to their desired location, permissible deviations of a group of anchor bolts and instructions for the required hole clearance are specified for base plates.

More interesting for connections with embedded anchor plates in concrete structures are the functional tolerances given in Annex D Tab. 2.20, see Fig. 8.1.

The European standard EN13670:2011 Execution of concrete structures contains in Chapter 10 also information to geometrical tolerances, which are for buildings of importance, such as structural safety. Two tolerance classes are defined, in which in generally the tolerance class 1 (reduced requirements) applies. The application of the tolerance class 2 is intended mainly in connection with structural design according to EN1992-1-1:2004 Appendix A. Fig. 8.2 (Fig. 2 in EN13670:2011) provides limits of permissible deviations from the vertical of walls and pillars. Deviations of these components have decisive influence on the steel structures to be connected there (if required).
<table>
<thead>
<tr>
<th>No</th>
<th>Criterion</th>
<th>Parameter</th>
<th>Permitted deviation $\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Foundation level</td>
<td>Deviation $\Delta$ from specified level</td>
<td>$-15 \text{ mm} \leq \Delta \leq +5 \text{ mm}$</td>
</tr>
<tr>
<td>2</td>
<td>Vertical wall</td>
<td>Deviation $\Delta$ from specified position at support point for steel component</td>
<td>$\Delta = \pm 25 \text{ mm}$</td>
</tr>
<tr>
<td>3</td>
<td>Pre-set foundation bolt where prepared for adjustment</td>
<td>Deviation $\Delta$ from specified location and protrusion: - Location at tip - Vertical protrusion $\Delta_p$</td>
<td>$\Delta_y, \Delta_z = \pm 10 \text{ mm}$ $-5 \text{ mm} \leq \Delta_p \leq 25 \text{ mm}$</td>
</tr>
<tr>
<td>4</td>
<td>Pre-set foundation bolt where not prepared for adjustment</td>
<td>Deviation $\Delta$ from specified location, level and protrusion: - Location at tip - Vertical protrusion $\Delta_p$ - Vertical protrusion $\Delta_x$</td>
<td>$\Delta_y, \Delta_z = \pm 3 \text{ mm}$ $-5 \text{ mm} \leq \Delta_p \leq 45 \text{ mm}$ $-5 \text{ mm} \leq \Delta_x \leq 45 \text{ mm}$</td>
</tr>
<tr>
<td>5</td>
<td>Steel anchor plate embedded in concrete</td>
<td>Deviations $\Delta_x, \Delta_y, \Delta_z$ from the specified location and level</td>
<td>$\Delta_x, \Delta_y, \Delta_z = \pm 10 \text{ mm}$</td>
</tr>
</tbody>
</table>

Fig. 8.1 Functional tolerances – concrete foundations and support, Tab. D.2.20 in EN1090-2:2008
<table>
<thead>
<tr>
<th>No</th>
<th>Type of deviation</th>
<th>Description</th>
<th>Permitted deviation $\Delta$ Tolerance class 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td></td>
<td>Inclination of a column or wall at any lever in a single or a multi-storey building $h \leq 10$ m $h &gt; 10$ m</td>
<td>The larger of $15$ mm or $h/400$ $25$ mm or $h/800$</td>
</tr>
<tr>
<td>b</td>
<td></td>
<td>Deviation between centres</td>
<td>The larger of $t/300$ or $15$ mm But not more than $30$ mm</td>
</tr>
<tr>
<td>c</td>
<td></td>
<td>Curvature of a column or wall between adjacent storey levels</td>
<td>The larger of $t/30$ or $15$ mm But not more than $30$ mm</td>
</tr>
<tr>
<td>d</td>
<td>$\sum_h$=sum of height of storeys considered</td>
<td>Location of a column or a wall at any storey level, from a vertical line through its intended centre at base level in a multi-storey structure $n$ is the number of storeys where $n &gt; 1$</td>
<td>The smaller of $50$ mm or $\frac{\sum h_i}{(200n^3)}$</td>
</tr>
</tbody>
</table>

Fig. 8.2 Permissible deviations from the vertical of walls and pillars, abridged Fig. 2 in EN13670:2011

Geometric tolerances, which are in terms of suitability for use and the accuracy of fit for the building of importance, are regulated in the informative Annex G, unless regulated otherwise, the tolerances of Annex G apply, see Fig. 8.3. It is assumed that tolerances contained therein relate to geometrical sizes, which have only a limited influence on the bearing capacity. Fig. 8.1 shows the permissible deviations of built in components in all directions, compare EN1090-2:2008 D. 2.20 line 5.
An assessment of the impact of the tabular listed permissible limits on connections with embedded anchor plates will be in the next Chapter 8.2.

### 8.2 Recommended tolerances

For deviations from fixtures (anchoring) of the target location, relatively low values are allowed in the previously mentioned standards, ±10 mm in each direction, see EN1090-2:2008, or ± 20 mm in the plains and ± 10 mm perpendicular to the surface, see EN13670:2011. Tolerances for angular misalignments of the anchor plates to their installation levels are not available.

However, in EN 13670 Fig. 2d for multi-story buildings clearly greater deviations of the upper floors to the vertical are allowed. For example, the permissible horizontal displacement of the top-level of a floor from the target location is for a seven-story building with a floor height of 3.50 m.

\[
\sum h_i / (200 \ n^{1/2}) = 46 \text{ mm} \tag{8.1}
\]

If the building is made of prefabricated concrete elements, the concrete anchor plate - even with exact location within the individual component - may exhibit the same displacement from the target location as the above shown deviations.

Therefore, the deviations defined directly for anchor plates by ± 10 mm seem to be hardly feasible. Much larger deviations have to be expected. If necessary, special tolerances for the location of the built in components have to be defined. EN13670:2011 describes another principle of tolerance definition, in which the allowable deviation of any point of construction compared to a theoretical target location over a fixed value is defined in Chapter 10.1 cl 5. A recommendation for the maximum permissible deviation is ± 20 mm.

Definitely, connecting elements between steel and concrete structures must be able to compensate tolerances. Considering the previous explanations, a development of joints for taking deviations of the anchor plate from the theoretical target location of ± 20 to 25 mm is recommended. Fig. 8.4 and 8.5 show exemplary a connections with and without the possibility to compensate geometrical derivations.
The following methods is used to compensate certain displacements of build in components to the target location. Depending on the loading, priority direction of the loads, the most appropriate solution has to be chosen.

**Tolerance absorption in the longitudinal direction of the profile:**
- Bolted connection with end plate and scheduled filler plates
- Bolted connection with base plate and grouting
- Cleat / console
- Beam / pillar scheduled with overlength; adjustment and welding on site
- Buttstrap with overlength; adjustment and welding on site
- Buttstrap with slot holes

**Tolerance absorption perpendicular to the longitudinal direction of profile:**
- Additional steel plate with threaded bolts; welded on site; beam / pillar with end plate
- Anchor plate with threaded bolts; head plate with oversized holes
- Buttstrap; welding on site
9 WORKED EXAMPLES

9.1 Pinned base plate

Check the resistance of the column base. The column of HE 200 B section, a concrete foundation size 850 x 850 x 900 mm, a base plate thickness 18 mm, steel S 235 and concrete C 12/15, $\gamma_{Mc} = 1.50$, $\gamma_{M0} = 1.00$.

Fig. 9.1 Designed column base

Step 1 Concrete design strength

The stress concentration factor should be calculated, see Chap. 3.3. The minimum values for $a_1$ (or $b_1$) are taken into account

$$a_1 = b_1 = \min \left\{ \begin{array}{l} a + 2a_r = 340 + 2 \cdot 255 = 850 \\ 3a = 3 \cdot 340 = 1020 \\ a + h = 340 + 900 = 1240 \end{array} \right\} = 850 \text{ mm}$$

The condition $a_1 = b_1 = 850 > a = 340 \text{ mm}$ is satisfied, and therefore

$$k_j = \frac{a_1 \cdot b_1}{a \cdot b} = \frac{850 \cdot 850}{340 \cdot 340} = 2.5$$

The concrete design strength is calculated from the equation

$$f_{jd} = \frac{\beta_j F_{Rd, u}}{b_{eff} f_{lef}} = \frac{\beta_j A_{c0} f_{cd} \sqrt{A_{c1}}}{A_{c0}} = \beta_j f_{cd} k_j = 0.67 \cdot \frac{12.0}{1.5} \cdot 2.5 = 13.4 \text{ MPa}$$

Step 2 Flexible base plate

The flexible base plate is replaced by a rigid plate, see the following picture Fig. 9.2.

The strip width is

$$c = t \sqrt{\frac{f_y}{3 \cdot f_{jd} \cdot \gamma_{M0}}} = 18 \cdot \sqrt{\frac{235}{3 \cdot 13.4 \cdot 1.00}} = 43.5 \text{ mm}$$
Fig. 9.2 Effective area under the base plate

The effective area of the base plate of H shape is calculated as a rectangular area minus the central areas without contact such that;

$$A_{\text{eff}} = \min(b; b_c + 2c) \cdot \min(a; h_{e\text{ff}} + 2c) - \max[\min(b; b_c + 2c) - t_w - 2c; 0] \cdot \max(h_c - 2t_f - 2c; 0)$$

$$A_{\text{eff}} = (200 + 2 \cdot 43.5) \cdot (200 + 2 \cdot 43.5)$$
$$- (200 + 2 \cdot 43.5 - 9 - 2 \cdot 43.5) \cdot (200 - 2 \cdot 15 - 2 \cdot 43.5)$$

$$A_{\text{eff}} = 82369 - 15853 = 66516 \text{ mm}^2$$

**Step 3 Design resistance**

The design resistance of the column base in compression is

$$N_{\text{Rd}} = A_{\text{eff}} \cdot f_{\text{id}} = 66516 \cdot 13.4 = 891 \cdot 10^3 \text{N}$$

**Comments**

The design resistance of the column footing is higher than the resistance of the column base

$$N_{\text{pl,Rd}} = \frac{A_c \cdot f_{\gamma}}{\gamma_{M0}} = \frac{7808 \cdot 235}{1.00} = 1835 \cdot 10^3 \text{N} > N_{\text{Rd}}$$

where $A_c$ is area of the column. The column base is usually designed for column resistance, which is determined by column buckling resistance.

It is expected, that the grout will not affect the column base resistance. The grout has to be of better quality or the thickness has to be smaller than

$$0.2 \min(a; b) = 0.2 \cdot 340 = 68 \text{ mm}$$

The steel packing or the levelling nuts is placed under the base plate during the erection. It is recommended to include the packing/nuts in the documentation.
9.2 Moment resistant base plate

In the following example the calculation of the moment resistance and the bending stiffness of the column base at Fig. 9.3 is shown. The Column HE 200 B is loaded by a normal force $F_{sd} = 500$ kN. The concrete block C25/30 of size $1600 \times 1600 \times 1000$ mm is designed for particular soil conditions. The base plate is of $30$ mm thickness and the steel strength is S235. Safety factors are considered as $\gamma_M = 1.50; \gamma_M = 1.15, \gamma_M = 1.00$; and $\gamma_M = 1.25$. The connection between the base plate and the concrete is carried out through four headed studs of $22$ mm diameter and an effective embedment depth of $150$ mm, see Fig. 9.3. The diameter of the head of the stud is $40$ mm. The supplementary reinforcement for each headed stud consists of two legged $12$ mm diameter stirrups on each side of the stud. Consider $f_{uk} = 470$ MPa for studs and design yield strength of the supplementary reinforcement as $f_{yd, re} = \frac{f_{uk, re}}{\gamma_{Ms}} = \frac{500}{1.15} = 435$ MPa.

\[ a = 420 \quad a_f = 590 \]
\[ e = 50 \quad b_f = 590 \]
\[ b = 420 \quad p = 240 \]
\[ e_c = 60 \quad r_b = 160 \]

Fig. 9.3 Designed column base

Step 1 Base plate resistance

1.1 Component base plate in bending and headed studs in tension

Lever arm, for fillet weld $a_{wf} = 6$ mm is

\[ m = 60 - 0.8 \cdot a_{wf} \cdot \sqrt{2} = 60 - 0.8 \cdot 6 \cdot \sqrt{2} = 53.2 \text{ mm} \]

The minimum T-stub length in base plates where the prying forces not taken into account, is

\[ l_{eff,1} = \min \begin{cases} 4 m + 1.25 e_a = 4 \cdot 53.2 + 1.25 \cdot 50 = 275.3 \\ 2 \pi m = 2 \pi \cdot 53.2 = 334.3 \\ b \cdot 0.5 = 420 \cdot 0.5 = 210 \end{cases} \]

\[ l_{eff,1} = 210 \text{ mm} \]

The effective length of headed studs $L_b$ is taken as
The resistance of T-stub with two headed studs is

\[ F_{T,1-2,Rd} = \frac{2 L_{\text{eff},1} t^2 f_y}{4 m \gamma_{M0}} = \frac{2 \cdot 210 \cdot 30^2 \cdot 235}{4 \cdot 532 \cdot 1.00} = 417.4 \text{kN} \]

The resistance is limited by tension resistance of two headed studs M 22, the area in tension \( A_s = 303 \text{ mm}^2 \).

\[ F_{T,2,Rd} = 2 \cdot B_{t,Rd} = 2 \cdot \frac{0.9 \cdot f_{\text{uk}} \cdot A_s}{\gamma_{M2}} = 2 \cdot \frac{0.9 \cdot 470 \cdot 303}{1.25} = 205.1 \text{kN} \]

### 1.2 Component base plate in bending and concrete block in compression

To evaluate the compressed part resistance is calculated the connection factor as

\[ a_1 = b_1 = \min \left\{ \begin{array}{l} a + 2 a_r = 420 + 2 \cdot 590 = 1600 \\ 3a = 3 \cdot 420 = 1260 \\ a + h = 420 + 1000 = 1420 \end{array} \right\} = 1260 \text{ mm} \]

and \( a_1 = b_1 = 1260 > a = b = 420 \text{ mm} \)

The above condition is fulfilled and

\[ k_j = \sqrt{\frac{a_1 \cdot b_1}{a \cdot b}} = \sqrt{\frac{1260 \cdot 1260}{420 \cdot 420}} = 3.00 \]

The grout is not influencing the concrete bearing resistance because

\[ 0.2 \cdot \min(a; b) = 0.2 \cdot \min(420; 420) = 84 \text{ mm} > 30 \text{ mm} = t_g \]

The concrete bearing resistance is calculated as

\[ f_{jd} = \frac{2}{3} \cdot \frac{k_j \cdot f_{ck}}{\gamma_{Mc}} = \frac{2}{3} \cdot \frac{3.00 \cdot 25}{1.5} = 33.3 \text{ MPa} \]

From the force equilibrium in the vertical direction \( F_{Sd} = A_{\text{eff}} \cdot f_{jd} - F_{t,Rd} \), the area of concrete in compression \( A_{\text{eff}} \) in case of the full resistance of tension part is calculated

\[ A_{\text{eff}} = \frac{F_{Sd} + F_{Rd,3}}{f_{jd}} = \frac{500 \cdot 10^3 + 205.1 \cdot 10^3}{33.3} = 21174 \text{ mm}^2 \]

The flexible base plate is transferred into a rigid plate of equivalent area. The width of the strip \( c \) around the column cross section, see Fig. 9.4, is calculated from

\[ c = t \cdot \sqrt{\frac{f_y}{3 \cdot f_{jd} \cdot \gamma_{M0}}} = 30 \cdot \sqrt{\frac{235}{3 \cdot 33.3 \cdot 1.00}} = 46.0 \text{ mm} \]
1.3 Assembly for resistance

The active effective width is calculated as

\[
b_{\text{eff}} = \frac{A_{\text{eff}}}{b_c + 2c} = \frac{21174}{200 + 2 \cdot 46.0} = 72.5 \text{ mm} < t_r + 2c = 15 + 2 \cdot 46.0 = 107.0 \text{ mm}
\]

The moment resistance of the column base is

\[
M_{\text{Rd}} = F_{\text{T,3,Rd}} \cdot r_t + A_{\text{eff}} \cdot f_{\text{yd}} \cdot r_c
\]

\[
M_{\text{Rd}} = 205.1 \cdot 10^3 \cdot 160 + 21174 \cdot 33.3 \cdot 109.8 = 110.2 \text{ kNm}
\]

Under acting normal force \(N_{\text{sd}} = 500 \text{ kN}\) the moment resistance in bending is \(M_{\text{Rd}} = 110.2 \text{ kNm}\).

1.4 The end of column resistance

The design resistance in poor compression is

\[
N_{\text{pl,Rd}} = \frac{A \cdot f_y}{\gamma_{\text{M0}}} = \frac{7808 \cdot 235}{1.00} = 1835 \cdot 10^3 \text{ N} > N_{\text{Rd}} = 500 \text{ kN}
\]

The column bending resistance

\[
M_{\text{pl,Rd}} = \frac{W_{\text{pl}} \cdot f_{\text{yk}}}{\gamma_{\text{M0}}} = \frac{642.5 \cdot 10^3 \cdot 235}{1.00} = 151.0 \text{ kNm}
\]

The interaction of normal force reduces moment resistance

\[
M_{\text{Ny,Rd}} = M_{\text{pl,Rd}} \left(1 - \frac{N_{\text{sd}}}{N_{\text{pl,Rd}}}\right) = 151.0 \cdot \left(1 - \frac{500}{1835}\right) = 124.2 \text{ kNm}
\]

The column base is designed on acting force only (not for column resistance).
**Step 2 Base plate stiffness**

**2.1 Component base plate in bending and headed stud in tension**

The component stiffness coefficients for headed studs and base plate are calculated

\[
k_b = 2.0 \cdot \frac{A_t}{I_b} = 2.0 \cdot \frac{303}{219.5} = 2.8 \text{ mm}
\]

\[
k_p = \frac{0.425 \cdot I_{\text{eff}}}{m^3} = \frac{0.425 \cdot 210 \cdot 30^3}{53.2^3} = 16.0 \text{ mm}
\]

![Fig. 9.5 The T stub in compression](image)

**2.2 Component base plate in bending and concrete block in compression**

For the stiffness coefficient the T-stub in compression, see Fig. 9.5, is

\[
a_{eq} = t_f + 2.5 \cdot t = 15 + 2.5 \cdot 30 = 90 \text{ mm}
\]

\[
k_c = \frac{E_c}{1.275 \cdot E_s} \cdot \sqrt{a_{eq} \cdot b_c} = \frac{31000}{1.275 \cdot 210000} \cdot \sqrt{90 \cdot 200} = 15.5 \text{ mm}
\]

**2.3 Assembly of component tensile stiffness coefficient to base plate stiffness**

The lever arm of component in tension \(z_t\) and in compression \(z_c\) to the column base neutral axes is

\[
z_t = \frac{h_c}{2} + e_c = \frac{200}{2} + 60 = 160 \text{ mm}
\]

\[
z_c = \frac{h_c}{2} - \frac{t_f}{2} = \frac{200}{2} - \frac{15}{2} = 92.5 \text{ mm}
\]

The stiffness coefficient of tension part, headed studs and T stub, is calculated as

\[
k_t = \frac{1}{\frac{1}{k_b} + \frac{1}{k_p}} = \frac{1}{2.8 + 16.0} = 2.4 \text{ mm}
\]

For the calculation of the initial stiffness of the column base the lever arm is evaluated

\[
z = z_t + z_c = 160 + 92.5 = 252.5 \text{ mm}
\]

\[
a = \frac{k_c \cdot z_c - k_t \cdot z_t}{k_c + k_t} = \frac{15.5 \cdot 92.5 - 2.4 \cdot 160}{15.5 + 2.4} = 58.6 \text{ mm}
\]

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EN1993-1-8 cl 6.3

EN1993-1-1 cl 6.2.9

EN1993-1-8 cl 6.3
The bending stiffness is calculated for particular constant eccentricity

\[ e = \frac{M_{Rd}}{F_{sd}} = \frac{110.2 \cdot 10^6}{500 \cdot 10^3} = 220.4 \text{ mm} \]

as

\[ S_{j,ini} = e + a \cdot \frac{E_S \cdot z^2}{\mu \sum \frac{1}{k_i}} = 220.4 + 58.6 \cdot \frac{210 \, 000 \cdot 252.5^2}{1 \cdot \left( \frac{1}{2.4} + \frac{1}{15.5} \right)} = 21.981 \cdot 10^9 \text{ Nmm/rad} \]

\[ = 21,981 \text{ kNm/rad} \]

**Step 3 Anchorage resistance and stiffness**

As discussed in Chapter 3 Concrete components, the stiffness of anchorage is determined for separate components, failure modes, and then combined together. In this case, the anchorage is considered as a group of four headed studs with nominal stud diameter of 22 mm arranged in a way displayed in Fig. 9.6. Furthermore, supplementary reinforcement with the arrangement shown in Fig. 9.6 is considered.

Due to moment loading on the anchor group generated by the lateral loads only one side studs will be subjected to tension loads. Therefore in the following example, two studs are considered while evaluating the behaviour of the anchor group. Here, diameter of the reinforcing bar is considered as 12 mm and the effective embedment depth of the stud is considered as 150 mm, distance from of the head to the concrete surface.

![Fig. 9.6 Headed studs and supplementary reinforcement configuration](image)

**3.1 Component S – Steel failure of headed stud**

Component S comprises of evaluating the design load-displacement response of the headed studs under tensile loading, when they undergo steel failure. Only two anchors will be considered in tension. From Eq. (3.3) and (3.4) is calculated the load and the stiffness as
\[ N_{ Rd,s } = \frac{n \cdot \pi \cdot d_{ s,\text{nom}}^2 \cdot f_{ uk} }{4 \cdot Y_{mp}} = \frac{2 \cdot \pi \cdot 22^2 \cdot 470}{4 \cdot 1.5} = 238 \text{ 216 N} = 238.2 \text{ kN} \]  

\[ k_{ s1 } = \frac{A_{s,\text{nom}} \cdot E_{s}}{L_{h}} = \frac{n \cdot \pi \cdot d_{ s,\text{nom}}^2 \cdot E_{s}}{4 \cdot L_{h}} = \frac{2 \cdot \pi \cdot 22^2 \cdot 210 \text{ 000}}{4 \cdot 150} = \]  

\[ = 1 \text{ 064 371 N/} \text{mm} = 1 \text{ 064.4 kN/mm} \text{ for } N_{\text{act}} < 238.2 \text{ kN} \]

\[ k_{ s2 } = 0; \text{ } N_{\text{act}} = 238.2 \text{ kN} \]

Hence, the load-displacement curve for the spring is obtained as shown in Fig. 9.7.

![Fig. 9.7 Load-displacement behaviour of spring representing component S](image)

3.2 Component C – Concrete cone failure

Component CC comprises of evaluating the design load-displacement response of the headed studs under tensile loading, when they undergo concrete cone failure. The critical edge distance \( c_{cr,N} = 1.5 \text{ h}_{ef} = 225 \text{ mm} \). Using Eqs (3.7) through (3.9), it is

\[ N_{ Rd } = N_{ Rk,c } \cdot \psi_{ A,N \cdot \psi_{ s,N \cdot \psi_{ re,N } } } / Y_{Mc} \]  

\[ N_{ Rk,c } = k_{n} \cdot h_{ef}^{1.5} \cdot f_{ck}^{0.5} = 12.7 \cdot 150^{1.5} \cdot 25^{0.5} = 116.7 \text{ kN} \]  

\[ \psi_{A,N} = \frac{A_{c,N}}{A_{c,N}^0} = (1.5 \cdot 150^2 + 240 + 1.5 \cdot 150) \cdot (1.5 \cdot 150 + 1.5 \cdot 150) = 690 \cdot 450 \]  

\[ = 9 \cdot 150^2 = 1.53 \]  

Since maximum edge distance, \( c < c_{cr,N} = 225 \text{ mm} \), hence \( \psi_{ s,N } = 1.0 \)

There is no closely spaced reinforcement, hence, \( \psi_{ re,N } = 1.0 \)

Therefore, \( N_{ Rd,c } = 116.7 \cdot 1.53 \cdot 1.0 \cdot \frac{1.0}{1.5} = 119.0 \text{ kN} \)

The stiffness of the descending branch \( k_{c,de} \) for the design is described with the following function

\[ k_{c,de} = \alpha_c \sqrt{f_{ck} h_{ef} \cdot \psi_{A,N \cdot \psi_{ s,N \cdot \psi_{ re,N } } } = -537 \sqrt{25 \cdot 150 \cdot 1.53 \cdot 1.0 \cdot 1.0 = -50.31 \text{ kN/mm} } \]

The displacement corresponding to zero load is \( \frac{119.0}{50.31} = 2.37 \text{ mm} \)
The load-displacement curve for the spring is shown in Fig. 9.9.

3.3 Component RS – Steel failure of stirrups

Component RS comprises of evaluating the design load-displacement response of the stirrups, when they undergo steel failure. The design load for yielding of the stirrups is given as Eq. (3.17)

\[ N_{\text{Rd,sl}} = A_{s,\text{sl}} \cdot f_{yd,\text{sl}} = n_{\text{re}} \cdot \pi \cdot \left(\frac{d_{s,\text{re}}^2}{4}\right) \cdot f_{yd,\text{sl}} \]

For each stud, two stirrups with two legs on either side of the headed stud are provided. Therefore, for two headed studs in tension, the total number of the legs of the stirrups in tension is 8. Hence,

\[ N_{\text{Rd,sl}} = 8 \cdot \left(\frac{\pi}{4} \cdot 12^2\right) \cdot 435 = 393.6 \text{ kN} \]

\[ \delta_{\text{Rd,sl}} = \frac{2 \cdot N_{\text{Rd,sl}}}{\alpha_s \cdot f_{ck} \cdot d_{s,\text{re}}^2 \cdot n_{\text{re}}^2} = \frac{2 \cdot 393578^2}{12 \cdot 100 \cdot 25 \cdot 12^4 \cdot 8^2} = 0.77 \text{ mm} \]

Stiffness as a function of displacement is given as Eq. (3.18)
The load-displacement curve for the spring is shown in Fig. 9.10

Fig. 9.10 Load-displacement behaviour of spring representing component RS

3.4 Component RB – Bond failure of stirrups

Component RB comprises of evaluating the design load-displacement response of the stirrups under tensile loading, when they undergo bond failure. The design anchorage capacity of the stirrups is given by Eq. (3.21). Assuming a cover of 25 mm to stirrups and considering the distance between the stud and the stirrup as 50 mm, $l_1$ is calculated as CEN/TC1992-4-1:2009

\[ l_1 = 150 - 25 - 0.7 \cdot 50 = 90 \text{ mm} \]

Considering $f_{bd}$ for C25/30 grade concrete is $2.25 \cdot 1.8 \cdot 1.0 \cdot 1.0 = 2.7 \text{ N/mm}^2$, see Eq (8.2) cl 8.4.2.(2) in EN1992:2004, it is

\[ N_{Rd,b,re} = \sum n_{re} \cdot l_1 \cdot \pi \cdot d_{s,re} \cdot \frac{f_{bd}}{\alpha} = 8 \cdot 90 \cdot \pi \cdot 12 \cdot \frac{2.7}{0.49} = 149565 \text{ N} = 149.6 \text{ kN} \]

The corresponding displacement is obtained using Eq. (3.20) as

\[ \delta_{Rd,b,re} = \frac{2 \cdot N_{Rd,b,re}^2}{\alpha_s \cdot f_{ck} \cdot d_{s,ref} \cdot n_{ref}^2} = \frac{2 \cdot 149565^2}{12100 \cdot 25 \cdot 12^4 \cdot 8} = 0.11 \text{ mm} \]

It may be noted that since $N_{Rd,b,re} < N_{Rd,s,re}$, bond failure of stirrups is the governing failure mode for the stirrups.

Stiffness as a function of displacement is given as

\[ k_{s,rel} = \frac{n_{ref}^2 \cdot \alpha_s \cdot f_{ck} \cdot d_{s,ref}^4}{\sqrt{2} \cdot \delta} = \frac{\sqrt{8^2 \cdot 12100 \cdot 25 \cdot 12^4 \cdot \delta}}{\sqrt{2} \cdot \delta} = 448023 \frac{\sqrt{\delta}}{N/mm} \]

for $\delta < \delta_{Rd,b,re}$

\[ k_{s,re2} = 0 \text{ for } \delta \geq \delta_{Rd,s,re} \]
The load-displacement curve for the spring is shown in Fig. 9.11.

![Load-displacement curve](image)

**Fig. 9.11 Load-displacement relation of spring representing component RB**

### 3.5 Component P - Pull out failure of the headed stud

For the first range, \( N > N_{Rd,c} \) using Eqs (3.26) through (3.30), it is

\[
k_p = \frac{a_p}{k_2} \cdot \frac{k_A}{k_2}
\]

\[
a = 0.5 \cdot (d_h - d_s) = 0.5 \cdot (40 - 22) = 9 \text{ mm}
\]

\[
k_a = \sqrt[5]{\frac{5}{a}} \geq 1.0; \text{ hence, } k_a = 1.0
\]

\[
k_A = \frac{0.5}{\sqrt{d_s^2 + m \cdot (d_h^2 - d_s^2)}} - 0.5 \cdot d_h = 0.5 \cdot \sqrt{22^2 + 9 \cdot (40^2 - 22^2)} - 0.5 \cdot 40 = 31.30
\]

\[
k_2 = 600 (\text{assuming uncracked concrete})
\]

\[
k_p = \frac{a_p}{k_2} \cdot \frac{k_A}{k_2} = 0.25 \cdot \frac{1.0 \cdot 31.30}{600} = 0.0130
\]

Thus, using Eq. (3.24), it is

\[
\delta_{Rd,p,1} = k_p \cdot \left( \frac{N_{Rd,c}}{A_h \cdot f_{ck} \cdot n} \right)^2 = 0.0130 \cdot \left( \frac{119.0 \cdot 10^3}{\pi / 4 \cdot (40^2 - 22^2) \cdot 25 \cdot 2} \right)^2 = 0.096 \text{ mm}
\]

In second range, using Eq. (3.25), it is

\[
\delta_{Rd,p,2} = 2k_p \cdot \left( \frac{\min \left( N_{Rd,p} ; N_{Rd,se} \right)}{A_h \cdot f_{ck} \cdot n} \right)^2 - \delta_{Rd,p,1}
\]

Eq. (3.31) yields

\[
N_{Rd,p} = n \cdot p_{uk} \cdot A_h / \gamma_{Mc}
\]

\[
N_{Rd,se} = \min (N_{Rd,se} ; N_{Rd,se}) = \min (393.6 ; 149.6) = 149.6 \text{ kN}
\]

The typical value of \( p_{uk} \) is considered as \( 12 \cdot f_{ck} = 12 \cdot 25 = 300 \text{ MPa} \). Hence, it is

\[
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\]
The stiffness as a function of displacement is obtained using equations (3.34) and (3.35) as:

\[
\begin{align*}
N_{\text{Rd,p}} &= 2 \cdot 300 \cdot \frac{\pi}{4} \cdot \frac{(40^2 - 22^2)}{1.5} = 350.6 \text{ kN} \\
\delta_{\text{Rd,p,2}} &= 2 \cdot 0.0130 \cdot \left( \frac{149.565}{\pi \cdot (40^2 - 22^2) \cdot 25 \cdot 2} \right)^2 - 0.096 = 0.21 \text{ mm}
\end{align*}
\]

The load-displacement curve for the spring is shown in Fig. 9.12.

![Fig. 9.12 Load-displacement behaviour of spring representing component P](image)

3.6 Interaction of components Concrete and Stirrups

Once the concrete breakout occurs, the load is transferred to the stirrups and the load shared by concrete decreases with increasing displacement. The load carried by the combined component concrete + stirrups corresponding to any given displacement is given by Eq. (3.59) as

\[
N_{\text{act}} = N_{\text{Rd,c}} + k_{c,de} \delta + \min(n_{re} d_{s,\text{re}}^2 \sqrt{\frac{\alpha_s f_{ck} \delta}{2}}; N_{\text{Rd,s,re}}; N_{\text{Rd,b,re}})
\]

Hence, for a given displacement \(\delta\) [mm] the load [kN] carried by combined concrete and stirrups is given as

\[
N_{\text{act}} = 119.0 - 50.31 \cdot \delta + \min(448.023 \sqrt{\delta}; 393.6; 149.6)
\]
The load-displacement curve for the spring is shown in Fig. 9.13.

Fig. 9.13 Load-displacement behaviour of spring representing combined component concrete and stirrups

Interaction of all components:

The combined load-displacement behaviour combining all the components is displayed in Figure 9.14

Fig. 9.144 Load-displacement behaviour obtained by combining all the components

Notes

- The resistance of the anchoring by the headed studs is limited by its threaded part, which represents a ductile behaviour.
- The resistance of the base plate is limited by the tension resistance of two headed studs M 22, 205.1 kN. Under the serviceability limit state SLS is required resistance of the concrete cone, 119.0 kN. The elastic behaviour is expected till the 2/3 of the bending resistance of the base plate, which comply, \( \frac{2}{3} \cdot 417.4 = 314.3 \) kN.
Fig. 9.155a The column base resistance is compared to the column resistance for different base plate thickness.

- The column base resistance is compared to the column resistance for different base plate thickness, see Fig. 9.15a. For plate P 30 are shown the major points of the diagram, e.g. the pure compression, the highest bending resistance, in case of coincidence of the neutral axis and the axis of symmetry of cross-section, the pure bending and the pure tension.
- A conservative simplification may be applied by placing the concrete reaction on the axes of the compressed flange only see Fig. 9.15b. This model is uneconomical and not often used for prediction of resistance, but applied for stiffness prediction.

Fig. 9.16b The column base resistance calculated by the simplified prediction, the contact force under the compressed flange only, is compared to the application of the of the full contact area.

- The stiffness of the anchoring by the headed studs corresponds to the expected stiffness calculated by a simplified conservative method based on the embedded effective length. The component stiffness coefficients for headed studs is estimated.
as
\[ k_b = 2.0 \cdot \frac{A_s}{k_b} = 2.0 \cdot \frac{A_s}{\text{min}(h_{\text{eff}}, b)} = 2.0 \cdot \frac{303}{150} = 4.04 \text{ mm} \]

and the deformation for acting force 300 kN is
\[ \delta_{300} = \frac{F_{\text{Ed}}}{E \cdot k_b} = \frac{300}{210 \cdot 10^3 \cdot 4.04} = 0.35 \text{ mm}. \]

For headed stud is predicted, see Fig. 9.13, more precise value reached 0.22 mm.

- The classification of the column base according to its bending stiffness is evaluated in comparison to column bending stiffness. For column length \( L_c = 4 \text{ m} \) and its cross-section HE 200 B is relative bending stiffness
\[ \bar{S}_{j,\text{ini}} = S_{j,\text{ini}} \cdot \frac{L_c}{E_s \cdot I_c} = 21.981 \cdot 10^9 \cdot \frac{4000}{210 \cdot 10^6 \cdot 56.96 \cdot 10^6} = 7.53 \]  

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cl 6.3

The designed column base is sway for braced as well as non-sway frames because
\[ \bar{S}_{j,\text{ini}} = 7.53 < 12 = \bar{S}_{j,\text{ini},\text{EC2,n}}; \bar{S}_{j,\text{ini}} = 7.53 < 30 = \bar{S}_{j,\text{ini},\text{EC3,s}} \]

- The influence of tolerances and size of welds, see EN 1090-2 and Chapter 8, is not covered in above calculation.
9.3 Stiffened base plate

Calculate the moment resistance of the column base shown in Fig. 9.17. Column HE 200 B is loaded by normal force $F_{sd} = 1100$ kN. Concrete block C16/20 of size 1600 x 1600 x 1000 mm is design for particular soil conditions. Base plate of thickness 30 mm, steel S235, $\gamma_{Mc} = 1.50$; $\gamma_{M0} = 1.00$; and $\gamma_{M2} = 1.25$.

![Fig. 9.17 Designed column base](image)

**Step 1 Component in tension**

Resistance of component base plate in bending and headed studs in tension. Anchor stud lever arm, for fillet weld $a_{wT} = 6$ mm is

$$m = 70 - 0.8 \cdot a_{wT} \cdot \sqrt{2} = 70 - 0.8 \cdot 6 \cdot \sqrt{2} = 63.2 \text{ mm}$$

The T-stub length, in base plates are the prying forces not taken into account, is:

$$l_{eff,1} = \min \left\{ \begin{array}{l}
4 m + 1.25 e_1 = 4 \cdot 63.2 + 1.25 \cdot 110 = 390.3 \\
2 \pi m = 2 \pi \cdot 63.2 = 397.1 \\
b = 0.5 \cdot 320 \cdot 0.5 = 160 \\
2 m + 0.625 e_1 + 0.5 w = 2 \cdot 63.2 + 0.625 \cdot 110 + 0.5 \cdot 132 = 261.2 \\
2 m + 0.625 e_1 + e_2 = 2 \cdot 63.2 + 0.625 \cdot 110 + 0.94 = 289.2 \\
2 \pi m + 4 e_2 = 2 \pi \cdot 63.2 + 4 \cdot 94 = 773.1 \\
2 \pi m + 2 w = 2 \pi \cdot 63.2 + 2 \cdot 132 = 661.1
\end{array} \right\}$$

$$l_{eff,1} = 160 \text{ mm}$$

The effective length of headed studs $L_b$ is taken as

$$L_b = 8 \cdot d + t_g + t + \frac{t_n}{2} = 8 \cdot 24 + 30 + 30 + \frac{19}{2} = 261.5 \text{ mm}$$

The resistance of T-stub with two headed studs is
The resistance is limited by tension resistance of two headed studs M 24 with the area in tension $A_s = 353$ mm

$$F_{T,Rd} = 2 \cdot B_{t,Rd} = 2 \cdot \frac{0.9 \cdot f_{ub} \cdot A_s}{Y_{M2}} = 2 \cdot \frac{0.9 \cdot 360 \cdot 353}{1.25} = 183.0\text{ kN}$$

\textbf{Step 2 Component in compression}

The connection concentration factor is calculated as

$$a_1 = \min \left\{ \begin{array}{c} a_1 + 2 a_r = 560 + 2 \cdot 520 = 1 600 \\ 3 a_1 = 3 \cdot 560 = 1 680 \\ a_1 + h = 560 + 1 000 = 1 560 \end{array} \right\} = 1 560\text{ mm}$$

$$b_1 = \min \left\{ \begin{array}{c} b_1 + 2 b_r = 320 + 2 \cdot 640 = 1 600 \\ 3 b_1 = 3 \cdot 320 = 960 \\ b_1 + h = 320 + 1 000 = 1 320 \end{array} \right\} = 960\text{ mm}$$

and $a_1 = 1560 > a_1 = 560\text{ mm} \quad b_1 = 960 > b_1 = 320 a_1$

The above condition is fulfilled and

$$k_j = \frac{a_1 \cdot b_1}{a \cdot b} = \frac{1 560 \cdot 960}{560 \cdot 320} = 2.89$$

The grout is not influencing the concrete bearing resistance because

$$0.2 \cdot \min (a; b) = 0.2 \cdot \min (560; 320) = 64\text{ mm} > 30\text{ mm} = t_g$$

The concrete bearing resistance is calculated as

$$f_{ld} = \frac{2}{3} \cdot \frac{k_j \cdot f_{ck}}{Y_{Mc}} = \frac{2}{3} \cdot \frac{2.89 \cdot 16}{1.5} = 20.6\text{ MPa}$$

From the force equilibrium in the vertical direction $F_{Sd} = A_{eff} \cdot f_{ld} - F_{t,Rd}$, is calculated the area of concrete in compression $A_{eff}$ in case of the full resistance of tension part.

$$A_{eff} = \frac{F_{Sd} + F_{Rd,3}}{f_{ld}} = \frac{1 100 \cdot 10^3 + 183 \cdot 10^3}{20.6} = 62 282\text{ mm}^2$$

The flexible base plate is transferred into a rigid plate of equivalent area. The width of the strip $c$ around the column cross section, see Fig. 9.18, is calculated from

$$c = t \cdot \sqrt{\frac{f_y}{3 \cdot f_{ld} \cdot Y_{M0}}} = 30 \cdot \sqrt{\frac{235}{3 \cdot 20.6 \cdot 1.00}} = 58.5\text{ mm}$$
Fig. 9.18 The effective area under the base plate

The effective area is

$$A_{\text{eff,1}} = l_s \cdot (2c + t_s) = 120 \cdot (2 \cdot 58.5 + 12) = 15480 \text{ mm}^2$$

$$A_{\text{eff,2}} = (2c + 200) \cdot (2c + t_f) = (2 \cdot 58.5 + 200) \cdot (2 \cdot 58.5 + 15) = 41844 \text{ mm}^2$$

$$A_{\text{eff,3}} = A_{\text{eff}} - (A_{\text{eff,1}} + A_{\text{eff,2}}) = 62282 - (15480 + 41844) = 4958 \text{ mm}^2$$

The active effective width is calculated from known area in compression

$$b_{\text{eff}} = \frac{A_{\text{eff,3}}}{2c + t_w} = \frac{4958}{2 \cdot 58.5 + 9} = 39.3 \text{ mm}$$

**Step 3 Assembly for resistance**

The gravity centre of effective area

$$x_t = \frac{A_{\text{eff,1}} \cdot x_{t1} + A_{\text{eff,2}} \cdot x_{t2} + A_{\text{eff,3}} \cdot x_{t3}}{A_{\text{eff}}}$$

$$= \frac{15480 \cdot \frac{l_s}{2} + 41844 \cdot \left( l_s + \frac{2c + t_f}{2} \right) + 4958 \cdot \left( l_s + 2c + t_f + \frac{b_{\text{eff}}}{2} \right)}{62282}$$

$$= \frac{15480 \cdot 60 + 41844 \cdot \left( 120 + \frac{2 \cdot 58.5 + 15}{2} \right) + 4958 \cdot \left( 120 + 2 \cdot 58.5 + 15 + \frac{39.3}{2} \right)}{62282}$$

$$= 161.5 \text{ mm}$$

The lever arm of concrete to the column axes of symmetry is calculated as

$$r_c = \frac{h_c}{2} + 120 + c + \left( b_{\text{eff}} - \frac{53}{2} \right) - x_t = \frac{200}{2} + 120 + 58.5 + (39.3 - 26.5) - 161.5 =$$
The lever arm of concrete to the column axes of symmetry is calculated as
\[ r_t = \frac{h_c}{2} + 70 + \left(\frac{53}{2} - b_{\text{eff}}\right) = 170 + (26.5 - 39.3) = 157.2 \, \text{mm} \]

Moment resistance of column base is
\[ M_{\text{Rd}} = F_{T3,\text{Rd}} \cdot r_t + A_{\text{eff}} \cdot f_{\text{id}} \cdot r_c \]
\[ M_{\text{Rd}} = 183 \cdot 10^3 \cdot 157.2 + 62 \, 282 \cdot 20.6 \cdot 129.8 = 195.3 \, \text{kNm} \]

Under acting normal force \( N_{\text{sd}} = 1 \, 100 \, \text{kN} \) is the moment resistance
\[ M_{\text{Rd}} = 195.3 \, \text{kNm} \]

**Step 4 Resistance of the end of column**

The design resistance in poor compression is
\[ N_{\text{pl,Rd}} = \frac{A \cdot f_y}{\gamma_{\text{Mo}}} = \frac{(A_{\text{HE200B}} + 2 \cdot l_s \cdot t_s) \cdot 235}{100} = \frac{(7 \, 808 + 2 \cdot 120 \cdot 12) \cdot 235}{100} = 2 \, 511.7 \, \text{kN} \]
\[ > N_{\text{Rd}} = 1 \, 100 \, \text{kN} \]

and column bending resistance
\[ M_{\text{pl,Rd}} = \frac{W_{\text{pl}} \cdot f_{yk}}{\gamma_{\text{Mo}}} \]
\[ W_{\text{pl}} = W_{\text{pl,s}} + W_{\text{pl,HEB}} = 2 \cdot l_s \cdot t_s \cdot z_s + 642.5 \cdot 10^3 = 2 \cdot 12 \cdot 120 \cdot 160 + 642.5 \cdot 10^3 = 1 \, 103.3 \cdot 10^3 \, \text{mm}^3 \]
\[ M_{\text{pl,Rd}} = \frac{W_{\text{pl}} \cdot f_{yk}}{\gamma_{\text{Mo}}} = \frac{1 \, 103.3 \cdot 10^3 \cdot 235}{100} = 259.3 \, \text{kNm} \]

The interaction of normal force reduces moment resistance
\[ M_{\text{Ny,Rd}} = M_{\text{pl,Rd}} \cdot \frac{1 - \frac{N_{\text{sd}}}{N_{\text{pl,Rd}}}}{1 - 0.5 \cdot \frac{A - 2 \cdot b_{t_f}}{A}} = 259.3 \cdot \frac{1 - \frac{1 \, 100}{2 \, 511.7}}{1 - 0.5 \cdot \frac{7 \, 808 - 2 \cdot 200 \cdot 15}{7 \, 808}} = 164.8 \, \text{kNm} \]

The column base is designed on acting force even for column resistance.

**Note**

The resistance of the base plate is limited by the tension resistance of two headed studs \( M_{24} = 183.0 \, \text{kN} \). The elastic behaviour is expected till the 2/3 of the bending resistance of the base plate; \( 2/3 \cdot 267.7 = 178.5 \, \text{kN} \), which comply for the bending moment at SLS about \( 195.3 \cdot 178.5/183.0 \, \text{kNm} \).
9.4 Column base with anchor plate

Evaluate the resistance of the column base shown in Fig. 9.19 using component method. The Column HE 200 B is loaded by the normal force $F_{Ed} = 45$ kN and by the bending moment $M_{Ed} = 20$ kNm. The concrete block designed for the particular soil conditions is made out of concrete strength C30/37 and has dimensions of 1600 x 1600 x 1000 mm. The base plate thickness is 30 mm and the anchor plate 10 mm. The steel grade is S355 and the safety factors are considered as $\gamma_{Mc} = 1.50; \gamma_{M0} = 1.00$ and $\gamma_{M2} = 1.25$.

![Fig. 9.19 Designed column base with anchor plate](image)

**Procedure**

The calculation follows the Component method procedure for the column bases:

1 Components in tension
   1.1 Threaded studs in tension
   1.2 Punching of the anchor plate under threaded studs
   1.3 Base plate in bending
   1.4 Threaded studs in shear and bearing
   1.5 Headed studs in tension
   1.6 Punching of the anchor plate above the headed studs
1.7. Concrete cone failure without reinforcement
1.8. Concrete cone failure with reinforcement
1.9. Pull out failure of headed studs
1.10. T stub of the anchor plate in bending
1.11. Anchor plate in tension
1.12. Headed studs in shear
1.13. Pry-out failure of headed stud
1.14. Reduction of vertical resistance
    of the threaded stud (tensile and punching resistance) and
    the headed studs (tensile resistance, concrete cone failure, stirrups failure, bond
    failure the threaded stud)
    Reduction of horizontal resistance
    of the threaded stud (shear and bearing resistance) and
    the headed studs (shear and pry out resistance)
1.15. Interaction in shear and tension for threaded and the headed studs

2 Component in compression
3 Assembly for resistance
   3.1 Base plate resistance
   3.2 End column resistance
   3.3 Elastic resistance for Serviceability limit state
4 Connection stiffness
   4.1 Component`s stiffness
   4.2 Assembly for stiffness

**Step 1 Components in tension**

**Step 1.1 Threaded studs in tension**

The resistance of the component threaded studs in tension, with \( d = 22 \text{ mm} \), strength 8.8, \( f_{ub} = 800 \text{ N/mm}^2 \), with number of studs is \( n = 2 \), area of one stud is \( A_s = 303 \text{ mm}^2 \) and coefficient \( k_2 = 0.9 \), is

\[
F_{L,Rd,2} = \frac{n \cdot k_2 \cdot A_s \cdot f_{ub}}{Y_m} = \frac{2 \cdot 0.9 \cdot 303 \cdot 800}{1.25} = 349.1 \text{ kN}
\]

The resistance of one stud is 174.5 kN.

**Step 1.2 Punching of the anchor plate under threaded studs**

The resistance in punching of the anchor plate, for \( f_u = 510 \text{ MPa} \) and the effective width of studs weld \( a_w = 1 \text{ mm} \), is

\[
\text{EN1993-1-8 Tab. 3.41}
\]
The resistance of one stud is 177.6 kN.

**Step 1.3 Base plate in bending**

The base plate has thickness $t_{p2} = 30$ mm, width $a_{p2} = 250$ mm, yield strength $f_{y} = 355$ N/mm$^2$, $m_2 = 33.2$ mm, $e_a = 40$ mm, $e_b = 75$ mm, and $p_2 = 100$ mm, see in Fig. 9.18. Headed stud lever arm for fillet weld $a_{wf} = 6$ mm is

$$m = 40 - 0.8 \cdot a_{wf} \cdot \sqrt{2} = 40 - 0.8 \cdot 6 \cdot \sqrt{2} = 33.2 \text{ mm}$$

The T-stub length, in base plate are the prying forces not taken into account, is

$$l_{\text{eff},2} = \min \left\{ \begin{array}{l}
4m + 1.25e_a = 4 \cdot 33.2 + 1.25 \cdot 40 = 182.9 \\
2\pi m = 2 \pi \cdot 33.2 = 208.7 \\
b \cdot 0.5 = 250 \cdot 0.5 = 125.0 \\
2m + 0.625e_a + 0.5p = 2 \cdot 33.2 + 0.625 \cdot 40 + 0.5 \cdot 100 = 141.4 \\
2m + 0.625e_a + e_b = 2 \cdot 33.2 + 0.625 \cdot 40 + 75 = 166.4 \\
2\pi m + 4e_b = 2\pi \cdot 33.2 + 4 \cdot 75 = 508.7 \\
2\pi m + 2p = 2\pi \cdot 33.2 + 2 \cdot 100 = 408.7
\end{array} \right. $$

$$l_{\text{eff},2} = 125 \text{ mm}$$

Resistance of rigid plate T-stub in bending is verified for three possible failure modes

**Mode 1**

$$F_{\text{T,1,Rd},2} = \frac{4 \cdot l_{\text{eff},2} \cdot m_{\text{pl,1,Rd},2}}{m} = \frac{4 \cdot l_{\text{eff},2} \cdot t_{p2}^2 \cdot f_{y}}{4 \cdot Y_{M0}} = \frac{4 \cdot 125 \cdot 30^2 \cdot 355}{4 \cdot 33.2} = 1202.5 \text{ kN}$$

**Mode 2**

$$F_{\text{T,2,Rd},2} = \frac{2 \cdot l_{\text{eff},2} \cdot m_{\text{pl,2,Rd},2} + \sum F_{\text{T,Rd} \cdot n}}{m + n} = \frac{2 \cdot l_{\text{eff},2} \cdot t_{p2}^2 \cdot f_{y} + \sum F_{\text{T,Rd} \cdot n}}{4 \cdot Y_{M0}} + \frac{\sum F_{\text{T,Rd} \cdot n}}{m + n} = \frac{2 \cdot 125 \cdot 30^2 \cdot 355}{4 \cdot 1.0} + 349 \cdot 10^3 \cdot 40}{33.2 + 40} = 463.5 \text{ kN}$$

**Mode 3**
\[
\sum F_{t,Rd} = \min\left( F'_{t,Rd}; F_{b,Rd,V} \right) = \min\left( 349.1; 355.2 \right) = 349.1 \text{ kN}
\]
\[
F_{T,3,Rd,2} = \sum F_{t,Rd} = 349.1 \text{ kN}
\]

Decisive is Mode 3 with failure in threaded studs in tension \( F_{t,3,Rd} = 349.1 \text{ kN}. \)

**Step 1.4 Threaded studs in shear and bearing**

Threaded studs have diameter \( d = 22 \text{ mm}, \) \( d_0 = 24 \text{ mm}, \) base plate thickness \( t_{p2} = 30 \text{ mm}, \) coefficient \( e_1 = 40 \text{ mm}, \) \( e_2 = 75 \text{ mm}, \) tensile strength \( f_u = 510 \text{ N/mm}^2, \) \( f_{ub} = 800 \text{ N/mm}^2, \) area of one stud \( A_c = 303 \text{ mm}^2; \) \( \alpha_v = 0.6; \) \( \gamma_{M2} = 1.25 \) see in Fig. 9.18.

\[
F_{v,Rd} = \frac{n \cdot \alpha_v \cdot f_{ub} \cdot A_c}{\gamma_{M2}} = \frac{2 \cdot 0.6 \cdot 800 \cdot \pi \cdot \left( \frac{22}{2} \right)^2}{1.25} = 291.9 \text{ kN}
\]

The resistance of one stud is 146.0 kN.

\[
F_{b,Rd,2} = \frac{n \cdot k_1 \cdot \alpha_b \cdot f_u \cdot d \cdot t}{\gamma_{M2}} = \frac{2 \cdot 2.5 \cdot 0.56 \cdot 510 \cdot 22 \cdot 30}{1.25} = 754.0 \text{ kN}
\]

The resistance of one stud is 377.0 kN.

where

\[
k_1 = \min \left\{ \frac{2.8 e_2}{d_0} - 1.7; 2.5 \right\} = \min \left\{ \frac{2.8 \cdot 75}{24} - 1.7; 2.5 \right\} = \min\{7.05; 2.5\} = 2.5
\]

\[
\alpha_b = \min \left\{ \frac{f_{ub}}{f_u}; 1.0; \frac{e_1}{3d_0} \right\} = \min \left\{ \frac{800}{510}; 1.0; \frac{40}{3 \cdot 24} \right\} = \min\{1.57; 1.0; 0.56\} = 0.56
\]

**Step 1.5 Headed studs in tension**

The resistance of headed studs in tension, of diameter \( d = 22 \text{ mm} \) and material 8.8, with tensile strength \( f_{ub} = 800 \text{ N/mm}^2, \) two studs \( n = 2 \) and coefficient \( k_2 = 0.9; \) is

\[
F'_{t,Rd} = \frac{n \cdot k_2 \cdot A_c \cdot f_{ub}}{\gamma_{M2}} = \frac{2 \cdot 0.9 \cdot \pi \cdot \left( \frac{22}{2} \right)^2 \cdot 800}{1.25} = 437.9 \text{ kN}
\]

The resistance of one stud is 219.0 kN.
**Step 1.6 Punching of the anchor plate above the headed studs**

The resistance in punching of the anchor plate, for $f_u = 510 \text{ N/mm}^2$ and the effective width of studs weld $a_w = 1 \text{ mm}$, is

$$F_{p,Rd,V} = \frac{n \cdot A_v \cdot f_{uk}}{\sqrt{3} \cdot \gamma_{M2}} = \frac{n \cdot t_{p1} \cdot l_{v,eff,1} \cdot f_{uk}}{\sqrt{3} \cdot \gamma_{M2}} = \frac{n \cdot t_{p1} \cdot 2 \pi \left( a_w + \frac{d_{stud}}{2} \right) \cdot f_{uk}}{\sqrt{3} \cdot \gamma_{M2}}$$

$$= \frac{2 \cdot 10 \cdot 2\pi \left( 1 + \frac{22}{2} \right) \cdot 510}{\sqrt{3} \cdot 1.25} = 355.2 \text{kN}$$

The resistance of one headed stud is 177.6 kN.

**Step 1.7 Concrete cone failure without reinforcement**

The resistance of concrete cone failure without reinforcement, for the concrete block made out of concrete strength C30/37, $f_{ck} = 30 \text{ N/mm}^2$, $k_1 = 12.7$; and length of headed studs $h_{ef} = 200 \text{ mm}$, is

$$N_{Rd} = N_{Rk,c}^0 \cdot \psi_{A,N} \cdot \psi_{S,N} \cdot \psi_{re,N}/\gamma_{Mc}$$

$$N_{Rk,c}^0 = k_1 \cdot h_{ef}^{1.5} \cdot f_{ck}^{0.5} = 12.7 \cdot 200^{1.5} \cdot 30^{0.5} \quad N = 196.8 \text{kN}$$

$$\psi_{A,N} = \frac{A_{c,N}}{A_{c,N}^0} = \frac{420000}{360000} = 1.17$$

$$A_{c,N}^0 = s_{cr,N}^2 = (2 \cdot c_{cr,N})^2 = (2 (1.5 \cdot h_{ef}))^2 = (2 (1.5 \cdot 200))^2 = 360000 \text{ mm}^2$$

$$A_{c,N} = \left( (1.5 \cdot h_{ef}) \cdot 2 \right) \cdot (1.5 \cdot h_{ef} + p + 1.5 \cdot h_{ef}) =$$

$$= (1.5 \cdot 200) \cdot 2 \cdot (1.5 \cdot 200 + 100 + 1.5 \cdot 200) = 420000 \text{ mm}^2$$

Since maximum edge distance is $c < c_{cr} = 1.5 \cdot h_{ef} = 300 \text{ mm}$ and $\psi_{S,N} = 1.0$

There is no closely spaced reinforcement and $\psi_{re,N} = 1.0$

$$N_{Rk,c} = 196.8 \cdot 1.17 \cdot 1.0 \cdot 1.0 = 230.3 \text{kN}$$

$$N_{Rd,c} = \frac{N_{Rk,c}}{\gamma_{Mc}} = \frac{230.3}{1.5} = 153.5 \text{kN}$$

**Step 1.8 Concrete cone failure with reinforcement**

For concrete cone failure with reinforcement, with diameter of headed studs $d = 22 \text{ mm}$ and diameter of stirrups $d_s = 8 \text{ mm}$, is factor for support of reinforcement
\[
\psi_{\text{supp}} = 2.5 - \frac{x}{h_{\text{ef}}} = 2.5 - \frac{\frac{d}{2} + d_{s,a} + \frac{d_{s,t}}{\tan 35^\circ}}{h_{\text{ef}}} = 2.5 - \frac{\frac{d}{2} + (5 \cdot \frac{d}{2} - \frac{d}{2}) + \left(\frac{d_s}{2} + 10\right) \tan 35^\circ}{h_{\text{ef}}}
\]

\[
= 2.5 - \frac{\frac{22}{2} + (5 \cdot \frac{8}{2} - \frac{22}{2}) + \left(\frac{8}{2} + 10\right) \tan 35^\circ}{200} = 2.3
\]

and resistance

\[
N_{Rd,\text{max}} = \frac{\psi_{\text{supp}} \cdot N_{Rc,\text{c}}}{\gamma_{M_c}} = \frac{2.3 \cdot 230.2}{1.5} = 353.0 \text{ kN}
\]

with

\[
k_{c,de} = \alpha_c \cdot \sqrt{f_{ck} \cdot h_{\text{ef}}} \cdot \psi_{A,N} \cdot \psi_{s,N} \cdot \psi_{f,N} = -537 \cdot \sqrt{30 \cdot 200 \cdot 1.17 \cdot 1.0 \cdot 1.0} = -48.7 \text{ kN/mm}
\]

where

\[
\alpha_c = -537 \text{ is factor of component concrete break out in tension}
\]

Yielding of reinforcement will occur for

\[
N_{Rd,1} = N_{Rd,s,\text{re}} + N_{Rd,c} + \delta_{Rd,s} \cdot k_{c,de} =
\]

\[
= A_{s,\text{re}} \cdot \frac{f_{yks}}{\gamma_{M_s}} + N_{Rd,c} + \frac{2 \cdot N_{Rd,s,\text{re}}}{\alpha_s \cdot f_{ck} \cdot d_{s,\text{re}}^2 \cdot (n \cdot n_{re})^2} \cdot k_{c,de} =
\]

\[
= n \cdot n_{re} \cdot \pi \cdot \left(\frac{d_{s,\text{re}}^2}{4}\right) \cdot \frac{f_{yks}}{\gamma_{M_s}} + N_{Rd,c} + \frac{2 \cdot \left(n \cdot n_{re} \cdot \pi \cdot \left(\frac{d_{s,\text{re}}^2}{4}\right) \cdot \frac{f_{yks}}{\gamma_{M_s}}\right)^2}{\alpha_s \cdot f_{ck} \cdot d_{s,\text{re}}^2 \cdot (n \cdot n_{re})^2} \cdot k_{c,de} =
\]

\[
= 2 \cdot 4 \cdot \pi \cdot \left(\frac{8^2}{4}\right) \cdot \frac{500}{1.15} + 153.5 + \frac{2 \cdot \left(2 \cdot 4 \cdot \pi \cdot \left(\frac{8^2}{4}\right) \cdot \frac{500}{1.15}\right)^2}{12100 \cdot 30 \cdot 8^4 \cdot (2 \cdot 4)^2} \cdot (-48.7) =
\]

\[
= 174.8 + 153.5 + 0.642 \cdot (-48.7) = 297.0 \text{ kN}
\]

where

\[
\alpha_s = 12 \text{ 100} \text{ is factor of the component stirrups}
\]

\[
n_{re} = 4 \text{ is total number of legs of shafts}
\]

\[
N_{Rd,s,\text{re}} \text{ is design tension resistance of the stirrups for tension failure [N]}
\]

\[
d_{s,\text{re}} = 8 \text{ mm is nominal diameter of the stirrup}
\]

\[
d_p = 25 \text{ mm is the covering}
\]

\[
f_{yks} = 500 \text{ N/mm}^2 \text{ is design yield strength of the stirrups}
\]

\[
\gamma_{M_s} = 1.15 \text{ is the partial safety factor}
\]

\[
l_1 \text{ is anchorage length [mm]}
\]

Anchorage failure resistance of the of reinforcement is
\[ N_{Rd,2} = N_{Rd,b, re} + N_{Rd,c} + \delta_{Rd,b} \cdot k_{c,de} = \sum n_{re} \cdot l_1 \cdot \pi \cdot d_{s, re} \cdot \frac{f_{bd}}{\alpha} + N_{Rd,c} + \delta_{Rd,b} \cdot k_{c,de} = \]

\[ = n \cdot n_{re} \cdot l_1 \cdot \pi \cdot d_s \cdot \frac{f_{bd}}{\alpha} + N_{Rd,c} + \frac{2 \cdot N_{Rd,b, re}^2}{\alpha_s \cdot f_{ck} \cdot d_{s, re} \cdot n_{re}^2} \cdot k_{c,de} = \]

\[ = n \cdot n_{re} \cdot \left( h_{ef} - d_p - d_{s, t} - \frac{d_{s,a}}{1.5} \right) \cdot \pi \cdot d_s \cdot \frac{2.25 \cdot \eta_1 \cdot \eta_2 \cdot f_{ck;0.05}}{\alpha \cdot \gamma_{Mc}} + N_{Rd,c} \]

\[ + \frac{2 \cdot \left( n \cdot n_{re} \cdot l_1 \cdot \pi \cdot d_s \cdot \frac{f_{bd}}{\alpha} \right)^2}{\alpha_s \cdot f_{ck} \cdot d_{s, re} \cdot n_{re}^2} \cdot k_{c,de} = \]

\[ = n \cdot n_{re} \cdot \left( h_{ef} - d_p - \left( \frac{d_s}{2} + 10 \right) - \frac{\frac{d_s}{2} - \frac{d}{2}}{1.5} \right) \cdot \pi \cdot d_s \cdot \frac{2.25 \cdot \eta_1 \cdot \eta_2 \cdot f_{ck;0.05}}{\alpha \cdot \gamma_{Mc}} + N_{Rd,c} + \]

\[ 2 \left( n \cdot n_{re} \cdot \left( h_{ef} - d_p - \left( \frac{d_s}{2} + 10 \right) - \frac{\frac{d_s}{2} - \frac{d}{2}}{1.5} \right) \cdot \pi \cdot d_s \cdot \frac{2.25 \cdot \eta_1 \cdot \eta_2 \cdot f_{ck;0.05}}{\alpha \cdot \gamma_{Mc}} \right)^2 \]

\[ \cdot \frac{\alpha_s \cdot f_{ck} \cdot d_{s, re} \cdot n_{re}^2}{\alpha_s \cdot f_{ck} \cdot d_{s, re} \cdot n_{re}^2} \cdot k_{c,de} = \]

\[ = 2 \cdot 4 \cdot \left( 200 - 25 - \left( \frac{8}{2} + 10 \right) - \frac{\frac{8}{2} - \frac{22}{2}}{1.5} \right) \cdot \pi \cdot 8 \cdot \frac{2.25 \cdot 1.0 \cdot 1.0 \cdot 2.0}{0.49 \cdot 1.5} + 153.5 \]

\[ + \frac{2 \cdot \left( 2 \cdot 4 \cdot \left( 200 - 25 - \left( \frac{8}{2} + 10 \right) - \frac{\frac{8}{2} - \frac{22}{2}}{1.5} \right) \cdot \pi \cdot 8 \cdot \frac{2.25 \cdot 1.0 \cdot 1.0 \cdot 2.0}{0.49 \cdot 1.5} \right)^2}{12100 \cdot 30 \cdot 8^4 \cdot (2 \cdot 4)^2} \cdot (-48.7) \]

\[ = 190.8 + 153.5 + 0.765 \cdot (-48.7) = 307.0 \text{ kN} \]

where

- \( l_1 \) is anchorage length [mm]
- \( d_s \) is diameter of stirrups [mm]
- \( \alpha = 0.7 \cdot 0.7 = 0.49 \) is factor for hook effect and large concrete cover
- \( f_{bd} \) is for C30/37 grade concrete is \( 2.25 \cdot \frac{2.0}{1.5} \cdot 1.0 \cdot 1.0 = 3.0 \text{ N/mm}^2 \)
- \( \eta_1 = 1.0 \) is coefficient of bond conditions for vertical stirrups
- \( \eta_2 = 1.0 \) is coefficient of bond conditions for dimension ≤ 32 mm
- \( \eta_2 = 1.0 \) is coefficient of bond conditions for dimension ≥ 32 mm

The resistance of concrete cone failure with reinforcement is

\[ \min(N_{Rd,max}; N_{Rd,1}; N_{Rd,2}) = \min(353.0; 297.0; 307.0) = 297.0 \text{ kN} \]
Step 1.9 Pull-out failure of headed studs

The resistance of pull-out failure of headed studs, with diameter of stud $d = 22$ mm, diameter of stud’s head $d_h = 37$ mm, concrete C30/37 with compressive strength $f_{ck} = 30$ N/mm² and the characteristic ultimate bearing pressure at ultimate limit state under the headed of stud $p_{uk} = 12 \cdot f_{ck}$ N/mm², is

$$N_{Rk,p} = n \cdot p_{uk} \cdot A_h = n \cdot 12 \cdot f_{ck} \cdot \frac{\pi}{4} \left(d_h^2 - d^2\right) = 2 \cdot 12 \cdot 30 \cdot \frac{\pi}{4} \left(37^2 - 22^2\right) = 500.5 \text{ kN}$$

$$N_{Rd,p} = \frac{N_{Rk,p}}{\gamma_{Mc}} = \frac{500.5}{1.5} = 333.7 \text{ kN}$$

The resistance of one stud is 166.8 kN

Step 1.10 T stub of the anchor plate in bending

The resistance of component T-stub of the anchor plate in bending has thickness $t_{p1} = 10$ mm, yield strength $f_{yk} = 355$ N/mm², distance of threaded and headed stud $m_1 = 80$ mm, $e_{a1} = 50$ mm, $e_{b1} = 125$ mm and $p_1 = 100$ mm, see in Fig. 9.18.

Due to small thickness of the anchor plate are the prying forces for evaluation of the effective length of T stub taken into account as

Resistance of anchor plate T-stub in tension is verified for three failure modes, see in Fig. 9.19. For effective length of the T stub

$$l_{eff,1} = \min \begin{cases} 4 \cdot m_1 + 1.25 \cdot e_{a1} = 4 \cdot 80 + 1.25 \cdot 50 = 382.5 \\ 2 \pi \cdot m_1 = 2 \pi \cdot 80 = 502.7 \\ 5 \cdot m_1 \cdot d_1 \cdot 0.5 = 220 \cdot 0.5 = 110.0 \\ 2 \cdot m_1 + 0.625 \cdot e_{a1} + 0.5 \cdot p_1 = 2 \cdot 80 + 0.625 \cdot 50 + 0.5 \cdot 100 = 241.3 \\ 2 \cdot m_1 + 0.625 \cdot e_{a1} + e_{b1} = 2 \cdot 80 + 0.625 \cdot 50 + 93.8 = 285.0 \\ \pi \cdot m_1 + 2 \cdot e_{b1} = \pi \cdot 80 + 2 \cdot 93.8 = 721.4 \\ \pi \cdot m_1 + p_1 = \pi \cdot 80 + 100 = 351.3 \end{cases}$$

$$l_{eff,1} = 110.0 \text{ mm}$$

Fig. 9.19 T-stub in tension and forces in the individual failure modes
Mode 1

\[
F_{T1,Rd,ap} = \frac{4 \cdot l_{eff,1} \cdot m_{pl,Rd,1}}{m} = \frac{4 \cdot l_{eff,1} \cdot \frac{t_{p,1}^2 \cdot f_{yk}}{4 \cdot Y_{M0}}}{m} = \frac{4 \cdot 110.0 \cdot \frac{10^2 \cdot 355}{4 \cdot 1.0}}{80} = 48.8 \text{ kN}
\]

Mode 2

\[
F_{T2,Rd,ap} = \frac{2 \cdot l_{eff,1} \cdot m_{plz,Rd,2} + \sum F_{T,Rd} \cdot n}{m + n} = \frac{2 \cdot l_{eff,1} \cdot \frac{t_{p,1}^2 \cdot f_{yk}}{4 \cdot Y_{M0}} + \sum F_{T,Rd} \cdot n}{m + n} = \frac{2 \cdot 110.0 \cdot \frac{10^2 \cdot 355}{4 \cdot 1.0} + 297.0 \cdot 10^3 \cdot 50}{80 + 50} = 129.1 \text{ kN}
\]

Mode 3

\[
\sum F_{T,Rd} = \min \left(F_{T,Rd,1}'; F_{p,Rd,V_1}; N_{Rd,1}; N_{Rd,p} \right) = \min \left(437.9; 355.2; 297.0; 333.7 \right) = 297.0 \text{ kN}
\]

\[
F_{T3,Rd,ap} = \sum F_{T,Rd} = 297.0 \text{ kN}
\]

Mode 1 is decisive for the thin plate, 48.8 kN, see in Fig. 9.20.

---

**Fig. 9.20 Vertical forces \( F_v \) and vertical deformation \( \delta \) of T stub**

---

**Step 1.11 Anchor plate in tension**

The anchor plate in tension resistance is

\[
F_{T,ap,Rd} = A_{ap,1} \cdot \frac{f_{yk}}{Y_{M0}} = t_{p,1} \cdot b_{ap,eff} \cdot \frac{f_{yk}}{Y_{M0}} = 10 \cdot 2 \cdot \left(22 + 2 \cdot \sqrt{2} \cdot 1\right) \cdot \frac{355}{1.0} = 176.3 \text{ kN}
\]

where

\[
b_{ap,eff} = n_1 \cdot \left(d_1 + 2 \cdot \sqrt{2} \cdot a_w\right)
\]

studs weld effective thickness \( a_w = 1 \text{ mm} \)
Step 1.12 Headed studs in shear

The shear resistance of headed studs, with material 8.8, strength $f_{ub} = 800$ N/mm$^2$, $\alpha_v = 0.6$; $\gamma_{M2} = 1.25$; is

$$F_{v,Rd} = \frac{n \cdot \alpha_v \cdot f_{ub} \cdot A_s}{\gamma_{M2}} = \frac{2 \cdot 0.6 \cdot 800 \cdot \pi \cdot \left(\frac{22}{2}\right)^2}{1.25} = 291.9 \text{ kN}$$

The resistance of one stud is 146.0 kN.

Step 1.13 Pry-out failure of headed stud

The resistance in pry-out failure of headed studs for is

$$V_{Rd,CP} = 2 \cdot N_{Rd,c} = 2 \cdot 153.5 = 307.0 \text{ kN}$$

Step 1.14 Reduction of resistance in the vertical/horizontal direction

For the calculation of plastic deformation is used model of continues beam with three plastic hinges at supports and under applied load, see in Fig. 9.21.

![Fig. 9.21 Model of continues beam with three plastic hinges](image)

$$A = \min(F_{T,1,Rd,1}; F_{T,2,Rd,1}; F_{T,3,Rd,1}) = \min (48.8; 126.1; 296.7) = 48.8 \text{ kN}$$
Plastic deformation is calculated, see Fig. 9.21, for moment resistance

\[
M_{pl} = \frac{b_{p1} \cdot t_{p1}^2 \cdot f_{yk}}{4} \cdot \frac{350 \cdot 10^2}{4} \cdot 355 \cdot \frac{1}{1} = 3.1 \text{kNm}
\]

\[
l_c = \frac{1}{12} \cdot b_{p1} \cdot t_{p1}^3 = \frac{1}{12} \cdot 350 \cdot 10^3 = 29.2 \cdot 10^3 \text{ mm}^4; l_b = \infty
\]

\[
\delta_T = \frac{1}{E_b} \cdot \frac{1}{6} \cdot b^2 \cdot M_{pl} + \frac{1}{E I_c} \cdot \frac{1}{3} \cdot b \cdot c \cdot M_{pl} =
\]

\[
= \frac{1}{210 \ \text{mm}^4} \cdot \frac{1}{6} \cdot 232.5^2 \cdot 3106 + \frac{1}{210 \ \text{mm}^4} \cdot \frac{1}{3} \cdot 232.5 \cdot 127.5 \cdot 3106 = 0 + 5.2
\]

\[
= 5.2 \text{ mm}
\]

with distance between threaded stud and headed stud \(a = 80 \text{ mm}\) as

\[
\delta_{T,pl} = 1.48 \delta_T = 7.8 \text{ mm}
\]

\[
\delta_{p,tot} = \delta_{T,pl} + \sqrt{a_{ap}^2 - a^2} = \sqrt{(a + \Delta a)^2 - a^2} = \delta_{T,pl} + \sqrt{\left(a + \frac{a F_{p,Rd}}{t_{p1} b_{ap,eff} E}\right)^2 - a^2} =
\]

\[
\Delta M I \quad \text{Eq. (4.43)}
\]

\[
= 7.8 + \sqrt{\left(80 + \frac{80 \cdot 8.88 \cdot 355}{210 \cdot 10^3}\right)^2 - 80^2} = 13.9 \text{ mm}
\]

For the plastic deformation at resistance of the anchor plate punching under the threaded studs \(F_{p,Rd} = 176.28 \text{ kN}\) and \(F_{p,Rd,V} = A + \frac{F_{p,Rd}}{(a + \Delta a)} = 79.0 \text{ kN}\)

The acting horizontal force for this deformation is
For the resistance of headed studs in shear $V_{rd} = 291.9$ kN is assumed the linear proportion between the axial and horizontal forces, see in Fig. 9.22. The resistance in tension is calculated as

$$F_{p,1,Rd} = F_{T,pl} + \frac{F_{t,p,Rd} - F_{T,pl}}{F_{p,Rd,H}} \cdot V_{rd} = 48.8 + \frac{79.0 - 48.8}{454.3} \cdot 291.9 = 68.2\text{ kN}$$

and deformation for $F_{p,1,Rd} = 68.2$ kN, see in Fig. 9.20, is

$$\delta_{p,1} = \delta_{T,pl} + \frac{F_{p,1,Rd} - F_{T,pl}}{F_{t,p,Rd} - F_{T,pl}} \cdot \delta_{p,tot} = 7.8 + \frac{68.2 - 48.8}{79.0 - 48.8} \cdot 13.9 = 16.7\text{ mm}$$

![Fig. 9.22 Acting vertical $F_v$ and horizontal $F_H$ forces to the anchor plate](image)

The acting force in headed studs in case of the membrane action in the anchor plate

$$N_{HS,1} = A + Q = 68.2 + 39.1 = 107.3\text{ kN}$$

### Step 1.15 Interaction in shear and tension for trenched and headed studs

For the threaded studs is the interaction in shear and tension

$$\frac{F_{v,Ed}}{F_{v,Rd}} + \frac{F_{LED}}{1.4 \cdot F_{t,Rd}} \leq 1$$

$$\frac{291.9 + (107.3 - 48.8) \cdot \left(\frac{220 + 165.9}{140 + 165.9}\right)}{1.4 \cdot 349.1} \leq 1.00$$

1.15 is not $\leq 1$
For the headed studs is the interaction in shear and tension

\[
\frac{F_{v,Ed}}{F_{v,Rd}} + \frac{F_{L,Ed}}{1.4 \cdot F_{L,Rd}} \leq 1
\]

\[
\frac{291.9}{291.9} + \frac{107.3 - 48.8}{1.4 \cdot 437.9} \leq 1
\]

1.10 is not \( \leq 1 \)

For anchoring of headed stud in concrete is the interaction in shear and tension

\[
\left( \frac{F_{v,Ed}}{F_{v,Rd}} \right)^3 + \left( \frac{F_{L,Ed}}{F_{L,Rd}} \right)^3 \leq 1
\]

\[
\left( \frac{291.9}{306.1} \right)^3 + \left( \frac{107.3 - 48.8}{296.7} \right)^3 \leq 1
\]

1.02 is not \( \leq 1 \)

The full capacity in shear is not achieve due to headed stud resistance. By reducing the acting forces to 80\% it is for interaction of the threaded stud

\[
\frac{233.5}{291.9} + \frac{(107.3 - 48.8) \cdot \left( \frac{220 + 165.9}{140 + 165.9} \right)}{1.4 \cdot 349.1} \leq 1
\]

0.95 \( \leq 1 \)

and for the headed stud

\[
\frac{233.5}{291.9} + \frac{107.3 - 48.8}{1.4 \cdot 437.9} \leq 1
\]

0.86 \( \leq 1 \)

and for anchoring of headed stud in concrete

\[
\left( \frac{233.5}{306.1} \right)^3 + \left( \frac{107.3 - 48.8}{296.7} \right)^3 \leq 1
\]

0.71 \( \leq 1 \)
Step 2 Component in compression

The component base plate in bending and concrete block in compression is calculated for the strength of the concrete block, C30/37, $f_{ck} = 30 \text{ N/mm}^2$, and $\gamma_{Mc} = 1.5$.

The connection concentration factor is

$$a_1 = \min \left\{ \begin{array}{l}
a_1 + 2a_t = 250 + 2 \cdot 675 = 1600 \\
3a_1 = 3 \cdot 250 = 750 \\
a_1 + h = 250 + 1000 = 1250
\end{array} \right\} = 750 \text{ mm}$$

$$b_1 = \min \left\{ \begin{array}{l}
b_1 + 2b_t = 360 + 2 \cdot 620 = 1600 \\
3b_1 = 3 \cdot 360 = 1080 \\
b_1 + h = 360 + 1000 = 1360
\end{array} \right\} = 1080 \text{ mm}$$

and $a_1 = 750 > a_2 = 250 \text{ mm}$, $b_1 = 1080 > b_2 = 360 \text{ mm}$

The above condition is fulfilled and

$$k_j = \frac{a_1 \cdot b_1}{a \cdot b} = \sqrt{\frac{1080 \cdot 750}{250 \cdot 360}} = 3.00$$

The concrete bearing resistance is calculated as

$$f_{jd} = \frac{2}{3} \cdot \frac{k_j \cdot f_{ck}}{\gamma_{Mc}} = \frac{2}{3} \cdot \frac{3.00 \cdot 30}{1.5} = 40.0 \text{ N/mm}^2$$

From the force equilibrium in the vertical direction $F_{sd} = A_{eff} \cdot f_{jd} - F_{t,Rd}$, is calculated the area of concrete in compression $A_{eff}$ in case of the full resistance of tension part

$$A_{eff} = \frac{F_{sd} + F_{rd} \cdot 3}{f_{jd}} = \frac{-45 \cdot 10^3 + 107.3 \cdot 10^3}{40.0} = 1557 \text{ mm}^2$$

The flexible base plate is transferred into a rigid plate of equivalent area. The width of the strip $c$ around the column cross section, see Fig. 9.23a, is calculated from

$$c = (t_{p1} + t_{p2}) \cdot \sqrt{\frac{f_y}{3 \cdot f_{jd} \cdot \gamma_{M0}}} = (30 + 10) \cdot \sqrt{\frac{355}{3 \cdot 40.0 \cdot 1.00}} = 68.8 \text{ mm}$$
Step 3 Assembly for resistance

Step 3.1 Column base resistance

The active effective width is calculated as

\[
b_{\text{eff}} = \frac{A_{\text{eff}}}{a_{p2} + 2 \cdot t_{p1}} = \frac{1557}{270} = 5.8 \text{ mm} < t_f + 2 \cdot c = 15 + 2 \cdot 68.8 = 152.6 \text{ mm}
\]

The lever arm of concrete to the column axes of symmetry, see Fig. 9.23b, is calculated as

\[
r_c = \frac{h_c}{2} + c - \frac{b_{\text{eff}}}{2} = \frac{200}{2} + 68.8 - \frac{5.8}{2} = 165.9 \text{ mm}
\]

The moment resistance of the column base is

\[
M_{Rd} = F_{T,\text{min}} \cdot r_t + A_{\text{eff}} \cdot f_{\text{yd}} \cdot r_c
\]

\[
F_{T,\text{min}} = 107.3 \cdot \frac{220 + 165.9}{140 + 165.9} = 135.3 \text{ kN}
\]

\[
M_{Rd} = 135.3 \cdot 10^3 \cdot 140 + 1557 \cdot 40 \cdot 165.9 = 29.3 \text{ kNm}
\]

Under acting normal force \( N_{\text{sd}} = -45 \text{ kN} \) the moment resistance in bending is

\[
M_{Rd} = 29.3 \text{ kNm}.
\]
3.2 End of column resistance

The design resistance in poor compression is

$$N_{pl,Rd} = \frac{A \cdot f_y}{Y_{M0}} = \frac{7808 \cdot 355}{1.00} = 2\,772 \cdot 10^3 \text{N} > N_{Rd} = -45 \text{kN}$$

EN1993-1-1 cl 6.2.5

The column bending resistance

$$M_{pl,Rd} = \frac{W_{pl} \cdot f_{yk}}{Y_{M0}} = \frac{642.5 \cdot 10^3 \cdot 355}{1.00} = 228.1 \text{kNm}$$

EN1993-1-1 cl 6.2.9

The interaction of normal force reduces moment resistance (this interaction is valid for compression load only)

$$M_{Ny,Rd} = M_{pl,Rd} \cdot \frac{1 - \frac{N_{sd}}{N_{pl,Rd}}}{1 - 0.5 \cdot \frac{A - 2 \cdot b \cdot t_y}{A}} = 228.1 \cdot \frac{1 - \frac{0}{2772}}{1 - 0.5 \cdot \frac{7808 - 2 \cdot 200 \cdot 15}{7808}} = 258.0 \text{kNm}$$

EN1993-1-8 cl 6.3

$$M_{Ny,Rd} = 228.1 \text{kNm}$$

The column base is designed on acting force only not for column resistance.

Step 3.3 Elastic resistance for Serviceability limit state

The resistance of the base plate is limited by the T stub resistance, 48.8 kN. The elastic-plastic behaviour is expected by reaching the bending resistance of the anchor plate T stub; 87.9 kN, which comply for the bending moment at SLS as 22.7 kNm.

Step 4 Connection stiffness

4.1 Component’s stiffness

The component’s stiffness coefficients are calculated as in Worked example 9.2. The additional component is the anchor plate in bending and in tension and the component threaded stud. In compression are transferring the forces both plates under the column, the base and anchor plates.

The component base plate in bending and the threaded studs in tension

The stiffness coefficient for the threaded stud is assumed as

$$k_{p2} = 2.0 \cdot \frac{A_s}{L_b} = 2.0 \cdot \frac{303}{49.5} = 12.2 \text{mm}$$

EN1993-1-8 cl. 6.3

The component stiffness coefficients for base plate is calculated as

$$k_{p2} = \frac{0.425 \cdot L_{beff} \cdot t^3}{m^3} = \frac{0.425 \cdot 125 \cdot 30^3}{33.2^3} = 39.2 \text{mm}$$

EN1993-1-8 cl. 6.3
Component base and anchor plates and concrete block in compression

The stiffness coefficient for concrete block in compression, see Fig. 9.23c, is calculated for

\[ a_{eq} = t_f + 2.5 \times t = 15 + 2.5 \times 40 = 115 \text{ mm} \]

where thickness \( t = t_1 + t_2 = 10 + 30 = 40 \text{ mm} \)

\[ k_c = \frac{E_c}{1.275 \times E_s} \cdot \sqrt{a_{eq} \times b_c} = \frac{33 \, 000}{1.275 \times 210 \, 000} \cdot \sqrt{115 \times 200} = 18.7 \text{ mm} \]

Component anchor plate in bending and in tension

The component stiffness coefficients for anchor plate is calculated from the bending of the anchor plate as

\[ k_{p1} = \frac{0.85 \times L_{\text{eff}} \times t^3}{m^3} = \frac{0.85 \times 110.0 \times 10^3}{(80 - 2 \times \frac{22}{2})^3} = 0.5 \text{ mm} \]

Component headed stud in tension

The component stiffness coefficients for headed studs is calculated as

\[ k_{h1} = \frac{n \times A_{s, \text{nom}}}{L_b} = \frac{2 \times \pi \times 22^2}{4 \times 8 \times 22} = 4.3 \text{ mm} \]

4.2 Assembly for stiffness

The coefficients of the initial stiffness in elongation are assembled to rotational stiffness as in Worked example 9.2. The additional component is the anchor plate in bending and in tension only.
The lever arm of components, see Fig. 9.23d, in tension $z_t$ and in compression $z_c$ to the column base neutral axes are

$$z_t = \frac{h_c}{2} + e_c = \frac{200}{2} + 40 = 140 \text{ mm}$$

$$z_c = \frac{h_c}{2} - \frac{t_f}{2} = \frac{200}{2} - \frac{15}{2} = 92.5 \text{ mm}$$

The stiffness of tension part, studs, T stubs and concrete parts, is calculated from the stiffness coefficient for base plate and threaded studs

$$k_{t2} = \frac{1}{k_{b2} + \frac{1}{k_{p2}}} = \frac{1}{12.2 + \frac{1}{39.2}} = 9.33 \text{ mm}$$

from the stiffness coefficient for anchor plate and headed studs

$$k_{t1} = \frac{1}{k_{p1} + \frac{1}{k_{b1}}} = \frac{1}{0.5 + 4.3} = 0.43 \text{ mm}$$

based on eccentricity

$$k_{t1,\text{eff}} = \frac{z}{z + 80} \cdot k_{t1} = \frac{232.5}{312.5} \cdot 0.43 = 0.32 \text{ mm}$$

where

$$z = z_t + z_c = 140 + 92.5 = 232.5 \text{ mm}$$

with the effective stiffness coefficient in tension in position of threaded stud

$$k_t = \frac{1}{k_{t1} + \frac{1}{k_{t2}}} = \frac{1}{0.32 + 9.33} = 0.31 \text{ mm}$$

For the calculation of the initial stiffness of the column base the lever arm is evaluated

$$z = 232.5 \text{ mm} \quad \text{ and}$$

$$a = \frac{k_c \cdot z_c - k_t \cdot z_t}{k_c + k_t} = \frac{18.7 \cdot 92.5 - 0.31 \cdot 140}{18.7 + 0.31} = 88.7 \text{ mm}$$

The bending stiffness is calculated for particular constant eccentricity

$$e = \frac{M_{\text{ld}}}{F_{\text{sd}}} = \frac{20 \cdot 10^6}{45 \cdot 10^3} = 444 \text{ mm}$$

as
Summary

Moment rotational diagram at Fig. 9.23e sums up the behaviour of column base with anchor plate for loading with constant eccentricity.

\[ S_{j,ini} = \frac{e}{e + a} \cdot \frac{E_s \cdot z^2}{\mu \sum \frac{1}{K_i}} = \frac{444}{444 + 88.7} \cdot \frac{210 \, 000 \cdot 232.5^2}{1 \cdot \left( \frac{1}{0.31} + \frac{1}{18.7} \right)} = 2 \, 888 \cdot 10^6 \text{ Nmm/rad} \]

\[ = 2 \, 888 \text{ kNm/rad} \]

EN1993-1-8
cl. 6.3.4

Fig. 9.23e Moment rotational diagram of column base with anchor plate for loading with constant eccentricity
9.5 Simple steel to concrete joint

In this example the calculation of a simple steel-to-concrete joint is demonstrated. A girder is connected to a concrete wall by a simple joint. The load capacity of the joint will be raised by the use of additional reinforcement. The example does only include the design calculation of the joint. The verification of the concrete wall is not included and the local failure of the concrete wall due to the tension force caused by the eccentricity of the shear load is not considered.

Overview about the system

In this example a steel platform is installed in an industrial building. The main building is made of concrete. The system consists of concrete walls and concrete girders. An extra platform is implemented in the building in order to gain supplementary storage room.

The platform consists of primary and secondary girders. The primary girders are made of HE400A and they are arranged in a grid of 4.00 m. On one side they are supported on the concrete wall, on the other side they are supported by a steel column. The concrete wall and the steel beam are connected by a pinned steel-to-concrete joint.

Fig. 9.24 Side view on structure

Structural system and design of the girder

The structural system of the primary girder is a simply supported beam with an effective length of 9.4 m. The cross section of the girder is HE400A. The girder carries load applied to a width $a = 4.0$ m which is the distance to the next girder, see Fig. 9.25

Fig. 9.25 structural system

Load on the girder

<table>
<thead>
<tr>
<th>Description</th>
<th>Load (kN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self-weight of the girder with connection</td>
<td>2.0</td>
</tr>
<tr>
<td>Floor and other girders</td>
<td>4.0 m · $1.0 \frac{kN}{m^2} = 4.0$ kN/m</td>
</tr>
<tr>
<td>Dead load</td>
<td>6.0 kN/m</td>
</tr>
</tbody>
</table>
Live load \(4.0 \text{ m} \cdot 5.0 \frac{\text{kN}}{\text{m}^2} = 20.0 \text{ kN/m}\)

**Design forces**

**Maximum shear load**

\[ V_{z,Ed} = 9.4 \text{ m} \cdot \frac{1.35 \times 6.0 \frac{\text{kN}}{\text{m}} + 1.5 \times 20.0 \frac{\text{kN}}{\text{m}}}{2} = 179 \text{ kN} \approx 180 \text{ kN} \]

**Maximum bending moment**

\[ M_{y,Ed} = (9.4 \text{ m})^2 \cdot \frac{1.35 \times 6.0 \frac{\text{kN}}{\text{m}} + 1.5 \times 20.0 \frac{\text{kN}}{\text{m}}}{8} = 420 \text{ kNm} \]

**Verification of the girder section**

**Next to the joint**

\[ V_{z,Ed} = 180 \text{ kN} \leq V_{pl,z,Rd} = 777.8 \text{ kN} \]

**In the middle of the girder**

\[ M_{y,Ed} = 420 \text{ kNm} \leq M_{pl,y,Rd} = 602.1 \text{ kNm} \]

The girder is stabilized against lateral torsional buckling by the secondary girders, which have a distance of 1.0 m. Lateral torsional buckling is not examined in this example. The example only includes the design calculation of the joint. The verification of the concrete wall is not included.

**Overview of the joint**

![Fig. 9.26 Joint geometry](EN1993-1-1)
In the following an overview of the joint geometry is given.

**Connected girder**: HE400A, S235

**Concrete**: C30/37 \((f_{ck,cube} = 37 \text{ N/mm}^2, \text{cracked})\)

**Stirrups**: 4 x 8 mm / B500A (two per headed stud)

**Butt straps**: 150 x 250 x 20 mm / S235

**Anchor plate**: 300 x 250 x 25 mm / S235

**Headed Studs**

\[
d = 22 \text{ mm} \\
h = 150 \text{ mm} / \text{S235J2} + \text{C470}
\]

**Bolt connection**: 2 x M24 10.9

**Shear load of the joint**: \(V_{Ed} = 180 \text{ kN}\)

**Connection between the girder HE400A and the anchor plate**

The small torsion moment caused by the eccentricity between the girder and the butt straps is transferred into the girder HE400A and from this primary girder to the secondary girders. The eccentric connection induces bending and shear stresses in the butt strap. In the following they are determined:

\[
M_{Ed} = V_{Ed} \cdot 0.1 = 18 \text{ kNm} \\
\tau_v = 1.5 \cdot \frac{V_{Ed}}{A_V} = 1.5 \cdot \frac{180}{5000} = 54.0 \leq 135.6 \text{ N/mm}^2 \\
\sigma = \frac{M_{Ed}}{W} = \frac{18}{250^2 \cdot 20 / 6} = 86.4 \leq 235.0 \text{ N/mm}^2
\]

The maximum forces don`t appear at the same place.

**Edge distances**

\[
e_1 = 65 \text{ mm} > 1.2 \cdot d_0 = 1.2 \cdot 26 = 31.2 \text{ mm} \\
e_2 = 50 \text{ mm} > 1.2 \cdot d_0 = 1.2 \cdot 26 = 31.2 \text{ mm} \\
p_1 = 120 \text{ mm} > 2.2 \cdot d_0 = 2.2 \cdot 26 = 57.2 \text{ mm}
\]

**Shear resistance of the bolts**: 

EN 3-1-8

Table 3.3
F_{V,Rd} = \alpha_V \cdot A_S \cdot \frac{f_{ub}}{\gamma_{M2}} = 0.6 \cdot 353 \cdot \frac{1000}{1.25} = 169.4 \text{kN} \quad \text{EN 3-1-8 Table 3.4}

V_{Rd,1} = n_V \cdot F_{V,Rd} = 2 \cdot 169.4 = 338.8 \text{kN}

**Bearing resistance of the butt strap:**

\[ V_{Rd,2} = 286.8 \text{kN} \]

\[ F_{b,Rd} = \frac{k_1 \cdot \alpha_b \cdot f_u \cdot d \cdot t}{\gamma_{M2}} = \frac{2.5 \cdot 0.83 \cdot 360 \cdot 24 \cdot 20}{1.25} = 286.8 \text{kN} \quad \text{EN 3-1-8 Table 3.4}

\[ k_1 = \min \left[ 2.8 \frac{e_2}{d_0} - 1.7 ; 1.4 \frac{p_2}{d_0} - 1.7 ; 2.5 \right] = \min [3.68; -; 2.5] \]

\[ \alpha_b = \min \left[ \frac{e_1}{3 \cdot d_0} ; \frac{f_{ub}}{f_u} ; 1.0 \right] = \min [0.83; 2.78; 1.0] \]

**Bearing resistance of the beam web:**

\[ V_{Rd,3} = 190.1 \text{kN} \]

\[ F_{b,Rd} = \frac{k_1 \cdot \alpha_b \cdot f_u \cdot d \cdot t}{\gamma_{M2}} = \frac{2.5 \cdot 1.0 \cdot 360 \cdot 24 \cdot 11}{1.25} = 190.1 \text{kN} \quad \text{EN 3-1-8 Table 3.4}

\[ k_1 = \min \left[ 2.8 \frac{e_2}{d_0} - 1.7 ; 1.4 \frac{p_2}{d_0} - 1.7 ; 2.5 \right] = \min [3.68; -; 2.5] \]

\[ \alpha_b = \min \left[ \frac{e_1}{3 \cdot d_0} ; \frac{f_{ub}}{f_u} ; 1.0 \right] = \min [-; 2.78; 1.0] \]

\[ V_{Rd} = \min [V_{Rd,1}; V_{Rd,2}; V_{Rd,3}] = 190.1 \text{kN} \geq V_{Ed} = 180 \text{kN} \]

**Welding of the butt straps to the anchor plate**

A welding seam all around with \( a_w = 7 \text{ mm} \) is assumed. Following stresses in the welding seam can be determined:

\[ a_w = 2 \cdot 7 = 14 \text{ mm} \]

\[ l_{eff} = 250 \text{ mm} \]

\[ W_{el,w} = \frac{a_w \cdot l_{w,eff}^2}{6} = \frac{14 \cdot 250^2}{6} = 145.8 \cdot 10^3 \text{ mm}^2 \]

\[ \sigma_{w,Rd} = \frac{f_u}{\beta_w \cdot \gamma_{M2}} = \frac{360}{0.8 \cdot 1.25} = 360 \text{ N/mm}^2 \]

**Shear stresses caused by shear load and eccentricity:**

\[ \tau_{II} = \frac{V_{Ed}}{2 \cdot a_w \cdot l_{w,eff}} = \frac{180}{2 \cdot 7 \cdot 250} = 51.4 \text{ N/mm}^2 \]

\[ \sigma_w = \frac{M_{Ed}}{W} = \frac{18}{145.8} = 123.5 \text{ N/mm}^2 \]

\[ \sigma_\perp = \sigma_w \cdot \sin 45^\circ = 123.5 \cdot \sin 45^\circ = 87.3 \leq \frac{0.9 \cdot f_u}{\gamma_{M2}} = 259.2 \text{ N/mm}^2 \]

**Interaction caused by bending and shear stresses:**

\[ \sigma_{w,Ed} = \sqrt{\sigma_\perp^2 + 3(\tau_{II}^2 + \tau_{II}^2)} = \sqrt{87.3^2 + 3(87.3^2 + 51.4^2)} = 195.0 \leq \sigma_{w,Rd} = 360 \text{ N/mm}^2 \]
Design of the connection to the concrete

The anchor plate has the geometry 300 x 250 x 25 mm S235

Headed studs
\[ d = 22 \text{ mm} \]
\[ h = 150 \text{ mm S350 C470} \]

Stirrups (for each headed stud)
\[ 4 \cdot 8 \text{ mm B 500 A} \]

The verification of the design resistance of the joint is described in a stepwise manner. The eccentricity \( e_v \) and the shear force \( V_{Ed} \) are known.

Step 1 Evaluation of the tension force caused by the shear load

If the joint is loaded in shear the anchor row on the non-loaded side of the anchor plate is subjected to tension. In a first step the tension load has to be calculated. Therefore the height of the compression area has to be assumed.

Shear load of the connection
\[ V_{Ed} = 180 \text{ kN} \]

Resistance due to friction
\[ V_f = C_{Ed} \cdot 0.2 = N_{Ed,2} \cdot 0.2 \]

Thickness plate
\[ t_p = 25 \text{ mm} \]

Diameter anchor
\[ d = 22 \text{ mm} \]

Eccentricity
\[ e_v = 100 \text{ mm} \]

Calculation of \( N_{Ed,2} \)
\[
N_{Ed,2} = \frac{V_{Ed} \cdot (e_v + d + t_p) - V_f \cdot d}{z} \\
N_{Ed,2} \cdot (1 + \frac{0.2 \cdot d}{z}) = \frac{V_{Ed} \cdot (e_v + d + t_p)}{z} \\

The height of the compression zone is estimated to \( x_c = 20 \text{ mm} \)

With \( x_c \) the lever arm
\[ z = 40 + 220 - \frac{x}{2} = 40 + 220 - \frac{20}{2} = 250 \text{ mm} \]

and
\[
N_{Ed,2} \cdot \left(1 + \frac{0.2 \cdot 22}{250}\right) = \frac{V_{Ed} \cdot (100 + 22 + 25)}{250} \\
\]

From this the tension force result \( N_{Ed,2} = 104.0 \text{ kN} \)

Step 2 Verification of the geometry of the compression zone

The tension component of the joint \( N_{Ed,2} \) forms a vertical equilibrium with the compression force \( C_{Ed} \) under the anchor plate on the loaded side. The next step of the calculation is to prove that the concrete resistance is sufficient for the compression force and that the assumption of the compression area was correct.

Calculation of the compression force
\[
\sum N: C_{Ed} = N_{Ed,2} = 104.0 \text{ kN} \\

Height of the compression zone is
\[ f_{cd} = f_{ck} \cdot \frac{\alpha}{\gamma_{Mc}} = 17 \text{ N/mm}^2 \]

where \(\alpha = 0.85\)

The compression forces are causing a bending moment in the anchor plate. To make sure that the anchor plate is still elastic, only the part of the anchor plate is activated which is activated with elastic bending only.

\[ b_{\text{eff}} = t_{bs} + 2 \cdot t_p \cdot \frac{f_y}{\sqrt{3 \cdot f_{yd} \cdot \gamma_{M0}}} = 20 + 2 \cdot 25 \cdot \frac{235}{\sqrt{3 \cdot 17 \cdot 1.0}} = 127 \text{ mm} \]

\[ x_c = \frac{C_{Ed}}{b \cdot 3 \cdot f_{cd}} = \frac{104.0}{127 \cdot 3 \cdot 17} = 16 \text{ mm} \]

Instead of the regular width \(b\) of the anchor plate the effective width \(b_{\text{eff}}\) is used. The calculated \(x_c = 16\) mm is smaller than the predicted value of \(x_c = 20\) mm. That means that the lever arm was estimated slightly too small. This is on the safe side, so the calculation may be continued.

**Step 3 Evaluation of the tension resistance**

### 3.1 Steel failure of the fasteners

Calculation of the characteristic failure load of the headed studs on the non-loaded side:

\[ N_{Rd,s} = n_a \cdot A_s \cdot \frac{f_{uk}}{\gamma_{Mp}} = 2 \cdot 380 \cdot \frac{470}{1.5} \cdot 10^{-3} = 238.1 \text{ kN} \]

where

- Characteristic ultimate strength \(f_{uk} = 470 \text{ N/mm}^2\)
- Characteristic yield strength \(f_{yk} = 350 \text{ N/mm}^2\)
- Number of headed studs in tension \(n_a = 2\)
- Cross section area of one shaft \(A_s = \pi \cdot \frac{d^2}{4} = 380 \text{ mm}^2\)
- Partial safety factor \(\gamma_{Mp} = 1.2 \cdot \frac{f_{uk}}{f_{yk}} = 1.5\)

### 3.2 Pull-out failure

If the concrete strength is too low or the load bearing area of the headed stud is too small, pull-out failure might occur.

\[ N_{Rd,p} = n \cdot \frac{P_{uk}}{\gamma_{Mc}} \cdot A_h = n \cdot \frac{P_k \cdot f_{ck}}{\gamma_{Mc}} \cdot \frac{\pi}{4} \cdot (d_h^2 - d_{s,nom}^2) = 2 \cdot \frac{12 \cdot 30}{1.5} \cdot \frac{\pi}{4} \cdot (35^2 - 22^2) = 279.4 \text{ kN} \]

where

- Factor considering the head pressing \(P_k = 12 \cdot f_{ck}\)
- Partial safety factor \(\gamma_{Mc} = 1.5\)

### 3.3 Concrete cone failure
A pure concrete cone failure should not occur because of the reinforcement, but this failure load has to be calculated so that the resistance may be combined with the resistance of the stirrups.

\[ N_{Rd,c} = N_{Rk,c}^0 \cdot \psi_{A,N} \cdot \psi_{S,N} \cdot \psi_{re,N} / \gamma_{Mc} \]

\[ N_{Rk,c}^0 = k_1 \cdot h_{ef} \cdot f_{ck}^{0.5} = 12.7 \cdot 165^{1.5} \cdot 30^{0.5} = 147.4 \text{ kN} \]

\[ \psi_{A,N} = \frac{A_{c,N}}{A_{c,N}^0} = \frac{319\,275}{245\,025} = 1.3 \] \( \text{DM I} \)

\[ A_{c,N}^0 = s_{cr,N}^2 = (2 \cdot c_{cr,N})^2 = (2 \cdot 1.5 \cdot h_{ef})^2 = (2 \cdot 1.5 \cdot 165)^2 = 245\,025 \text{ mm}^2 \]

\[ N_{Rk,c} = 147.4 \cdot 1.3 \cdot 1.0 \cdot 1.0 = 191.6 \text{ kN} \]

\[ N_{Rd,c} = \frac{N_{Rk,c}}{\gamma_{Mc}} = \frac{191.6}{1.5} = 127.7 \text{ kN} \] \( \text{DM I} \)

\[ \text{where} \]

- Effective anchorage depth \( h_{ef} = h_n + t_{AP} - k = 165 \text{ mm} \)
- Factor for close edge \( \psi_{S,N} = 1.0 \) \( \text{Eq. (3.8)} \)
- Factor for small reinforcement spacing \( \psi_{re,N} = 1.0 \) \( \text{Eq. (3.9)} \)

\[ \text{Actual projected area} \]

\[ A_{c,N} = (2 \cdot 1.5 \cdot h_{ef}) \cdot (2 \cdot 1.5 \cdot h_{ef} + s_1) = (2 \cdot 1.5 \cdot 165) \cdot (2 \cdot 1.5 \cdot 165 + 150) = 319\,275 \text{ mm}^2 \] \( \text{Eq. (3.11)} \)

\[ \gamma_{Mc} = 1.5 \]

\[ \text{Partial safety factor} \]

\[ \text{3.4 Concrete cone failure with reinforcement} \]

With reinforcement one of the three below described failure modes will occur.

\[ \text{3.5 Concrete failure} \]

\[ N_{Rk,cs} = \psi_{supp} \cdot N_{Rk,u,c} = 2.26 \cdot 191.6 = 433.0 \text{ kN} \]

\[ N_{Rd,cs} = \frac{N_{Rk,cs}}{\gamma_{Mc}} = \frac{433.0}{1.5} = 288.7 \text{ kN} \] \( \text{DM I} \)

\[ \text{where} \]

- Factor for support of reinforcement \( \psi_{supp} = 2.5 - \frac{x}{h_{ef}} = 2.26 \) \( \text{Ch. 3.2.4} \)

\[ \text{Distance between the anchor axis and the crack on the surface} \]

\[ x = \frac{d_{nom}}{2} + d_{sa} + \frac{d_{st}}{\tan 35^\circ} = 40 \text{ mm} \] \( \text{DM I} \)

\[ \text{Distance of hanger reinforcement to the face of the anchor shaft} \]

\[ d_{sa} = 5 \cdot \frac{d}{2} = 9 \text{ mm} \]

\[ \text{Distance axis of the reinforcement to the concrete surface} \]

\[ d_{st} = \frac{d}{2} + 10 = 14 \text{ mm} \]

\[ \gamma_{Mc} = 1.5 \]

\[ \text{3.6 Yielding of reinforcement} \]

\[ N_{Rd,re,1} = N_{Rd,sa} + N_{Rd,c} + \delta_{Rd,sa} \cdot k_{cde} \]

\[ N_{Rd,sa} = 174.8 + 127.7 + 0.642 \cdot 49.1 = 271.0 \text{ kN} \] \( \text{DM I} \)

\[ \text{where} \]

- Normal force of hanger reinforcement
Deformation of reinforcement at yielding

\[
\delta_{\text{Rd,re}} = \frac{2 \cdot (A_{s,y} \cdot f_{s,yd})^2}{\alpha_s \cdot f_{ck} \cdot d_{s,re}^4 \cdot (n \cdot n_{re})^2} = \frac{2 \cdot (174.8 \cdot 10^3)^2}{12100 \cdot 30 \cdot 8^4 \cdot (2 \cdot 4)^2} = 0.642 \text{ mm}
\]

Stiffness concrete break out

\[
k_{c,de} = \alpha_c \cdot \sqrt{f_{ck} \cdot h_{ef} \cdot \psi_{AN} \cdot \psi_{s,N} \cdot \psi_{re,N}} = -537 \cdot \sqrt{30 \cdot 165 \cdot 1.3 \cdot 1.0 \cdot 1.0} = -49.1 \text{ kN/mm}
\]

Partial safety factor \( \gamma_{Ms} = 1.15 \)

3.7 Anchorage failure of the reinforcement

\[
N_{\text{Rd,ref}} = N_{\text{Rd,b,ref}} + N_{\text{Rd,c}} + \delta_{\text{Rd,b,ref}} \cdot k_{c,de}
\]

\( N_{\text{Rd,ref}} = 147.7 + 127.7 + 0.459 \cdot (-49.1) = 252.8 \text{ kN} \)

where

- Anchorage force of all hanger legs
  \[
  N_{\text{Rd,b,ref}} = n \cdot n_{re} \cdot l_1 \cdot \pi \cdot d_s \cdot \frac{f_{bd}}{\alpha}
  \]
  \[
  N_{\text{Rd,b,ref}} = 2 \cdot 4 \cdot 120 \cdot \pi \cdot 8 \cdot \frac{3 \cdot 0.49}{12100} \cdot 10^{-3}
  = 147.7 \text{ kN}
  \]

- Anchorage length of the hanger
  \[
  l_1 = h_{ef} - d_p - d_{s,t} - \frac{d_{sa}}{1.5} = 165 - 25 - 14 - \frac{9}{1.5} = 120 \text{ mm}
  \]

- Dist. hanger reinforcement to the face of the anchor shaft:
  \[
  d_{sa} = 5 \cdot \frac{d_s}{2} - \frac{d}{2} = 5 \cdot \frac{8}{2} - \frac{12}{2} = 9 \text{ mm}
  \]

- Dist. axis of the reinforcement to the concrete surface
  \[
  d_{st} = \frac{d_s}{2} + 10 = 14 \text{ mm}
  \]

Bond strength

\[
f_{bd} = 2.25 \cdot \eta_1 \cdot \eta_2 \cdot \frac{f_{ck}}{\gamma_{Mc}} = 2.25 \cdot 1 \cdot 1 \cdot \frac{2}{1.5} = 3.0 \text{ N/mm}^2
\]

where \( \eta_1 \) is coefficient of bond conditions, \( \eta_1 = 1.0 \) for vertical stirrups and 0.7 for horizontal stirrups, \( \eta_2 = 1.0 \) for dimension ≤ 32 mm and (132 - dimension)/100 for dimension ≥ 32 mm

Hook \( \alpha = 0.49 \)

Def. of the reinforcement at bond failure

\[
\delta_{\text{Rd,b,ref}} = \frac{2 \cdot (N_{\text{Rd,b,ref}})^2}{\alpha_s \cdot f_{ck} \cdot d_{s,ref}^4 (n \cdot n_{re})^2} = \frac{2 \cdot (147.7 \cdot 10^3)^2}{12100 \cdot 30 \cdot 8^4 \cdot (2 \cdot 4)^2} = 0.459 \text{ mm}
\]

Partial safety factor \( \gamma_{Mc} = 1.5 \)

The decisive component of the three failure modes of the concrete cone failure with reinforcement is the anchorage failure of the reinforcement. The anchors have a tension resistance of \( N_{\text{Rd,u}} = N_{\text{Rd,ref}} = 252.8 \text{ kN} \)

Step 4 Evaluation of the shear resistance

4.1 Steel failure of the fasteners
\[ F_{v,Rd} = \frac{n_{av} \cdot 0.6 \cdot f_{uk} \cdot A_s}{\gamma_{M2}} = \frac{2 \cdot 0.6 \cdot 470 \cdot \pi \left(\frac{22}{2}\right)^2}{1.25} = 171.5 \text{kN} \]

4.2 Pry-out failure
\[ V_{Rd,CP} = k_3 \cdot N_{Rd,cc+group} = 2 \cdot 184.9 = 369.9 \text{kN} \]

where
Min. component concrete failure
\[ N_{Rd+group} = \min\{N_{Rd,cc}; N_{Rd,cre}; N_{Rd,cre}; N_{Rd,cc,group}\} \]
\[ \min\{288.7 \text{kN}; 271.0 \text{kN}; 252.8 \text{kN}, 184.9 \text{kN}\} \]
Partial safety factor \( \gamma_{Mc} = 1.5 \)

According to the Technical Specifications the factor \( k_3 \) is taken as 2.0. There are not yet made examinations how the resistance \( V_{Rd,CP} \) may be calculated taking account of the reinforcement. Therefore \( N_{Rd,cc+hr} \) is determined as the minimum value of the concrete cone failure with reinforcement \( (N_{Rk,u,\text{max}}, N_{Rd,u,1}, N_{Rd,u,2}) \) and the concrete cone failure of the whole anchor group without considering additional reinforcement \( (N_{Rd,u,c}) \). \( N_{Rd,u,c} \) is calculated in the following.

\[ N_{Rk,u,\text{group}} = N_{0,u,c} \cdot \frac{A_{c,N}}{A_{c,N}^0} \cdot \Psi_{se,N} \cdot \Psi_{re,N} \cdot \Psi_{ec,N} \]

\[ N_{Rk,u,\text{group}} = 147.4 \cdot \frac{461175}{245025} \cdot 1.0 \cdot 1.0 \cdot 1.0 = 277.4 \text{kN} \]

\[ N_{Rd,u,\text{group}} = \frac{N_{Rk,u,\text{c}}}{\gamma_{Mc}} = \frac{277.4 \text{kN}}{1.5} = 184.9 \text{kN} \]

where
\[ N_{0,u,c} = k_1 \cdot f_{ck}^{0.5} \cdot h_{ef}^{1.5} = 12.7 \cdot 30^{0.5} \cdot 165^{1.5} \cdot 10^{-3} = 147.4 \text{kN} \]
Effective anchorage depth \( h_{ef} = h_n - t_{AP} = 150 - 10 + 25 = 165 \text{mm} \)
Factor for close edge \( \Psi_{se,N} = 1.0 \)
Factor for small reinforcement spacing \( \Psi_{re,N} = 1.0 \)
Factor for eccentricity of loading \( \Psi_{ec,N} = 1.0 \)
Reference projected area \( A_{c,N}^0 = s_{crN}^2 = 495^2 = 245025 \text{mm}^2 \)
Actual projected area \( A_{c,N} = (s_{crN} + s_2) \cdot (s_{crN} + s_1) \)
\[ = (495 + 220) \cdot (495 + 150) = 461175 \text{mm}^2 \]

Step 5 Verification of interaction conditions

5.1 Interaction of tension and shear for steel failure
Shear load in the headed studs on the non-loaded side is
\[ V_{Ed,2} = V_{Ed} - V_{Rd,5} - V_{f} = 180 - 190.1 - 20.8 = -31.0 \text{kN} \]
All loads is taken by the front anchor. No load for the back anchor and

\[ \left(\frac{N_{Ed,2}}{N_{Rd,5}}\right)^2 + \left(\frac{V_{Ed,2}}{V_{Rd}}\right)^2 \leq 1 \]
\[ \left(\frac{104.0}{238.1}\right)^2 + \left(\frac{0}{171.5}\right)^2 = 0.19 \leq 1 \]
5.2 Interaction of tension and shear for concrete failure

Shear load in the headed studs on the non-loaded side is

\[ V_{Ed,2} = \frac{V_{Ed} - V_f}{2} = \frac{180 - 20}{2} = 80 \text{kN} \]

\[ \left( \frac{N_{Ed,2}}{N_{Rd,u}} \right)^{3/2} + \left( \frac{V_{Ed,2}}{V_{Ed}} \right)^{3/2} \leq 1 \]

\[ \left( \frac{104.0}{252.8} \right)^{3/2} + \left( \frac{80}{184.9} \right)^{3/2} = 0.57 \leq 1 \]

Note

Without the additional reinforcement there would be a brittle failure of the anchor in tension in concrete. The resistance of pure concrete cone failure with reinforcement is nearly two times the size of the resistance without reinforcement. With the additional reinforcement there is a ductile failure mode with reserve capacity.
9.6 Moment resistant steel-to-concrete joint

The steel-to-concrete connection is illustrated in Fig. 9.27. It represents the moment-resistant support of a steel-concrete-composite beam system consisting of a hot rolled or welded steel profile and a concrete slab, which can either be added in situ or by casting semi-finished precast elements. Beam and slab are connected by studs and are designed to act together. Whereas the advantage of the combined section is mostly seen for positive moments, where compression is concentrated in the slab and tension in the steel beam, it may be useful to use the hogging moment capacity of the negative moment range either as a continuous beam, or as a moment resistant connection. In this case, the reinforcement of the slab is used to raise the inner lever arm of the joint. The composite beam is made of a steel profile IPE 300 and a reinforced concrete slab with a thickness of 160 mm and a width of 700 mm. The concrete wall has a thickness of 300 mm and a width of 1450 mm. The system is subjected to a hogging bending moment $M_{E,d} = 150$ kNm. Tabs 9.1 and 9.2 summarize data for the steel-to-concrete joint.

![Fig. 9.27: Geometry of the moment resisting joint](image)

Tab. 9.1 Geometry for the steel-to-concrete joint

<table>
<thead>
<tr>
<th>Geometry</th>
<th>RC wall</th>
<th>RC Slab</th>
<th>Anchors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Anchor plate</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t$ [mm]</td>
<td>300</td>
<td>160</td>
<td>22</td>
</tr>
<tr>
<td>$b$ [mm]</td>
<td>1450</td>
<td>700</td>
<td>35</td>
</tr>
<tr>
<td>$h$ [mm]</td>
<td>1600</td>
<td>1550</td>
<td>200</td>
</tr>
<tr>
<td><strong>Reinforcement</strong></td>
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<td></td>
</tr>
<tr>
<td>$d$ [mm]</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$d_e$ [mm]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$l$ [mm]</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$l_a$ [mm]</td>
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<td></td>
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</tr>
<tr>
<td>$n_v$ [mm]</td>
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<td>16</td>
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</tr>
<tr>
<td>$n_v$ [mm]</td>
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<td>2</td>
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</tr>
<tr>
<td>$s_v$ [mm]</td>
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<td>120</td>
<td>50</td>
</tr>
<tr>
<td>$s_v$ [mm]</td>
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<td>10</td>
<td>200</td>
</tr>
<tr>
<td>$t$ [mm]</td>
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<td>14</td>
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</tr>
<tr>
<td>$t$ [mm]</td>
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</tr>
<tr>
<td><strong>Steel beam IPE 300</strong></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$h$ [mm]</td>
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</tr>
<tr>
<td>$b$ [mm]</td>
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</tr>
<tr>
<td>$t$ [mm]</td>
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</tr>
<tr>
<td>$t$ [mm]</td>
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<td>$l$ [mm]</td>
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<tr>
<td>$A_s$ [mm$^2$]</td>
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<tr>
<td><strong>Contact Plate</strong></td>
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<td>$t$ [mm]</td>
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<td>$b_{cp}$ [mm]</td>
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<td>$l_{cp}$ [mm]</td>
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<tr>
<td>$e_{f, cp}$ [mm]</td>
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<tr>
<td>$e_{h, cp}$ [mm]</td>
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<tr>
<td>$b_{ap}$ [mm]</td>
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<td></td>
</tr>
</tbody>
</table>

---

**Notes:**
- $M_{E,d} = 150$ kNm
- Steel profile IPE 300
- Concrete slab: $t = 160$ mm, $b = 700$ mm
- Concrete wall: $h = 300$ mm, $b = 1450$ mm
- Reinforcement details for slab and beam
- Anchor plate dimensions
- Contact plate dimensions
The part of the semi-continuous joint configuration, within the reinforced concrete wall, adjacent to the connection, is analyzed in this example. This has been denominated as “Joint Link”. The main objective is to introduce the behaviour of this component in the global analysis of the joint which is commonly disregarded.

### Tab. 9.2 Material of the steel-to-concrete joint

<table>
<thead>
<tr>
<th>Concrete wall</th>
<th>Concrete slab</th>
<th>Rebars wall</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{c,k,cube}$ [Mpa]</td>
<td>50</td>
<td>$f_{c,k,cube}$ [Mpa]</td>
</tr>
<tr>
<td>$f_{c,k,cyl}$ [Mpa]</td>
<td>40</td>
<td>$f_{c,k,cyl}$ [Mpa]</td>
</tr>
<tr>
<td>E [GPa]</td>
<td>36</td>
<td>E [GPa]</td>
</tr>
<tr>
<td>$f_{y,k}$ [Mpa]</td>
<td>3.51</td>
<td>$f_{y,k}$ [Mpa]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rebars Slab</th>
<th>Steel Plates</th>
<th>Anchors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{y,k}$ [Mpa]</td>
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<td>$f_{y,k}$ [Mpa]</td>
</tr>
<tr>
<td>$f_{u}$ [Mpa]</td>
<td>540</td>
<td>$f_{u}$ [Mpa]</td>
</tr>
<tr>
<td>$\varepsilon_{sry}$ [%]</td>
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<td>$\varepsilon_{sry}$ [%]</td>
</tr>
<tr>
<td>$\varepsilon_{sru}$</td>
<td>75</td>
<td>$\varepsilon_{sru}$</td>
</tr>
</tbody>
</table>

The design value of the modulus of elasticity of steel $E_s$ may be assumed to be 200 GPa.

![Fig. 9.28 Activated joint components](image)

In order to evaluate the joint behaviour, the following basic components are identified, as shown in Fig. 9.28:
- longitudinal steel reinforcement in the slab, component 1
- slip of the composite beam, component 2;
- beam web and flange, component 3;
- steel contact plate, component 4;
- components activated in the anchor plate connection, components 5 to 10 and 13 to 15;
- the joint link, component 11.

### Step 1 Component longitudinal reinforcement in tension

In this semi-continuous joint configuration, the longitudinal steel reinforcement bar is the only component that is able to transfer tension forces from the beam to the wall. In addition, the experimental investigations carried (Kuhlmann et al., 2012) revealed the importance of this component on the joint response. For this reason, the accuracy of the model to predict the joint response will much depend on the level of accuracy introduced in the modelling of this component. According to ECCS Publication Nº 109 (1999), the behaviour of the longitudinal steel reinforcement in tension is illustrated in Fig. 9.29.
The resistance of the component may then be determined as follows

\[ F_{sr} = A_{sr} f_{yr} \]

Since concrete grades of wall and slab are different it is possible to evaluate separately the stress-strain curve of the two elements. While the concrete is uncracked, the stiffness of the longitudinal reinforcement is considerably higher when compared with bare steel. Cracks form in the concrete when mean tensile strength of the concrete \( f_{ctm} \) is achieved. The stress in the reinforcement at the beginning of cracking (\( \sigma_{sr1} \)) is determined as follows.

\[
\sigma_{sr1,SLAB} = \frac{\sigma_{sr1,SLAB}}{\gamma_{Ms}} = \frac{f_{ctm,SLAB} \cdot k_c \left[ 1 + \rho \frac{E_s}{E_c} \right]}{Y_{Ms} \cdot \rho} = 2.87 \cdot 0.39 \frac{1}{1.15 \cdot 0.010} \left[ 1 + 0.010 \cdot 6.06 \right] Nmm^{-2}
\]

\[
\sigma_{sr1,WALL} = \frac{\sigma_{sr1,WALL}}{\gamma_{Ms}} = \frac{f_{ctm,WALL} \cdot k_c \left[ 1 + \rho \frac{E_s}{E_c} \right]}{Y_{Ms} \cdot \rho} = 3.51 \cdot 0.39 \frac{1}{1.15 \cdot 0.010} \left[ 1 + 0.010 \cdot 6.06 \right] Nmm^{-2}
\]

where: \( f_{ctm} \) is the tensile strength of the concrete; \( E_s \) and \( E_c \) are the elastic modulus of the steel reinforcement bar and concrete, \( k_c \) is a factor which allows using the properties of the steel beam section and \( \rho \) is the ratio between the area of steel reinforcement and the area of concrete flange expressed as follows:

\[
k_c = \frac{1}{1 + \frac{t_{slab}}{2 \cdot z_0}} = \frac{1}{1 + \frac{160}{2 \cdot 51.8}} = 0.39
\]

\[
\rho = \frac{A_{s,r}}{A_{c,slab}} = \frac{n_1 \cdot \pi \cdot \Phi h^2 / 4}{b_{eff,slab} \cdot t_{slab}} = \frac{1.206.4}{700 \cdot 160} = 0.010
\]

where: \( A_{c,slab} \) is the area of the effective concrete slab; \( A_{s,r} \) is the area of the longitudinal reinforcement within the effective slab width (in this example the width of the slab is fully effective); \( t_{slab} \) is the thickness of the concrete flange and \( z_0 \) is the vertical distance between the centroid of the uncracked concrete flange and uncracked unreinforced composite section, calculated using the modular ratio for short-term effects, \( E_s/E_c \).

\[
z_0 = x_{c,h} - \frac{t_{slab}}{2} = \left[ \frac{b_{eff} \cdot E_c}{E_s} \cdot \frac{t_{slab}}{2} + \left( \frac{t_{slab}}{2} + \frac{h_{IPE300}}{2} \right) \cdot A_{IPE300} \right] - \frac{t_{slab}}{2} = 51.8 \text{ mm}
\]

\[ \text{CEB-FIB Model Code (1990)} \]
where \( x_c, h \) is the dimension of the component concrete block in compression.

According to CEB-FIB Model Code (1990), the stress \( \sigma_{sr,d} \) and the increment of the reinforcement strain \( \Delta \varepsilon_{sr} \) are given by

\[
\Delta \varepsilon_{sr,SL} = \frac{f_{ctm,SL} \cdot k_c}{\gamma_s \cdot E_s \cdot \rho} = 0.00045 \\
\Delta \varepsilon_{sr,WA} = \frac{f_{ctm,WA} \cdot k_c}{\gamma_s \cdot E_s \cdot \rho} = 0.00056
\]

\[
\varepsilon_{sr,SL} = \frac{\sigma_{sr,1,SL}}{E_s} - \Delta \varepsilon_{sr,SL} = 3.0 \cdot 10^{-5} \\
\varepsilon_{sr,WA} = \frac{\sigma_{sr,1,WA} - \Delta \varepsilon_{sr,WA}}{E_s} = 3.6 \cdot 10^{-5}
\]

\[
\sigma_{sr,d,SL} = 1.3 \cdot \sigma_{sr,1,SL} = 126.2 \text{Nmm}^{-2} \\
\sigma_{sr,d,WA} = 1.3 \cdot \sigma_{sr,1,WA} = 154.3 \text{Nmm}^{-2}
\]

\[
\varepsilon_{sr,SL} = \varepsilon_{sr,1,SL} + \Delta \varepsilon_{sr,SL} = 4.9 \cdot 10^{-4} \\
\varepsilon_{sr,WA} = \varepsilon_{sr,1,WA} + \Delta \varepsilon_{sr,WA} = 5.9 \cdot 10^{-4}
\]

The yield stress and strain, \( f_{sy,k} \) and \( \varepsilon_{sm,y} \) are given by

\[
f_{sy,k,d} = \frac{f_{sy,k}}{\gamma_s} = \frac{400}{1.15} = 347.8 \text{Nmm}^{-2}
\]

\[
\varepsilon_{sm,y,SL} = \frac{f_{sy,k,d} - \sigma_{sr,d,SL}}{E_s} + \varepsilon_{sr,SL} = 1.6 \cdot 10^{-3}
\]

\[
\varepsilon_{sm,y,WA} = \frac{f_{sy,k,d} - \sigma_{sr,d,WA}}{E_s} + \varepsilon_{sr,WA} = 1.6 \cdot 10^{-3}
\]

The ultimate strain \( \varepsilon_{sm,u} \) is determined as follows, where the tension stiffening is also taken into account. The factor \( \beta_t = 0.4 \) takes into account the short-term loading; and for high-ductility bars, \( \delta \) is taken equal to 0.8.

\[
\varepsilon_{sm,u,SL} = \varepsilon_{sy} - \beta_t \Delta \varepsilon_{sr,SL} + \delta \left( 1 - \frac{\sigma_{sr,d,SL}}{f_{sy,k,d}} \right) (\varepsilon_{su} - \varepsilon_{sy}) = 4.4 \cdot 10^{-2}
\]

\[
\varepsilon_{sm,u,WA} = \varepsilon_{sy} - \beta_t \Delta \varepsilon_{sr,WA} + \delta \left( 1 - \frac{\sigma_{sr,d,WA}}{f_{sy,k,d}} \right) (\varepsilon_{su} - \varepsilon_{sy}) = 4.0 \cdot 10^{-2}
\]

where: \( \varepsilon_{sy} \) and \( f_{sy,k,d} \) are the yield strain and stress of the bare steel reinforcement bars, respectively; \( \varepsilon_{su} \) is the ultimate strain of the bare steel reinforcement bars.

Assuming the area of reinforcement constant, the force-deformation curve is derived from the stress-strain curve, where the reinforcement deformation should be evaluated as described above.

\( \Delta = \varepsilon \cdot l \)

The elongation length (l) to consider is equal to sum of the \( L_t \) (related to the slab) with \( h_c \) (related to the wall). Only in the determination of the ultimate deformation capacity, the length of the reinforcement bar is considered higher than this value, as expressed in the following:

\[
\rho < 0.8 \% \quad \Delta \varepsilon_{sr,u} = 2 \cdot L_t \varepsilon_{sm,u} \\
\rho \geq 0.8 \% \text{ and } a \leq L_t \quad \Delta \varepsilon_{sr,u} = (h_c + L_t) \varepsilon_{sm,u} \\
\rho \geq 0.8 \% \text{ and } a > L_t \quad \Delta \varepsilon_{sr,u} = (h_c + L_t) \varepsilon_{sm,u} + (a - L_t) \varepsilon_{sr,mu}
\]

where is

\[
L_t = \frac{k_c \cdot f_{ctm} \cdot \Phi}{4 \cdot \tau_{sm} \cdot \rho} = \frac{0.39 \cdot 2.87 \cdot 16}{4 \cdot 5.16 \cdot 0.01} = 81 \text{ mm}
\]

In the above expression, \( L_t \) is defined as the transmission length and represents the length of the reinforcement from the wall face up to the first crack zone which should form close to the joint. The parameter \( a \) is the distance of the first shear connector to the joint and \( h_c \).
is the length of the reinforcement up to the beginning of the bend. $\tau_{sm}$ is the average bond stress, given by

$\tau_{sm} = 1.8 \cdot f_{ctm}$

Forces can be evaluated considering minimum values of tensions found for slab and wall. Table 9.3 summarizes the results for the stress-strain and force-displacement curves.

<table>
<thead>
<tr>
<th>$\sigma_{SL}$ [N/mm²]</th>
<th>$\varepsilon_{SL}$ [-]</th>
<th>$\sigma_{WA}$ [N/mm²]</th>
<th>$\varepsilon_{WA}$ [-]</th>
<th>$F$ [kN]</th>
<th>$\Delta_r$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>97.1</td>
<td>3.0 $\cdot$ 10^{-5}</td>
<td>118.7</td>
<td>3.6 $\cdot$ 10^{-5}</td>
<td>117.1</td>
<td>0.0</td>
</tr>
<tr>
<td>126.2</td>
<td>4.9 $\cdot$ 10^{-4}</td>
<td>154.3</td>
<td>5.9 $\cdot$ 10^{-4}</td>
<td>152.3</td>
<td>0.1</td>
</tr>
<tr>
<td>347.8</td>
<td>1.6 $\cdot$ 10^{-3}</td>
<td>347.8</td>
<td>1.6 $\cdot$ 10^{-3}</td>
<td>419.6</td>
<td>0.3</td>
</tr>
<tr>
<td>469.5</td>
<td>4.4 $\cdot$ 10^{-2}</td>
<td>469.5</td>
<td>4.0 $\cdot$ 10^{-2}</td>
<td>566.5</td>
<td>5.7</td>
</tr>
</tbody>
</table>

**Step 2 Component slip of composite beam**

The slip of composite beam is not directly related to the resistance of the joint; however, the level of interaction between the concrete slab and the steel beam defines the maximum load acting on the longitudinal reinforcement bar. In EN 1994-1-1: 2010, the slip of composite beam component is not evaluated in terms of resistance of the joint, but the level of interaction is considered on the resistance of the composite beam. However, the influence of the slip of the composite beam is taken into account on the evaluation of the stiffness and rotation capacity of the joint. The stiffness coefficient of the longitudinal reinforcement should be affected by a reduction factor $k_{slip}$ determined according to Chap. 3.7.

According to (Aribert, 1995) the slip resistance may be obtained from the level of interaction as expressed in the following. Note that the shear connectors were assumed to be ductile allowing redistribution of the slab-beam interaction load.

$$F_{slip} = N \cdot P_{RK}$$

Where: $N$ is the real number of shear connectors; and $P_{RK}$ is characteristic resistance of the shear connectors that can be determined according to EN1994-1-1:2010 as follows

$$P_{RK} = \min \left( \frac{0.8 \cdot f_{u} \cdot \pi \cdot d^2}{\gamma_{MV} \cdot 4}, \frac{0.29 \cdot \alpha \cdot d^2 \cdot f_{ck} \cdot E_{cm}}{\gamma_{MV}} \right)$$

with

$$3 \leq \frac{h_{sc}}{d} \leq 4 \quad \alpha = 0.2 \left( \frac{h_{sc}}{d} + 1 \right)$$

$$\frac{h_{sc}}{d} > 4 \quad \alpha = 1$$

where $f_{u}$ is the ultimate strength of the steel shear stud; $d$ is the diameter of the shear stud; $f_{ck}$ is the characteristic concrete cylinder resistance; $E_{cm}$ is the secant modulus of elasticity of the concrete; $h_{sc}$ is the height of the shear connector including the head; $\gamma_{V}$ is the partial factor for design shear resistance of a headed stud.

$$P_{RK} = \min \left( \frac{0.8 \cdot 540 \cdot \pi \cdot 22^2}{1.25 \cdot 4}, \frac{0.29 \cdot 1 \cdot 22^2 \cdot \sqrt{30 \cdot 33}}{1.25} \right) = \min(486.5; 111.0) = 111.0 \text{ kN}$$

$$F_{slip} = 9 \cdot 111.0 = 999.0 \text{ kN}$$

Concerning the deformation of the component, assuming an uniform shear load distribution along the beam, an equal distribution of the load amongst the shear studs is expected.
The stiffness of the component is obtained as a function of the number of shear studs and of the stiffness of a single row of shear studs, as follows

\[ k_{\text{slip}} = N \cdot k_{sc} = 900 \text{ kN/mm} \]

where the stiffness of one shear connector \( k_{sc} \) may be considered equal to 100 kN/mm, see cl A.3(4) in EN 1994-1-1:2010.

**Step 3 Component beam web and flange in compression**

According to EN1993-1-8:2006, the resistance can be evaluated as follows

\[ M_{c,Rd} = \frac{W_{pl} \cdot f_{yk}}{\gamma_{M0}} = \frac{628400 \cdot 355}{1.0} = 223.0 \text{ kN} \]

\[ F_{c,fb,Rd} = \frac{223000}{(300 - 10.7)} = 771.1 \text{ kN} \]

The stiffness of this component may be neglected.

**Step 4 Component steel contact plate in compression**

According to EN1994-1-1:2010, the resistance can be evaluated as follows and the stiffness is infinitely rigid compared to rest of the connection.

\[ F_{cp} = f_{y,cp} A_{eff,cp} = 440 \cdot 200 \cdot 30 = 2640 \text{ kN} \]

**Step 5 Component T-stub in compression**

According to EC 1993-1-8:2006, the bearing width \( c \) can be calculated using the hypothesis of cantilever beam for all directions. It is an iterative process as the bearing width and the concrete bearing strength \( f_j \) are mutually dependent.

\[ c = t_{ap} \cdot \sqrt{\frac{f_y}{3 \cdot f_{jd} \cdot \gamma_{M0}}} \]

\[ f_{jd} = \frac{\beta_j F_{Rd,u}}{b_{eff} l_{eff}} = \frac{\beta_j A_{c0} f_{cd}}{A_{c0} \sqrt{A_{c1}}} = \beta_j f_{cd} k_j \]

where \( \beta_j \) is the foundation joint material coefficient and \( F_{Rd,u} \) is the concentrated design resistance force. Assuming an uniform distribution of stresses under the equivalent rigid plate and equal to the bearing strength of the concrete, the design compression resistance of a T-stub should be determined as follows

\[ F_{C,Rd} = f_{jd} \cdot b_{eff} \cdot l_{eff} \]

where \( b_{eff} \) and \( l_{eff} \) are the effective width and length of the T-stub flange, given by

\[ A_{eff} = \min \left( 2c + b_{cp} + b_{ap} \right) \cdot \left( c + l_{cp} + \min \left( c; e_{1,cp} \right) \right) = 69.4 \cdot 239.4 = 16625.9 \text{ mm}^2 \]

and \( f_{jd} \) is the design bearing strength of the joint.

Thus, \( c = 19.7 \text{ mm}; f_{jd} = 84.9 \text{ MPa}; l_{eff} = 69.4 \text{ mm}; b_{eff} = 239.4 \text{ mm}; F_c = 1411.0 \text{ kN} \)

The initial stiffness \( S_{ini} \) may be evaluated as follows

\[ S_{ini} = \frac{E_c \sqrt{A_{eff}}}{1.275} \]

\( c \) is given by \( c = 1.25 \cdot t_{ap} \) and \( b_{eff} \) and \( l_{eff} \) are given by

\[ A_{eff} = \min \left( 2.5 t_{ap} + b_{cp} + b_{ap} \right) \cdot \left( 1.25 t_{ap} + l_{cp} + \min \left( 1.25 t_{ap}, e_{1,cp} \right) \right) = 67.5 \cdot 237.5 = 16031 \text{ mm}^2 \]
Thus, $c = 18.7$ mm; $l_{\text{eff}} = 67.5$ mm; $b_{\text{eff}} = 237.5$ mm and $S_{\text{in,ij}} = 3575.0$ kN/mm

This value of the initial stiffness could be used for the calculation of the component of displacement related to the T-stub in compression.

**Step 6 Joint Link**

In the proposed model based on the STM principles, the properties of this diagonal spring are determined as follows:

- The resistance is obtained based on the strut and nodes dimension and admissible stresses within these elements, given in Tab. 3.2.
- The deformation of the diagonal spring is obtained by assuming a non-linear stress-strain relation for the concrete under compression, as defined in (Henriques, 2013).

In terms of resistance, the model is characterized by the resistance of the nodes at the edge of the diagonal strut. Accordingly, the maximum admissible stresses, see Tab. 3.2, and the geometry of these nodes define the joint link load capacity. It is recalled that failure is governed by the nodal regions and disregarded within the strut. Hence, the resistance of the nodes is obtained as follows.

6a) **Node N1**

The geometry of the node is defined in one direction by the bend radius of the longitudinal reinforcement and by the strut angle $\theta$ with the dimension $a$ Fig. 9.30. In the other direction (along the width of the wall), assuming the distance between the outer longitudinal overestimates the resistance of this node, since the analytical approach assumes that the stresses are constant within the dimension $brb$ and the stress field “under” the hook and along this dimension is non-uniform.

![Fig. 9.30 Definition of the width of node N1](image)

According to Henriques (2013), in order to obtain a more accurate approach, an analytical expression was derived to estimate an effective width “under” each reinforcement bar where constant stresses can be assumed. The basis of this analytical expression was a parametrical study performed by means of numerical calculations.

In order to obtain an expression which could approximate the effective width with sufficient accuracy, a regression analysis, using the data produced in the parametrical study, was performed. The effective width $b_{\text{eff,rb}}$ of the reinforcement is calculated as a function of the reinforcement bar diameter $drb$, the spacing of bars $s_{\text{rb}}$ and strut angle $\theta$ as follows

$$s_{\text{rb}} \geq 80 \text{ mm} \quad b_{\text{eff,rb}} = n \cdot 2.62 \cdot d_{rb}^{0.96} \cdot (\cos \theta)^{-1.05}$$

$$s_{\text{rb}} < 80 \text{ mm} \quad b_{\text{eff,rb}} = n \cdot 2.62 \cdot d_{rb}^{0.96} \cdot (\cos \theta)^{-1.05} \cdot \left(\frac{s_{\text{rb}}}{80}\right)^{0.61}$$

As in this case $s_{\text{rb}} > 80$ mm
\[ \theta = \arctan \left( \frac{Z}{b} \right) = \arctan \left( \frac{406.65}{300 - \frac{16}{2} - \frac{10}{2} - 30 \cdot 2} \right) = 1.06 \text{ rad} \]

\[ a = 2 \cdot r_{hook} \cdot \cos(\theta) = 2 \cdot 160 \cdot \cos(1.06) = 155.97 \text{ mm} \]

\[ b_{\text{eff,rb}} = 6 \cdot 2.62 \cdot d_{\text{rb}}^{0.96} \cdot (\cos \theta)^{-1.05} = 478.054 \text{ mm} \]

The node dimensions are determined from

\[ A_{N1} = b_{\text{eff,rb}} \cdot 2 \cdot r \cdot \cos \theta \]

where \( A_{N1} \) is the cross-section area of the diagonal concrete strut at node N1. Finally, the resistance of the node is given by

\[ F_{r,N1} = A_{N1} \cdot 0.75 \cdot v \cdot f_{cd} = 1252.7 \text{ kN} \]

\[ v = 1 - \frac{f_{ck,y}}{250} = 0.84 \]

6b) Node N2

The geometry of the node, on the concrete strut edge, is defined by the projection of the dimensions of the equivalent rigid plate, representing the anchor plate subjected to compression, in the direction of the concrete strut, see Fig. 9.31.

The node dimensions are determined from

\[ A_{N2} = \frac{l_{\text{eff}}}{\cos \theta} \cdot b_{\text{eff}} = 35041.3 \text{ mm}^2 \]

where: \( A_{N2} \) is the cross-section area of the diagonal concrete strut at node N2 where the admissible stresses have to be verified; \( l_{\text{eff}} \) and \( b_{\text{eff}} \) are the dimensions of the equivalent rigid plate determined according to the effective T-stub in compression. Considering the admissible stresses and the node dimensions, the resistance of the node is obtained

\[ F_{r,N2} = A_{N2} \cdot 3 \cdot v \cdot f_{cd} = 2354 \text{ kN} \]

![Fig. 9.31 Definition of the width of node N2](image)

6c) Joint link properties

The minimum resistance of the two nodes, N1 and N2, gives the resistance of the joint link in the direction of the binary force generated by the bending moment applied to the joint. Projecting the resistance in the horizontal direction, yields

\[ F_{c-T,\text{JL}} = F_{r,N1} \cdot \cos \theta = 610.6 \text{ kN} \]

According to (Henriques 2013), the deformation of the joint link is given by
$$\Delta_{L} = \left( 6.48 \times 10^{-8} \cdot F_{c-T, JL}^2 + 7.47 \times 10^{-5} \cdot F_{c-T, JL} \right) \cdot \cos \theta$$

Thus, considering 10 load steps, Tab. 9.4 summarizes the force-displacement curve.

**Tab. 9.4 Force-displacement for the Joint Link component**

<table>
<thead>
<tr>
<th>$F_h$ [kN]</th>
<th>$\Delta_h$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.00</td>
</tr>
<tr>
<td>61.1</td>
<td>0.00</td>
</tr>
<tr>
<td>122.1</td>
<td>0.00</td>
</tr>
<tr>
<td>183.2</td>
<td>0.01</td>
</tr>
<tr>
<td>244.2</td>
<td>0.01</td>
</tr>
<tr>
<td>305.3</td>
<td>0.01</td>
</tr>
<tr>
<td>366.3</td>
<td>0.02</td>
</tr>
<tr>
<td>427.4</td>
<td>0.02</td>
</tr>
<tr>
<td>488.5</td>
<td>0.03</td>
</tr>
<tr>
<td>549.5</td>
<td>0.03</td>
</tr>
<tr>
<td>610.6</td>
<td>0.03</td>
</tr>
</tbody>
</table>

**Step 7 Assembly of joint**

The simplified mechanical model represented in Fig. 9.32 consists of two rows, one row for the tensile components and another for the compression components. It combines the tension and compression components into a single equivalent spring per row.

$$F_{eq} = \min(F_{t} \text{ to } F_{n})$$

$$\Delta_{eq} = \sum_{i=1}^{N} \Delta_i$$

where index $i$ to $n$ represents all relevant components, either in tension or in compression, depending on the row under consideration.

According to the joint configuration, it is assumed that the lever arm is the distance between the centroid of the longitudinal steel reinforcement bar and the middle plane of the bottom flange of the steel beam. The centroid of the steel contact plate is assumed aligned with this reference point of the steel beam. Hence, the bending moment and the corresponding rotation follow from

$$M_j = \min(F_{eq,T}; F_{eq,C}; F_{JL}) \cdot h_r$$

$$\phi_j = \frac{\Delta_{eq,t} + \Delta_{eq,c} + \Delta_{L}}{h_r}$$

Thus

$F_{t,max} = 566.5 \text{ kN} \quad \text{Longitudinal rebar}$

$F_{c,max} = 610.6 \text{ kN} \quad \text{Joint link}$
Table 9.5 summarizes the main results in order to calculate the moment rotation curve, where $\Delta_r$ is the displacement of the longitudinal steel reinforcement, $\Delta_{\text{slip}}$ is related to the slip of composite beam through the coefficient $k_{\text{slip}}$, $\Delta_{T\text{-stub}}$ is the displacement of the T-stub in compression and $\Delta_{JL}$ is the displacement of the joint link.

<table>
<thead>
<tr>
<th>$F$ [kN]</th>
<th>$\Delta_r$ [mm]</th>
<th>$\Delta_{\text{slip}}$ [mm]</th>
<th>$\Delta_{T\text{-stub}}$ [mm]</th>
<th>$\Delta_{JL}$ [mm]</th>
<th>$\Phi$ [mrad]</th>
<th>$M_j$ [kNm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>117.1</td>
<td>0.01</td>
<td>0.13</td>
<td>0.03</td>
<td>0.00</td>
<td>0.17</td>
<td>0.40</td>
</tr>
<tr>
<td>152.3</td>
<td>0.09</td>
<td>0.17</td>
<td>0.04</td>
<td>0.01</td>
<td>0.30</td>
<td>0.73</td>
</tr>
<tr>
<td>419.6</td>
<td>0.27</td>
<td>0.47</td>
<td>0.12</td>
<td>0.02</td>
<td>0.88</td>
<td>2.06</td>
</tr>
<tr>
<td>566.5</td>
<td>5.68</td>
<td>0.63</td>
<td>0.16</td>
<td>0.03</td>
<td>6.36</td>
<td>15.53</td>
</tr>
</tbody>
</table>

Note

The resulting moment-rotation behaviour is shown in Fig. 9.33. The system is able to resist the applied load.

![Joint Bending Moment-rotation curve](image-url)
9.7 Portal frame

This example illustrates the design of a portal frame designed of columns with cross section HEB 180 and of a rafter with cross section IPE 270, as illustrated in Fig. 9.33. The stiffness of the connections and column bases is considered under design. The steel grade is S235JR, \( f_y = 235 \text{ N/mm}^2 \) and the profiles are class 1 sections. Safety factors are considered as \( \gamma_{M0} = 1.0 \) and \( \gamma_{M1} = 1.1 \).

Fig. 9.34 highlights position of loads and Tab. 9.2 synthetizes the loads values, while load case combinations are summarized in Tab. 9.3.
Tab. 9.2 Applied loads

Self-weight + dead loads
\[ \text{gr} = 0.5 \cdot 5.3 = 2.7 \text{ kN/m} \]
\[ g = 4.8 \text{ kN/m} \]
\[ s = 5.0 \text{ kN/m} \]
\[ q_1 = 3.0 \text{ kN/m}, \quad b = 2.6 \text{m (equipment)} \]
\[ Q_1 = 9.8 \text{ kN} \]
\[ w_0 = 0.8 \text{ kN/m} \]
\[ w_s = -3.9 \text{ kN/m} \]

Wind
\[ h_{w,D} = 0.8 \cdot 0.65-5.3 = 2.7 \text{ kN/m} \]
\[ w_{D} = 0.8 \text{ kN/m} \]
\[ h_{w,S} = 0.5 \cdot 0.65-5.3 = 1.7 \text{ kN/m} \]
\[ w_{S} = -3.9 \text{ kN/m} \]

Impact load (EN1991-1-7:2006)
\[ F_{d,x} = 100 \text{ kN (} h=1.45 \text{m)} \]

max \( Q_{\text{stab}} \approx (48+58) \times 0.85/200 < 0.5 \text{ kN} \)
(added in the wind load case)

Tab. 9.3 Load case combinations

<table>
<thead>
<tr>
<th>LC</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>((g+g_f) \cdot 1.35)</td>
</tr>
<tr>
<td>2</td>
<td>((g+g_f) \cdot 1.35 + s \cdot 1.5)</td>
</tr>
<tr>
<td>3</td>
<td>((g+g_f) \cdot 1.35 + s \cdot 1.5 + q_1 \cdot 1.5)</td>
</tr>
<tr>
<td>4</td>
<td>((g+g_f) \cdot 1.35 + s \cdot 1.5 + (w+w_0) \cdot 1.5 \cdot 0.6 + q_1 \cdot 1.5)</td>
</tr>
<tr>
<td>5</td>
<td>((g+g_f) \cdot 1.35 + s \cdot 1.5 + (w+w_0) \cdot 1.5 \cdot 0.6 + q_1 \cdot 1.5)</td>
</tr>
<tr>
<td>6</td>
<td>((g+g_f) \cdot 1.35 + s \cdot 1.5 + (w+w_0) \cdot 1.5 \cdot 0.6 + q_1 \cdot 1.5)</td>
</tr>
<tr>
<td>7</td>
<td>((g+g_f) \cdot 1.35 + s \cdot 1.5 + (w+w_0) \cdot 1.5 + q_1 \cdot 1.5)</td>
</tr>
<tr>
<td>8</td>
<td>((g+g_f) \cdot 1.0 + (w+w_s) \cdot 1.5)</td>
</tr>
<tr>
<td>9</td>
<td>((g+g_f) \cdot 1.0 + q_1 \cdot 1.0 + \text{truck} + s \cdot 0.2) (exceptional combination – impact load)</td>
</tr>
</tbody>
</table>

The main steps in order to verify a steel portal frame are the following:

Step 1 Global analysis of the steel structure, with fully restrained column bases. Provide internal forces and moments and the corresponding displacements under several loading condition.

Step 2 Verification of single elements

Step 3 Verification of the column-beam joint, in terms of stiffness and resistance.

Step 4 Verification of column base joint, taking into account an impact load

Step 5 Updating of internal forces and moments of the system considering the effective stiffness of the restraints

Step 1 Global analysis

From a 1st order elastic analysis the internal force diagrams envelope due to vertical and horizontal loads, Fig. 9.35 to 9.36 are obtained. Fig. 9.37 illustrates the structural displacement in case of wind load, in direction \( x \). For each combination is necessary to check whether 2nd order effects should be taken into account in the structural analysis by the following simplified expression for beam-and-column type plane frames

\[
\alpha_{cr} = \frac{H_{Ed}}{V_{Ed}} \cdot \left( \frac{h_i}{\delta_{H,Ed}} \right)\]

where:

- \( H_{Ed} \) is the total horizontal reaction at the of the storey
- \( V_{Ed} \) is the total vertical reaction at the bottom of the storey
- \( \delta_{H,Ed} \) is the relative horizontal displacement of the top storey
- \( h_i \) is the height of the storey

In this case, \( \alpha_{cr} \) is always greater than 10 and thus the first order analysis is enough.
Maximal deformation under variable load is 17 mm at the top.

Step 2 Verification of elements
Verifications are performed using the EC3 Steel Member Calculator for iPhone.

Column HEB 180 is verified as

<table>
<thead>
<tr>
<th>Acting forces from LC 6</th>
<th>Critical section resistance</th>
<th>Buckling resistance</th>
<th>Verification</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{\text{min,d}} = -80$ kN</td>
<td>$N_{c,Rd} = -1533$ kN</td>
<td>$N_{b,y,Rd} = -1394$ kN</td>
<td>$\varepsilon \leq 1$</td>
</tr>
<tr>
<td>$M_{Hy,d} = 51$ kNm</td>
<td>$M_{y,c,Rd} = 113.1$ kNm</td>
<td>$N_{b,z,Rd} = 581$ kN</td>
<td>$\varepsilon \leq 1$</td>
</tr>
<tr>
<td>$M_{By,d} = 45$ kNm</td>
<td>$V_{c,Rd} = 274$ kNm</td>
<td>$M_{b,Rd} = 102.8$ kNm</td>
<td>$\varepsilon \leq 1$</td>
</tr>
</tbody>
</table>

Beam IPE 270 is verified as

<table>
<thead>
<tr>
<th>Acting force from LC 4</th>
<th>Critical section resistance</th>
<th>Buckling resistance</th>
<th>Verification</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{\text{min,d}} = -19$ kN</td>
<td>$N_{c,Rd} = 1079.7$ kN</td>
<td>$M_{b,Rd} = 103.4$ kNm</td>
<td>$\varepsilon \leq 1$</td>
</tr>
<tr>
<td>$M_{Ey,d} = 61$ kNm</td>
<td>$M_{y,c,Rd} = 113.7$ kNm</td>
<td>$M_{I,Rd} = 581$ kN</td>
<td>$\varepsilon \leq 1$</td>
</tr>
<tr>
<td>$M_{By,d} = -51$ kNm</td>
<td>$V_{c,Rd} = 300.4$ kN</td>
<td>$M_{b,Rd} = 102.8$ kNm</td>
<td>$\varepsilon \leq 1$</td>
</tr>
</tbody>
</table>

Maximal deformation under variable load is 17 mm at the top.

Step 2 Verification of elements
Verifications are performed using the EC3 Steel Member Calculator for iPhone.
Step 3 Design of beam to column joint
The connection is illustrated in Fig. Fig. 9.39. The end plate has a height of 310 mm, a thickness of 30 mm and a width of 150 mm with 4 bolts M20 10.9.

**Design Values**
- $M_{y,Rd} = -70.7 \text{kNm} > -54.5 \text{kNm}$ (at $x = 0.09$ of supports axis)
- $V_{z,Rd} = 194 \text{kN}$

![Fig. 9.39 Design of beam-to-column joint](image)

The verification is performed using the ACOP software. The resulting bending moment – rotation curve is represented in Fig. 9.40.

![Fig. 9.40 The bending moment to rotation curve $M_j - \Phi_j$](image)

Step 4 Verification of the column base joint

**Main Data**
- Base plate of 360 x 360 x 30 mm, S235
- Concrete block of size 600 x 600 x 800 mm, C30/37
- Welds $a_{w,Fl} = 7 \text{ mm}$, $a_{w,St} = 5 \text{ mm}$
- The support with base plate is in a 200 mm deep of the foundation.

**Design Values**

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>LC</th>
<th>$N_{x,d}$ [kN]</th>
<th>$M_{y,d}$ [kNm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{min}$</td>
<td>6</td>
<td>-80</td>
<td>51</td>
</tr>
<tr>
<td>$M_{max}$</td>
<td>9</td>
<td>-31.6</td>
<td>95.6</td>
</tr>
</tbody>
</table>
Fig. 9.41 represents the designed column base. In the verification procedure, the following steps are accomplished:

a) calculation of the resistance of component base plate in bending and anchor bolts in tension;
b) evaluation of the area of concrete in compression,
c) calculation of the strip c around the column cross section,
d) calculation of moment resistant of column base,
e) check of the end of column,
f) evaluation of the bending stiffness component stiffness;
g) evaluation of the stiffness of tension part, bolts and T stub,
h) evaluation of the bending stiffness.

**4a) Resistance of component base plate in bending and anchor bolts in tension**

For anchor bolt lever arm, for fillet weld $a_{wf} = 7$ mm, it is

$m = 60 - 0.8 \cdot a_{wf} \cdot \sqrt{2} = 60 - 0.8 \cdot 7 \cdot \sqrt{2} = 52.1$ mm

The T-stub length, in base plates are the prying forces not taken into account, is

$$l_{eff,1} = \min \left( \begin{array}{l} 4 \cdot m + 1.25 \cdot e_a = 4 \cdot 52.1 + 1.25 \cdot 30 = 245.9 \\
4 \cdot \pi \cdot m = 4 \cdot \pi \cdot 52.1 = 654.7 \\
0.5 \cdot b = 0.5 \cdot 360 = 180 \\
2 \cdot m + 0.625 \cdot e_a + 0.5 \cdot p = 2 \cdot 52.1 + 0.625 \cdot 30 + 0.5 \cdot 240 = 243 \\
2 \cdot m + 0.625 \cdot e_a + e_b = 2 \cdot 52.1 + 0.625 \cdot 30 + 60 = 183 \\
2 \cdot \pi \cdot m + 4 \cdot e_b = 2 \cdot \pi \cdot 52.1 + 4 \cdot 60 = 567.4 \\
2 \cdot \pi \cdot m + 2 \cdot p = 2 \cdot \pi \cdot 52.1 + 2 \cdot 240 = 807.4 \\
\end{array} \right)$$

$L_b = 8 \cdot d + t = 8 \cdot 20 + 30 = 190$ mm

The effective length of anchor bolt $L_e$ is taken as

$$l_{eff,1} = 180 \text{ mm}$$

The resistance of T-stub with two anchor bolts is

$$F_{T,1-2,Rd} = \frac{2 \cdot l_{eff,1} \cdot t^2 \cdot f_y}{4 \cdot m \cdot \gamma_M} = \frac{2 \cdot 180 \cdot 30^2 \cdot 235}{4 \cdot 52.1 \cdot 1} = 365.4 \cdot 10^3 \text{N}$$

while the tension resistance of two anchor bolts M 20 for the area of threaded part of bolt

$$A_s = 314 \text{ mm}$$

$$F_{T,3,Rd} = 2 \cdot B_{t,Rd} = 2 \cdot \frac{0.9 \cdot f_{ub} \cdot A_s}{\gamma_M} = \frac{0.9 \cdot 360 \cdot 314}{1.25} = 162.8 \cdot 10^3 \text{N}$$

4b) To evaluate the **compressed part resistance** is calculated the connection concentration factor as
\[ a_1 = b_1 = \min \left\{ \begin{array}{l} a + 2 \cdot a_t = 360 + 2 \cdot 120 = 600 \\ 3 \cdot a = 3 \cdot 360 = 1080 \\ a + h = 360 + 800 = 1160 \end{array} \right\} = 600 \text{ mm} \]

and
\[ a_1 = b_1 = 600 \text{ mm} > \max(a, b) \]

The above condition is fulfilled and
\[ k_j = \frac{a_1 \cdot b_1}{a \cdot b} = \frac{600 \cdot 600}{360 \cdot 360} = 1.67 \]

The grout is not influencing the concrete bearing resistance because
\[ 0.2 \min(a; b) = 0.2 \cdot \min(360; 360) = 72 \text{ mm} > 30 \text{ mm} = t \]

The concrete bearing resistance is calculated as
\[ f_{ld} = \frac{2}{3} \frac{k_j \cdot f_{ck}}{Y_M} = \frac{2}{3} \frac{1.67 \cdot 30}{1.5} = 22.3 \text{ MPa} \]

for each load case, from the force equilibrium in the vertical direction \( F_{sd} = A_{eff} f_i - F_{trd} \), is calculated the area of concrete in compression \( A_{eff} \) in case of the full resistance of tension part.

\[ A_{eff-LC6} = \frac{F_{sd-LC6} + F_{Rd,1}}{f_{ld}} = \frac{8 \cdot 10^{3} + 365.4 \cdot 10^{3}}{22.3} = 19973.1 \text{ mm}^2 \]

\[ A_{eff-LC9} = \frac{F_{sd-LC9} + F_{Rd,1}}{f_{ld}} = \frac{316 \cdot 10^{3} + 365.4 \cdot 10^{3}}{22.3} = 17802.7 \text{ mm}^2 \]

4c) The flexible base plate is transferred into a rigid plate of equivalent area. The width of the strip \( c \) around the column cross section, see Fig. 9.40, is calculated from
\[ c = t \cdot \frac{f_y}{3 \cdot f_{ld} \cdot Y_M} = 30 \cdot \frac{235}{3 \cdot 22.3 \cdot 1} = 56.2 \text{ mm} \]

4d) The active effective width is calculated from known area in compression
\[ b_{eff-LC6} = \frac{A_{eff-LC6}}{b_c + 2 \cdot c} = \frac{19937.1}{180 + 2 \cdot 57.2} = 68.3 \text{ mm} < t_f + 2 \cdot c = 14 + 2 \cdot 56.2 = 126.4 \text{ mm} \]

\[ b_{eff-LC9} = \frac{A_{eff-LC9}}{b_c + 2 \cdot c} = \frac{17802.7}{180 + 2 \cdot 57.2} = 60.9 \text{ mm} < t_f + 2 \cdot c = 14 + 2 \cdot 56.2 = 126.4 \text{ mm} \]

The lever arms of concrete to the column axes of symmetry is calculated as
\[ r_{c-LC6} = \frac{h_c}{2} + c - \frac{b_{eff-LC6}}{2} = \frac{180}{2} + 56.2 - \frac{68.3}{2} = 112.1 \text{ mm} \]

\[ r_{c-LC9} = \frac{h_c}{2} + c - \frac{b_{eff-LC9}}{2} = \frac{180}{2} + 56.2 - \frac{60.9}{2} = 115.8 \text{ mm} \]

Moment resistances of column base are
\[ M_{Rd-LC6} = F_{T,1,Rd} \cdot r_t + A_{eff-LC6} \cdot f_{ld} \cdot r_{c-LC6} = 104.7 \text{ kNm} \]

---

Fig. 9.42 The effective area under the base plate
M_{Rd-LC9} = F_{T,L,Rd} \cdot r_b + A_{eff-LC9} \cdot f_{yd} \cdot r_{c-LC9} = 100.8 \text{ kNm}

4e) The end of column needs to be checked. The design resistance in pure compression is

\[ N_{pl,Rd} = \frac{A \cdot f_y}{\gamma_{M0}} = \frac{625 \times 235}{1.0} = 1533.4 \text{ kN} \]

and column bending resistance

\[ M_{pl,Rd} = \frac{W_{pl} \cdot f_y}{\gamma_{M0}} = \frac{481 \times 10^3 \times 235}{1.0} = 113.1 \text{ kNm} \]

The interaction of normal force changes moment resistance

\[ M_{Ny,Rd} = M_{pl,Rd} \cdot \left( 1 - \frac{N_{sd}}{N_{pl,Rd}} \right) = \frac{113.0 \cdot \left( 1 - \frac{80}{1533.4} \right)}{1 - 0.5 \cdot \frac{A - 2 \cdot b \cdot t_f}{A}} = 120.9 \text{ kNm} \]

4f) To evaluate the bending stiffness the particular component stiffness is calculated

\[ k_b = 2.0 \cdot \frac{A_s}{l_b} = 2.0 \cdot \frac{314}{190} = 3.3 \text{ mm} \]

\[ k_p = \frac{0.425 \cdot l_{\text{eff}} \cdot t^3}{m^3} = \frac{0.425 \cdot 180 \cdot 30^3}{52.1^3} = 14.6 \text{ mm} \]

Fig. 9.43 The T stub in compression

The concrete block stiffness is evaluated based on T-stub in compression, see Fig. 9.43

\[ a_{eq} = t_f + 2.5 \cdot t = 14 \cdot 2.5 \cdot 30 = 89 \text{ mm} \]

\[ k_c = \frac{E_c}{1.275 \cdot E_s} \cdot \sqrt{a_{eq} \cdot b_c} = \frac{33 \cdot 000}{1.275 \cdot 210000} \cdot \sqrt{89 \cdot 180} = 15.6 \text{ mm} \]

4g) The lever arm of component in tension \( z_t \) and in compression \( z_c \) to the column base neutral axes is

\[ r_t = \frac{b_c}{2} + e_c = \frac{180}{2} + 60 = 150 \text{ mm} \]

\[ z_c = \frac{h_c}{2} - \frac{t_f}{2} = \frac{180}{2} + \frac{14}{2} = 83 \text{ mm} \]

The stiffness of tension part, bolts and T stub, is calculated as

\[ k_t = \frac{1}{k_b + \frac{1}{k_p}} = \frac{1}{3.3 + 14.6} = 2.7 \text{ mm} \]

4h) For the calculation of the initial stiffness of column base is evaluated the lever arm

\[ r = r_t + z_c = 150 + 83 = 233 \text{ mm} \]

and

\[ a = \frac{k_c \cdot r_{c1} - k_t \cdot r_t}{k_c + k_t} = \frac{15.6 \cdot 83 - 2.7 \cdot 150}{15.6 + 2.7} = 43.26 \text{ mm} \]

The bending stiffness is calculated for particular constant eccentricity

\[ e_{LC6} = \frac{M_{Rd-LC6}}{F_{Sd-LC6}} = \frac{104.7 \cdot 10^6}{80.0 \cdot 10^3} = 1308.8 \text{ mm} \]
\[
e_{\text{LC-9}} = \frac{M_{\text{Rd-LC9}}}{F_{\text{Sd-LC9}}} = \frac{100.8 \cdot 10^6}{31.6 \cdot 10^3} = 3189.9 \text{ mm}
\]
as
\[
S_{j,\text{ini-LC6}} = \frac{e_{\text{LC-6}}}{e_{\text{LC-6}} + a} \cdot \frac{E_s \cdot r^2}{\mu \sum_{i=1}^{k} E_i} = \frac{1 \quad 308.8}{1 \quad 308.8 + 3 \quad 189.9} \cdot \frac{210 \quad 000 \cdot 233^2}{1 \cdot \left(1 \frac{1}{2.7} + 1 \frac{1}{15.6}\right)} = 25 \quad 301 \text{ kNm/rad}
\]
\[
S_{j,\text{ini-LC9}} = \frac{e_{\text{LC-9}}}{e_{\text{LC-9}} + a} \cdot \frac{E_s \cdot r^2}{\mu \sum_{i=1}^{k} E_i} = \frac{3 \quad 189.9}{3 \quad 189.9 + 3 \quad 189.9} \cdot \frac{210 \quad 000 \cdot 233^2}{1 \cdot \left(1 \frac{1}{2.7} + 1 \frac{1}{15.6}\right)} = 25 \quad 846 \text{ kNm/rad}
\]

These values of stiffness do not satisfy the condition about the rigid base
\[
S_{j,\text{ini}} \geq 30 \quad E \cdot I_b / L_b = 45 \quad 538 \text{ kNm/rad}
\]

**Step 5 Updating of internal forces and moments**

Steps 1 to 4 should be evaluated again considering internal forces obtained from a structural analysis taking into account the stiffness of column base, see Fig. 9.44. Tab. 9.4 summarizes results of the structural analysis of the two meaningful combinations \(N_{\text{min}}\) and \(M_{\text{max}}\).

![Fig. 9.44 Structural system with rotational springs](image)

**Table 9.4 Comparison of internal forces between the model with rigid column base joint and the model with the actual stiffness**

<table>
<thead>
<tr>
<th>Load case</th>
<th>Column base stiffness</th>
<th>Point A</th>
<th>Point B</th>
<th>Point C</th>
<th>Point D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[kN]</td>
<td>[kNm]</td>
<td>[kN]</td>
<td>[kNm]</td>
<td>[kN]</td>
</tr>
<tr>
<td>6</td>
<td>Rigid</td>
<td>-57.0</td>
<td>1.6</td>
<td>-54.0</td>
<td>27.7</td>
</tr>
<tr>
<td></td>
<td>Semi-rigid</td>
<td>-56.9</td>
<td>3.1</td>
<td>-53.3</td>
<td>24.3</td>
</tr>
<tr>
<td>9</td>
<td>Rigid</td>
<td>-31.6</td>
<td>95.6</td>
<td>-29</td>
<td>-18.7</td>
</tr>
<tr>
<td></td>
<td>Semi-rigid</td>
<td>-30.5</td>
<td>87.3</td>
<td>-27.9</td>
<td>-17.7</td>
</tr>
</tbody>
</table>

For the LC6 has been implemented a structural model with two rotational springs equal to 25 301 kNm/rad. For the LC9 the adopted rotational stiffness was equal to 25 846 kNm/rad. Due to the proximity of the stiffness value calculated in Step 4, it was reasonable to assumed in a simplified manner. The lower value of the stiffness in order to update the internal forces of the system.

As shown in the above table, the differences in terms of internal forces are negligible and therefore the single elements and the beam to column joint is considered verified. Tab. 9.4 synthetizes the updated properties of the column base joint.
The designed column base fulfils the asked requirements as shown in the Tab. 9.4.

<table>
<thead>
<tr>
<th>Load case</th>
<th>Column base stiffness</th>
<th>$A_{\text{eff}}$ [mm$^2$]</th>
<th>$b_{\text{eff}}$ [mm]</th>
<th>$r_c$ [mm]</th>
<th>$M_\text{rd}$ [kNm]</th>
<th>$S_{\text{linit}}$ [kNm/rad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>Rigid</td>
<td>19 973.1</td>
<td>68.3</td>
<td>112.1</td>
<td>104.7</td>
<td>25 301</td>
</tr>
<tr>
<td></td>
<td>Semi-rigid</td>
<td>20 008.0</td>
<td>68.4</td>
<td>112.0</td>
<td>104.8</td>
<td>25 268</td>
</tr>
<tr>
<td>9</td>
<td>Rigid</td>
<td>17 802.7</td>
<td>60.9</td>
<td>115.8</td>
<td>100.8</td>
<td>25 846</td>
</tr>
<tr>
<td></td>
<td>Semi-rigid</td>
<td>17 757.0</td>
<td>60.7</td>
<td>115.8</td>
<td>100.7</td>
<td>25 344</td>
</tr>
</tbody>
</table>
10 SUMMARY

This design manual summarises the reached knowledge in the RFCS Project RFSR-CT-2007-00051 New Market Chances for Steel Structures by Innovative Fastening Solutions between Steel and Concrete (INFASO). The material was prepared in cooperation of two teams of researchers one targeting on fastening technique modelling and others focusing to steel joint design from Institute of Structural Design and Institute of Construction Materials, Universität Stuttgart, Department of Steel and Timber Structures, Czech Technical University in Prague, and practitioners Gabinete de Informática e Projecto Assistido Computador Lda., Coimbra, Goldbeck West GmbH, Bielefeld, stahl+verbundbau GmbH, Dreieich and European Convention for Constructional Steelwork, Bruxelles.

The model of three types of steel to concrete connections with the headed studs on anchor plate are introduced. There are based on component method and enable the design of steel to concrete joints in vertical position, e.g. beam to column or to wall connections, and horizontal ones, base plates. The behaviour of components in terms of resistance, stiffness, and deformation capacity is summed up for components in concrete and steel parts: header studs, stirrups, concrete in compression, concrete panel in shear, steel reinforcement, steel plate in bending, threaded studs, anchor plate in tension, beam web and flange in compression and steel contact plate.

In the Chapters 5 and 6 are described the possibility of assembly of components behaviour into the whole joint behaviour for resistance and stiffness separately. The presented assembly enables the interaction of normal forces, bending moments and shear forces acting in the joint. The global analyses in Chapter 7 is taken into account the joint behaviour. The connection design is sensitive to tolerances, which are recapitulated for beam to column connections and base plates in Chapter 8. The worked examples in Chapter 9 demonstrates the application of theory to design of pinned and moment resistant base plates, pinned and moment resistance beam to column connections and the use of predicted values into the global analyses.
References

Standards and guidelines

CEN/TS1992-4-1, Design of fastenings for use in concrete – Part 4-2, Headed fasteners
EN1090-2, Execution of steel structures and aluminium structures, Part 2, Technical
EN13670, Execution of concrete structures, CEN, Brussels, 2011.
EN1991-1-1, Eurocode 1: Actions on structures, Part 1.1, General actions, Densities, self-
EN1991-1-1, Eurocode 1: Actions on structures, Part 1.7, General actions, Densities, self-
EN1992-1-1, Eurocode 2, Design of concrete structures, Part 1-7, General actions -
EN1993-1-8, Eurocode 3, Design of steel structures, Part 1-8, Design of joints, CEN,
EN1994-1-1, Eurocode 4, Design of composite steel and concrete structures, Part 1-1,
EN206-1, Concrete - Part 1, Specification, performance, production and conformity, CEN,
FIB Bulletin 58, Design of anchorages in concrete, Guide to good practice, International
federation for structural concrete, Lausanne, 2011.

Textbooks and publications

Aribert, J. M., Influence of Slip on Joint Behaviour, Connections in Steel Structures III,
Astaneh A. et al., Behaviour and design of base plates for gravity, wind and seismic loads, In
Bouwman L.P., Gresnigt A.M., Romeijn A., Research into the connection of steel base plates
to concrete foundations, TU-Delft Stevin Laboratory report 25.6.89.05/c6, Delft.
Bravery P.N.R., Cardington Large Building Test Facility, Construction details for the first
British Steel plc. The behaviour of multi-storey steel framed buildings in fire, European Joint
Research Programme, Swinden Technology Centre, South Yorkshire, 1999.
Demonceau J., Steel and composite building frames: Sway-response under conventional
loading and development of membrane effects in beam further to an exceptional
Demonceau J.F., Huvelle C., Comeliau L., Hoang L.V., Jaspart J.P., Fang C. et al,
Robustness of car parks against localised fire, European Comission, Final Report
Da Silva L. Simoes, Towards a consistent design approach for steel joints undergeneralized


Jaspart J.P., Recent advances in the field of steel joints - column bases and further configurations for beam-to-column joints and beam splices, Professorship Thesis, Department MSM, University of Liege, Belgium, 1997.


Wald F., Bouguin V., Sokol Z., Muzeau J.P., Component Method for Base Plate of RHS, Proceedings of the Conference Connections in Steel Structures IV: Steel Connections in the New Millenium, October 22-25, Roanoke 2000, s. IV/8- IV/816.


**Software**


**Sources**

This Design manual I was prepared based on Final report (Kuhlmann et al, 2012). The following Figs were worked up according the Eurocodes and its background materials: EN1090-2:2008 Figs 8.1-8.3, EN1992-1-1:2004 Fig. 3.5, CEB-FIP:1990 Fig. 4.11, (Gresnight et al, 2008) Fig. 4.20, (Steenhuis et al, 2008) Figs 3.6-3.9, (Wald et al, 2008) Figs 4.1-4.8.
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Design of Steel-to-Concrete Joints

Design Manual II

Deliverable of a project carried out with a financial grant from the Research Fund for Coal and Steel of the European Community.
Design of Steel-to-Concrete Joints

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The present document and others related to the research project INFASO RFSR-CT-2007-00051 “New Market Chances for Steel Structures by Innovative Fastening Solutions between Steel and Concrete and the successive dissemination project RFS2-CT-2012-00022 “Valorisation of Knowledge for Innovative Fastening Solution between Steel and Concrete, which have been co-funded by the Research Fund for Coal and Steel (RFCS) of the European Community, can be accessed for free on the following project partners’ web sites:

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Germany: www.uni-stuttgart.de/ke/
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<th>Title</th>
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1 Introduction

1.1 Introduction and structure of the document

The mixed building technology allows to utilise the best performance of all structural materials available such as steel, concrete, timber and glass. Therefore the building are nowadays seldom designed from only one structural material. Engineers of steel structures in practice are often faced with the question of economical design of steel-to-concrete joints, because some structural elements, such as foundations, stair cases and fire protection walls, are optimal of concrete. A gap in knowledge between the design of fastenings in concrete and steel design was abridged by standardized joint solutions developed in the INFASO project, which profit from the advantage of steel as a very flexible and applicable material and allow an intelligent connection between steel and concrete building elements. The requirements for such joint solutions are easy fabrication, quick erection, applicability in existing structures, high loading capacity and sufficient deformation capacity. One joint solution is the use of anchor plates with welded headed studs or other fasteners such as post-installed anchors. Thereby a steel beam can be connected by butt straps, cams or a beam end plate connected by threaded bolts on the steel plate encased in concrete. Examples of typical joint solutions for simple steel-to-concrete joints, column bases and composite joints are shown in Fig. 1.1.

![Fig. 1.1: Examples for steel-to-concrete joints: a) simple joint, b) composite joint, c) column bases](image)

The Design Manual II "Application in practice" shows, how the results of the INFASO projects can be simply applied with the help of the developed design programs. For this purpose the possibility of joint design with new components will be pointed out by using practical examples and compared with the previous realizations. A parametric study also indicates the effects of the change of individual components on the bearing capacity of the entire group of components. A detailed technical description of the newly developed components, including the explanation of their theory, can be found in the Design Manual I "Design of steel-to-concrete joints" [13].

Chapter 2 includes a description of the three design programs that have been developed for the connection types shown in Fig. 1.1. Explanations for the application in practice, the handling of results and informations on the program structure will be given as well as application limits and explanations of the selected static system and the components. Practical examples, which have been calculated by using the newly developed programs, are included in Chapter 3. These connections are compared in terms of handling, tolerances and the behaviour under fire conditions to joints calculated by common design rules. The significant increase of the bearing capacity of the "new" connections under tensile and/or bending stress result from the newly developed components "pull-out" and "concrete cone failure with additional reinforcement". Chapter 4 contains parameter studies in order to show the influence of the change of a single component on the entire group of components, and hence to highlight their effectiveness.
2 Program description

2.1 Restrained connection of composite beams

2.1.1 General

In the following the Excel sheet “Restrained connection of composite beams” (Version 2.0 Draft) [21] is presented. With this program the load bearing capacity (moment and shear) of a fully defined joint, composed of tensional reinforcement in slab and cast-in steel plate with headed studs and additional reinforcement at the lower flange of the steel section can be determined. The shear and the compression component, derived from given bending moment, are acting on a welded steel bracket with a contact plate in-between, as the loading position on the anchor plate is exactly given. The tensional component derived from given bending moment is transferred by the slab reinforcement, which is bent downwards into the adjacent wall. Attention should be paid to this issue as at this state of modelling the influence of reduced distances to edges is not considered. The wall with the cast-in steel plate is assumed to be infinite in elevation. In this program only headed studs are considered. Post installed anchors or similar have to be taken in further consideration.

2.1.2 Program structure
The Excel file is composed of two visible sheets. The top sheet contains full input, a calculation button and the resulting load bearing capacity of the joint with utilization of bending moment and shear (see Fig. 2.1). The second sheet gives the input data echo, with some additional calculated geometry parameters and the characteristic material properties. Subsequently it returns calculation values to allow checking the calculation flow and intermediate data. Other sheets are not accessible to the user. One contains data for cross sections (only hot rolled sections are considered), for headed studs and concrete. The three other sheets are used to calculate tension in studs, shear resistance, anchor plate assessment and stiffness values. Parameter and results are given in output echo (sheet 2). The user introduces data in cells coloured in light yellow. All drawings presented are used to illustrate the considered dimensions and theory used behind. They do neither change with input nor are drawn to scale. A check for plausibility will be executed for some input parameters, with warning but without abortion. The user has to interpret results on own responsibility and risk. The majority of the calculations are performed introducing formulae in the cells. However, when more complex calculation and iterative procedure is required, a macro is used to perform these calculation. The user has to press the corresponding 'Calculation' button. If any changes in the parameters are made the macro calculation should be repeated. By opening the worksheet the accessible input cells (in yellow) are preset with reasonable default values. They must be changed by the user. Hot rolled steel sections, steel and concrete grades, type and length of studs/reinforcement are implemented with the help of a dropdown menu to choose one of the given parameters. To model the stiffness according to the developed theory some additional information must be given in the top (input) sheet. The effective width and length of slab in tension, the reinforcement actually built in and the number and type of studs connection slab and steel beam. These information do not influence the load capacity calculations.

2.1.3 Input and output data and input data cells

The user inserts data only into cells coloured in light yellow. The accessible input cells are not empty but preset per default with reasonable values. They can be changed by the user. The units given in the input cells must not be entered, they appear automatically to remind the correct input unit.

Choice of appropriate code - whereas Eurocode EN 1992-1-1 [7] for design of reinforced concrete, EN 1993-1-1 [8] for design of steel and EN 1994-1-1 [10] for steel-concrete composite structures are the obligatory base for all users, the national annexes must be additionally considered. For purpose of design of connections to concrete it can be chosen between EN 1992-1-1 [7] in its original version and the appropriate (and possibly altered) values according to national annex for Germany, Czech Republic, Portugal, the UK, France and Finland. The input procedure should be self-explaining, in context with the model sketch on top of first visible sheet. According to this principal sketch of the moment resisting joint there are nine components and their input parameters necessary to define characteristics and geometry.

1. + 2. Composite beam of a hot rolled section of any steel grade acc. to EN 1993-1-1 [8] and a reinforced concrete slab of any concrete grade acc. to EN 1992-1-1 [7]. They are connected by studs and working as a composite structure according to EN 1994-1-1 [10]. This composite behaviour is only subject of this calculation because it’s flexibility due to slip influences the connection stiffness. Following selections can be made:

- Type of sections: Hot rolled sections IPE, HEA, HEB, HEM of any height
- Concrete grades: C20/25 until C50/60 acc. to EN 1992-1-1 [7]
- Reinforcement grade: Bst 500 ductility class B acc. to EN 1992-1-1 [7]

3. Concrete wall – the shear and bending moment are to be transferred into the infinite concrete wall with limited thickness. Per definition reinforcement and a cast-in steel plate are used. It can be chosen between:

- Concrete grades: C20/25 until C50/60 acc. to EN 1992-1-1 [7]
- Reinforcement grade: Bst 500 ductility class B acc. to EN 1992-1-1 [7]

4. Anchor plate with studs – at the bottom flange of the steel section an anchor plate is inserted into the concrete wall. Welded studs on the rear side transfer tensional (if any) and/or shear forces from top of
anchor plate into the concrete. The compression components are transferred directly by contact between the steel plate and the concrete.

- **Geometry of plate:** Thickness and 2D-dimensions and steel grade, single input values in [mm], input check: thickness ≥ 8mm is deemed to be ok.
- **Type of studs:** Köco resp. Nelson d19, d22, d25 regular or d19, d22 stainless steel, Peikko d19, d20 regular or d20, d25 reinforcement bar with head all data including steel grades from ETA-approval (e.g. the steel grades are considered automatically according to approval).
- **Length of studs:** 75 until 525 mm (from ETA-approval), input check: length less than wall thickness less coverage and plate thickness is deemed to be ok.
- **Distribution studs:** Number of studs (4,6,8) and inner distances, input check: distances to lay within plate are deemed to be ok.

5. **Steel bracket** is welded on top of the anchor plate and takes the shear force with small eccentricity and transfers it into the anchor plate /concrete wall.

- **Geometry of plate:** Thickness and 2D-dimensions, ‘nose’ thickness, input check: width and height less than anchor plate is deemed to be ok. Position/eccentricity is required in 6. Contact plate.

6. **Contact plate** – the contact plate is inserted force-fit between the end of the steel section and the anchor plate at lower flange level. The compression force component from negative (closing) bending moment is transferred on top of anchor plate.

- **Geometry of plate:** Thickness and 2D-dimensions, eccentricity of plate position in relation to anchor plate centre. Input check: width less than anchor plate and position within anchor plate is deemed to be ok.

7. **Reinforcement bars** – in the slab of the composite section. The tensional force component from negative (closing) bending moment is transferred into the wall and bent down on the rear face of the wall and anchored there. Whereas the necessary design reinforcement is calculated by the work sheet, for later use of stiffness calculation, the existing reinforcement in the slab of the composite beam is required. The bar diameter should be chosen in a way that reasonable spacings within the effective width result and that the bend can be installed within the wall. The length of tension zone is crucial for the stiffness evaluation and depends on the structural system. It must be chosen in accordance with codes or independent calculation results of the underlying model (Example: in case of beam simply supported and other side restrained it is ≈ 0.25*length, in case of cantilever beam it is 1.0*length)

- **Bars:** steel area [cm²], diameter and length of tension zone [cm], input check: reinforcement must be ≥ minimum design reinf. area, spacing of bars should be within interval 5-25 cm, bar curvature Ø*20 must fit into wall.

8. **Additional stirrups** – these optional stirrups are proposed as an effective means to improve the joint in case of tension forces (if any, only in case of small moments and large shear force with large eccentricity) in the stud. They are useful only in the upper row, and only under certain circumstances of the complete assemblage. Further information can be found in the parameter study in Chapter 4. Generally there is always a surface reinforcement in the front face of the wall. This may be optionally taken into account and will improve the capacity of the joint under certain circumstances of the complete assemblage.

- **Reinforcement:** bar-diameter of stirrups with legs each very close to studs (default: no stirrup, input range Ø8-14 mm), and surface reinforcement bar-diameter (Ø6-14 mm) and spacing (75-250 mm)
9. **Slab studs** – these studs are welded at the upper flange of the steel cross section and are the connection medium in the joint between steel and concrete sections to work as a composite structure according to EN 1994-1-1 [10]. Only the composite behaviour is subject of the calculation because it’s flexibility due to slip influences the connection stiffness.

- **Studs:** Diameter (Ø16 -25 mm) and length of studs (75-525 mm) of any kind, input check: length less than slab thickness less coverage.
- **Distribution studs:** Number of studs < 27 within length of tension zone, input check: spacing of studs should be within limits of EN 1994-1-1 [10].

10. **Loads** - a combination of shear force and bending moment must be given by overwriting the preset starting values. Design Forces with partial safety factors according to current codes are required. An evaluation of capacity of composite beam section is not executed at that point and must be done separately by the user!

- **Loading:** Shear force $V_{ed}$ [kN] and bending moment $M_{ed}$ [kNm] from external member calculation.

### 2.1.4 Calculation

As it is the characteristic of worksheet programming the calculation has to be updated any time if the user changes input parameters. This program does the same, starting with the preset values and recalculates any time the content of a cell (if necessary for the mechanical model) is changed, so any result is up to date. Due to the nonlinear characteristic of the compressed anchor plate on the top of the wall concrete at bottom of top sheet a calculation button is placed, which starts the complex update of the effective geometry (via Macro-programming). After any change of input parameters this button must be pushed for updating the complex evaluation of anchor plate behaviour. Even if differences often can be small, only the calculation starting with the calculation button yields the correct result regarding the presented model. Detailed results are given on second sheet. If the anchor plate is assumed to be rigid, the original dimensions can be used for further calculation. In case of a thin and flexible anchor plate reduced dimensions are returned and used for further calculation.

### 2.1.5 Output mask

The user inserts data only into cells coloured in light yellow. Any other cell is automatically (or by using the ‘Calculate’ button) updated with result data. At the bottom of top sheet (see Fig. 2.2) the load bearing capacity and utilization of the joint assemblage for tension and shear is given in terms of $V_{Rd}$ and $M_{Rd}$ resp. $V_{Sd}/V_{Rd}$ and $M_{Sd}/M_{Rd}$. The minimum tensional reinforcement (design) in the slab is given as information. On top of the second sheet (see Fig. 2.2) the input data from page one together with some additionally calculated geometry parameters and characteristic material properties are given. These are:

- **Steel section of the composite beam,** with characteristics geometric and steel grade values. The section is restricted to common hot rolled sections as available in Europe. Three predefined steel grades are available according to EN 1993-1-1 [8]. Attention should be paid, that these steel grades are only used for assessment of the composite beam and not for the anchoring of the joint.
- **Contact plate** – the contact plate is an interface between lower flange of the steel section and the anchor plate. Per definition the gravity centre is in one line with the centroid axis of the flange. By input of distance between upper edge of the anchor plate to upper edge of the contact plate the loading position is defined.
- **Steel bracket** – this bracket is the interface to carry the shear load. The position is defined exactly by input of contact plate because of direct contact. The eccentricity of shear loading, i.e. the position of vertical support of section flange is defined by subtracting half contact area ($t_{sb2}$ or $a_1$) from total thickness ($t_{sb}$) of the bracket.
- **The rectangular steel plate** is defined by three parameters. It is assumed to be flush with surface of concrete wall, where it is embedded. Three predefined steel grades are available according to EN 1993-1-1 [8]. The given steel grade applies for contact plate and steel bracket as well.
The parameters of headed studs are:

- The thickness and the steel grade with characteristic tensile resp. yield strength acc. to the European approvals of the stud types, the number of rows and columns with their corresponding distances of axis.

The parameters of the concrete parts are:

- Slab section – concrete grade with characteristic (cylindrical) strength according to EN 1992-1-1 [7], thickness together with distance of reinforcement (fixed to 40 mm), width of slab and area of reinforcement (given) are returned from input.
- Wall section – concrete grade with characteristic (cylindrical) strength according to EN 1992-1-1, wall thickness together with the distance of the reinforcement (fixed to 40 mm at both sides) are returned from input. The wall is assumed to be infinite, as close to edge anchoring is not considered. Two possible types can be built in as supplementary reinforcement. The orthogonal surface reinforcement of given diameter and spacing (distance to surface 40 mm) and the supplementary reinforcement close to the stud in tension to clearly enhance the capacity of the stud in concrete. These stirrups may be defined by arbitrary diameter, whereas the number of legs in that case is fixed to four in this program (i.e. two stirrups very close to each stud, positioned orthogonal to the wall surface).

Joint loading echo:

- Beneath the given external moment and shear loading (design forces) the resulting external design components tension in slab reinforcement (T_d) and compression on contact plate (C_d) are returned – these forces are equal in absence of external axial forces, as it is assumed in that model.
- The eccentricity of the shear force is calculated with geometric components of bracket, anchor plate thickness and - following common practice - the stud diameter. This yields a local moment acting on the anchor plate which is returned.
**Fig. 2.2: EXCEL output file 1**

**Restrained Connection of Composite Beams**

### Input Data Echo

<table>
<thead>
<tr>
<th>Profile</th>
<th>Contact plate</th>
<th>Bracket</th>
<th>Anchor Plate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth</td>
<td>h (mm)</td>
<td>d (mm)</td>
<td>b (mm)</td>
</tr>
<tr>
<td></td>
<td>71</td>
<td>248.6</td>
<td>110</td>
</tr>
<tr>
<td>Width</td>
<td>b (mm)</td>
<td>d (mm)</td>
<td>t (mm)</td>
</tr>
<tr>
<td></td>
<td>110</td>
<td>16.7</td>
<td>100</td>
</tr>
<tr>
<td>Height</td>
<td>h (mm)</td>
<td>b (mm)</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>779.6</td>
<td>110</td>
<td>25</td>
</tr>
</tbody>
</table>

**Input data values:**
- d: 385
- t: 0.255
- n: 2.5

### Result Data

**Anchor plate in compression + bending**

<table>
<thead>
<tr>
<th>Beam Type</th>
<th>Number</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>0.67</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>32.54</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>21.25</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>136.46</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>155.46</td>
<td></td>
</tr>
</tbody>
</table>

**Maximum for applied load:**
- N = 136.46
- M = 711.47

**Maximum for rigidity:**
- N = 124.85
- M = 509.46

**Verification:**
- Maximum for rigidity is OK.
- Maximum for applied load is OK.
2.1.6 Results of evaluation of individual components

2.1.6.1 General

In the following the results of evaluation of individual components are presented. In this specific joint constellation of moment resisting composite joint with closing, negative bending moment, generally concrete is in compression and the anchor plate is in bending. The calculation model relies on consideration of the activated components illustrated in the following Fig. 2.5. The Excel sheet evaluates the following components.

<table>
<thead>
<tr>
<th>Number</th>
<th>Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Slab reinforcement in tension</td>
</tr>
<tr>
<td>2</td>
<td>Beam web and flange in compression</td>
</tr>
<tr>
<td>3</td>
<td>Contact plate in compression</td>
</tr>
<tr>
<td>4</td>
<td>Bracket in bending (rotational spring)</td>
</tr>
<tr>
<td>5</td>
<td>Anchor in tension</td>
</tr>
<tr>
<td>6</td>
<td>Concrete in compression with anchor plate in bending</td>
</tr>
<tr>
<td>7</td>
<td>Concrete strut in compression.</td>
</tr>
</tbody>
</table>

Fig. 2.5: Components for joints with negative bending moment
2.1.6.2 Anchor plate in compression and bending

Any result of this component is derived from the nonlinear evaluation of combined compression force from negative bending moment and moment due to the eccentricity of shear force at the anchor plate. First the minimum thickness required to consider the anchor plate as rigid is determined. By comparing the minimum thickness previously calculated with the actual thickness of the anchor plate, the type of the plate under consideration is determined. If the plate is rigid, the real dimensions of the plate are used in the following calculations. If flexible, an equivalent rigid and smaller plate is determined. An iterative procedure using a macro is implemented. This macro will be started by pushing the ‘CALCULATE’ button at top sheet. Subsequently two extreme cases are considered:

- Maximum axial load under compression together with a given eccentricity moment from shear force X eccentricity.
- Maximum bending moment together with a given axial load under compression.

The given values are the same as in cells D/36 and D/33 respectively, the calculated values define the corresponding forces the anchor plate configuration can bear additionally. The fictive effective size of the plate is returned as well.

2.1.6.3 Tensile resistance of the upper row of anchors

In this part the resistance of the upper row of anchors, which possibly may be in tension, is evaluated. Different failure modes are possible. Tension resistance of studs is equal to the minimum resistance value of the five following components:

**Steel failure**

Steel failure is calculated according to EN 1992-4-2, Cl. 6.2.3 [2].

**Pull-out failure**

Pull out failure is calculated according to EN 1992-4-2, Cl. 6.2.4 [2]. Cracked concrete is assumed, though uncracked concrete of the wall is possible due to vertical loading in the wall; this must be separately assessed – in case the parameter may be 1,4 for uncracked concrete.

**Concrete cone failure without supplementary reinforcement (modified standard model)**

Cracked concrete is assumed, though uncracked concrete of the wall is possible due to vertical loading in the wall; this must be separately assessed – in case the parameter may be 1,4 for uncracked concrete.

**Concrete cone failure with supplementary hanger reinforcement**

If supplementary stirrups are used with a diameter according to the input on the top sheet and per definition with 2+2 legs, an additional resistance component can be evaluated, assuming the stirrup bar axis being 40 mm below the surface (see Fig. 2.6). This concrete cone failure mode depends fully on the behaviour of the stirrups. If steel yielding or steel bond failure occurs before reaching the concrete cone resistance, the resistance force will be the yielding or anchorage force instead.

- Yielding of stirrups
- Anchoring failure of stirrups

Fig. 2.6: Definition of distance x
Splitting failure

Due to the fact, that the wall per definition is indefinite only the minimum wall thickness must be checked. For long studs with large cones splitting is generally possible and must be assessed. The existence of a minimum surface reinforcement is sufficient to avoid splitting failure. This reinforcement should be determined in each orthogonal direction according to Eq. (2.1). The material safety factor used with reinforcement bars is $\gamma_s = 1.15$. If this condition is not fulfilled, the resistance force for splitting failure will be calculated.

$$A_s = 0.5 \cdot \sum \frac{N_{Ed}}{f_{yk}/\gamma_{S,\text{reinf}}}(2.1)$$

2.1.6.4 Diagonal concrete strut

Regarding the concrete part of the wall, for the bending moment a simple single diagonal compression strut has been assumed. In Fig. 2.7 this strut is represented by a dashed line.

2.1.6.5 Longitudinal steel reinforcement in tension

The longitudinal reinforcement of the concrete slab is the only element considered in the tension zone. Concrete is ignored. The tension force is calculated using a two-point-section with reinforcement in slab and compression point in the middle of lower flange, fulfilling equilibrium. The resistance of this first component is evaluated according to EN 1994-1-1, Cl. 4.4 [10] and is restricted to reinforcement within the effective width according to Cl. 4.2.

2.1.6.6 Shear components

In this part the shear resistance of the anchors is evaluated. Three resistance components can be determined for shear: friction, steel failure of the anchors and pry-out failure. Shear resistance of studs is equal to the minimum resistance value of the three components mentioned above.

Friction

In the compressed area a friction component acting opposite to the shear force is possible. Nevertheless the coefficient at that stage is set to zero, i.e. no friction.

Steel failure

Steel failure is calculated according to EN 1992-4-2, Cl. 6.3.3 [2].

Pry-out failure

Pry-out failure is calculated according to EN 1992-4-2, Cl. 6.3.4 [2].

Resulting shear resistance

The shear force which can be applied to the concrete wall is restricted by two mechanism – the minimum of these two will be the relevant design force under given geometrical circumstances.

- Pure shear: the shear resistance is derived from the fore mentioned considerations. This value is governed under usual circumstances, as found in real structures. This force is called $V_{Rd,N}$.
- Shear force with small eccentricity: the shear force can be limited as well by the resistance of the anchor plate. The maximum moment derived from eccentricity under a given compression force is evaluated in 2.1.6.2 cell D/36. Divided by the lever arm of the bracket the shear force called $V_{Rd,M}$ is defined.
2.1.6.7 Other steel components

In this part other steel components on top of the anchor plate might be assessed that should not fail even if they are not part of the model "anchor plate with headed studs". Under consideration are: steel contact plate, beam web and flange in compression and steel bracket. The first two are calculated using EN 1994-1-1 [10] and EN 1993-1-8 [9], respectively. The steel bracket is analysed comparing the acting bending moment with resisting bending moment, at the cross-section in contact with the anchor plate. Additionally one must assess the welding seams of the bracket as well. The assessment of these components is returning 'OK' or 'NOT OK'. The user has to decide what actions to be taken (e.g. changing geometry or material grades) to fulfil the requirements.

2.1.7 Global Results

2.1.7.1 Utilisation in terms of overall bending moment and shear

At the bottom of second sheet the load bearing capacity and utilization of the joint assemblage for tension and shear is given in terms of VR,d and MR,d resp VS,d/VR,d and MS,d/MR,d. These values are transferred to bottom of top sheet (see 2.1.5).

2.1.7.2 Interaction

In case of tension and shear in the stud additionally the combined action of both components must be assessed. As it is a rare situation due the governing compression force from closing moments, usually there is no limitation. If tension and shear forces have to be considered, Eq (2.2) can be applied according to EN 1992-4-2, Cl. 6.4.1 [10].

\[
\mu = \left( \frac{V_{S,d}}{V_{R,d}} \right)^{\alpha} + \left( \frac{T_{S,d}}{T_{R,d}} \right)^{\alpha} \leq 1
\]

(2.2)

As Exponent $\alpha = 2.0$ is taken in case of steel failure acc. to Cl. 6.4.1.1 or $\alpha = 1.5$ in case of other failure modes acc. to Cl. 6.4.1.2. In case of supplementary reinforcement which is designed for both failure modes tension and shear, the same $\alpha$ can be applied. For simplification, and according to the current status of European approvals for headed studs, the value $\alpha = 1.5$ is used.

2.1.7.3 Stiffness and ductility

Due to the character of the joint, the stiffness of the moment resisting joint (i.e. relation of overall bending moment to rotation) depends mainly on the nonlinear flexibility of steel/concrete bond, the slip of studs in slab and the behaviour of concrete shear panel in the wall which is activated by the bend of reinforcement, whereas the compression strain in the anchor plate is inferior. This approximate joint stiffness by $M^*lever$ arm/horizontal displacement in axis of reinforcement is given with two parameters:

- $S_{ini} =$ initial stiffness in unit [MNm/rad] gives the relation between bending moment and rotation of the connection in the very beginning. The incline represents the maximum elastic behaviour.
- $S_{sec} =$ Secant stiffness in unit [MNm/rad] gives the relation between the effective bending moment and the according, possibly nonlinear rotation of the connection. The incline is always equal (in case of small bending moment and elastic behaviour) or typically smaller than $S_{ini}$.

The term ductility is usually used in connections with energy consuming behaviour due to plasticity, if there is displacement which will not reset but will remain in case of load removal. So even if the descent of stiffness $S_{sec}$ points to nonlinearity it mostly will be a nonlinear elastic effect, which yields no ductility factor. In that case the cell will give the information 'elastic'.

2.1.7.4 Anchor plate and minimum tensional reinforcement

The type of anchor plate behaviour is given as information (rigid/flexible) and represents cell B45 of this sheet and the minimum tensional reinforcement (design) in the slab is given as information.
2.2 Slim anchor plate with headed studs – bending joints

2.2.1 General

With the program “slim anchor plates with headed studs - bending joints” (Version 2.0) [22] load carrying capacities of joints with minimum four and maximum six headed studs can be proved. The headed studs therefore have to be placed in two rows and the loading only can be considered in one direction (see Fig. 2.8). In the progress of the calculation the deformation behaviour of the anchor plate up to a kinematic chain is taken into consideration. At the end a moment-rotation curve can be obtained. The load carrying capacity of the tensional component can be increased by taking the supplementary reinforcement which is placed next to the headed studs into account. Compared to pure concrete cone failure the capacity of this component can be highly increased due to supplementary reinforcement. Within anchor plates, where the load carrying capacity of the tensional-, bending- or combined components is not governed due to failure of the steel components (anchor plate in bending, headed studs in tension) high increases in loading of the joint are possible. Additional the knowledge of the deformation behaviour of the joint can be used in the global analysis.

Fig. 2.8: Geometry of the joint with slim anchor plate

2.2.2 Program structure and static model

2.2.2.1 General

The design software is based on the EXCEL table calculation program with the integrated programming language VBA. Within the EXCEL file ten different spreadsheets for the in- and output, for the design of the different components, for the consideration of the joint in the global analysis and for a summary of the joint properties. Due to physical non-linear behaviour of the anchor plate under bending forces and the geometric non-linear effects based on the development of cinematic chains, the design approach is done iteratively with consideration of changes in the system. The geometric non-linear effect occurs due to the activation of the anchor plate due to tension forces and additional non-linear load-deformation behaviour of the single components. This is implemented in the VBA-program, which is accessing the input data from the different spreadsheets.

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2.2.2.2 Load-transfer of the vertical loads \( N \) and the bending moment \( M \) – static model at the beginning of the calculation process

The first model for the load transfer of the vertical loads \( N \) and the bending moments \( M \) is a continuous beam supported on single springs. The anchor plate is therefore modeled as a two-dimensional system. As the connected profile stiffens the anchor plate, this section is modeled with rigid members. Springs for compression are placed at the nodes 1 to 8 to reflect the behaviour of the concrete under compression. If the anchor plate is not in contact with the concrete surface and no compression forces in this place might occur, the springs can be neglected. Non-linear tensional springs are reflecting the load carrying behaviour of the headed stud with the supplementary reinforcement. Depending on the geometry of the anchor plate the tensional springs can be only placed on the nodes 2 and 7, 3 and 6 or 4 and 5. They are only activated if the distance between the anchor plate and the concrete surfaces increases. If not a spring which is simulating the compression forces of the concrete is placed at the same node. There are no hinges in the continuous beam at the beginning of the calculation, but within the calculation process plastic hinges might occur at the nodes 2, 3, 6 and 7. After each load step the boundary conditions of the supporting springs are adopted. The prying forces of the anchor plate are considered by the compression springs in the external nodes 1 and 8 (see Fig. 2.9).

![Diagram](image)

Fig. 2.9: Design model for vertical loads and bending moments

The calculation will be done by displacement method. Non-linear (physical) effects will be considered by an iterative calculation with continuous increase of load steps. For every load step the support conditions and the appearance of plastic hinges will be checked. In case of changing support conditions or appearance of plastic hinges the corresponding elements of the total stiffness matrix \( K \), the kinematic transformation matrix \( a \) and the vector of the external nodal forces \( P \) will be manipulated. In case of bending loads without tension forces \( (N \geq 0) \) the row of headed studs near to the compression zone is not considered as support spring for tension loads \( (c_s=0) \). Internal forces and global node deformations caused by bending moments and normal forces will be determined by using the displacement method, (Krätzig [18]).

\[
v = a \cdot V \tag{2.3}
\]

\[
s = k \cdot v + \dot{s} \tag{2.4}
\]

\[
P = a^T \cdot s \tag{2.5}
\]

\[
P = a^T \cdot k \cdot aV + a^T \cdot \dot{s} = K \cdot V + a^T \cdot \dot{s} \tag{2.6}
\]

\[
V = K^{-1} \cdot P - K^{-1} \cdot a^T \cdot \dot{s} \tag{2.7}
\]

\[
s = k \cdot a \cdot V + \dot{s} \tag{2.8}
\]
With:
- \( s \) Vector of internal element end forces;
- \( v \) Vector of internal element end displacements;
- \( P \) Vector of external nodal forces;
- \( V \) Vector of external nodal displacements;
- \( k \) Reduced stiffness matrix of all elements;
- \( a \) Kinematic transformation matrix;
- \( \dot{s} \) Vector of internal rigid-boundary element forces.

Non-linear material effects will be considered by manipulating the total stiffness matrix \( K \), the kinematic transformation Matrix \( a \) and the vector of the external nodal forces \( P \).

\[
K = K_{\text{sing}} + K_{\text{bound}} \quad (2.9)
\]

With:
- \( K_{\text{sing}} \) Stiffness matrix without boundary conditions and hinges at node 2, 3, 6 and 7;
- \( K_{\text{bound}} \) Stiffness matrix considering boundary conditions and reducing 0-Elements at the main diagonal caused by reducing hinges.

\[
P = P' + \Delta P \quad (2.10)
\]

With:
- \( P' \) Nodal forces caused by external loads;
- \( \Delta P \) Nodal forces caused by non-linear support springs and plastic hinges;
- \( a \) Varying some values to reduce the number of degrees of freedom at the nodes 2, 3, 6 and 7 in case of no plastic hinges.

The bearing reactions will be determined by multiplying the diagonal elements of \( K_{\text{bound}} \) by the corresponding deformations of \( V \) plus the nodal forces of \( P' \).

\[
C = K_{\text{bound}}^{ij} \cdot V^{ij} + \Delta P^{ij} \quad (2.11)
\]

\[
C_i = K_{\text{bound},1i} \cdot V_i + \Delta P_{1i}; \ldots; C_9 = K_{\text{bound},88} \cdot V_8 + \Delta P_8 \quad (2.12)
\]

\[
P = \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_{20} \end{bmatrix} \quad (20\times1) \quad V = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_{20} \end{bmatrix} \quad (20\times1) \quad s = \begin{bmatrix} M_1 \\ M_2 \\ \vdots \\ M_7 \\ M_8 \end{bmatrix} \quad (14\times1) \quad \dot{s} = \begin{bmatrix} M_1 \\ M_2 \\ \vdots \\ M_7 \\ M_8 \end{bmatrix} \quad (14\times1)
\]

\[
k^2 = \begin{bmatrix} 4EI_1 & 2EI_1 \\ 2EI_1 & 4EI_1 \\ \vdots & \vdots \\ 2EI_7 & 4EI_7 \\ l_1 & l_1 \\ \vdots & \vdots \\ l_7 & l_7 \end{bmatrix} \quad (2\times2)
\]

\[
k = \begin{bmatrix} 4EI_1 & l_1 & \ldots & 0 & 0 \\ l_1 & 4EI_1 & \ldots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \ldots & 4EI_7 & 2EI_7 \\ 0 & 0 & \ldots & 2EI_7 & 4EI_7 \end{bmatrix} \quad (14\times14)
\]
In case of no hinge at node 2, 3, 6 or 7 the marked values of the corresponding lines will be changed.

\[
K_{\text{sing}} = a^T \cdot k \cdot a \quad (20\times20)
\]

\[
K_{\text{Bound}} = \begin{bmatrix} K_{11} & \vdots & 0 & 0 \\
0 & \ddots & \vdots & 0 \\
\vdots & \ddots & \ddots & \vdots \\
0 & 0 & \ddots & 0 \\
0 & 0 & \ddots & K_{20\times20}
\end{bmatrix} \quad (20\times20)
\]

\[
K = K_{\text{sing}} + K_{\text{Bound}} \quad (20\times20)
\]

The loading that has been implemented by the engineer in the input worksheet is subdivided into 100 load steps and applied gradually to the system. After 100 load steps the entire load is applied to the statical system. It might happen, that a kinematic chain due to plastic hinges will occur and the beam series will fail before reaching the last sub step (singular stiffness matrix). In this cases the iteration will continue with a different system, which is described in the following.

2.2.2.3 Load-transfer of the vertical loads \(N\) and the bending moment \(M\) – static model after formation of a plastic chain

The anchor plate can be considered as a tension member after the formation of a plastic chain (see Fig. 2.10.) As a simplification the whole resultant tension force is assigned to the bar with the higher inclination. For each new load step the increase in loading of the normal force in the deformed system is determined. In the next step the elongation of the tensional bar and the entire deformation of the anchor plate is calculated. In general the load carrying capacity is limited due to the component resistance of the supports (headed studs). Due to the relatively low deformation of the anchor plate extreme horizontal forces will act at the supports of the membrane system.
Fig. 2.10: Model of the baseplate under tension and simplified calculation model

For the load transfer of the horizontal forces $V$ the friction forces between concrete and the anchor plate are considered on all joints with compression springs (see Fig. 2.11). The remaining forces as difference between friction part and applied shear load will be distributed among the headed studs according to the stiffness of the spring.

![Diagram](image)

Fig. 2.11: Design model for horizontal (shear) loads

### 2.2.3 EXCEL-Worksheets / VBA-Program

The whole design tool contains ten Microsoft Excel worksheets and one Microsoft Visual Basic program part. Visible for the user are only the worksheets “Input + Output” and “Design output”. The following schedule gives a short overview about the function of the different worksheets (see Tab. 2.1 and Tab. 2.2).

#### Tab. 2.1: Overview of all worksheets

<table>
<thead>
<tr>
<th>Name (Worksheet)</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>“Input + Output”</td>
<td>Chapter 2.2.7</td>
</tr>
<tr>
<td>“Design output”</td>
<td>Chapter 2.2.8</td>
</tr>
<tr>
<td>“Headed studs tension”</td>
<td>Determination of the deformation behaviour and the load bearing capacity of the component “headed studs in tension (considering additional reinforcement)”</td>
</tr>
<tr>
<td>“Headed studs shear”</td>
<td>Determination of the deformation behaviour and the load bearing capacity of the component “headed studs in shear”</td>
</tr>
<tr>
<td>“HS interaction tension-shear”</td>
<td>Determination of the load bearing capacity of headed studs under tension and shear loads</td>
</tr>
<tr>
<td>“Concrete member compression”</td>
<td>Determination of the deformation behaviour and the load bearing capacity of the component “Concrete member under compression loads”</td>
</tr>
<tr>
<td>“Steel plate bending”</td>
<td>Determination of the deformation behaviour and the load bearing capacity of the component “Steel plate under bending moments”</td>
</tr>
<tr>
<td>“Calculation core anchor plate”</td>
<td>Calculation of internal forces and bearing reactions by displacement method for every load step</td>
</tr>
<tr>
<td>“Data”</td>
<td>Data schedule for fixed values (materials, dimensions, partial factors, internal control parameters)</td>
</tr>
<tr>
<td>“Data temp”</td>
<td>Data schedule for temporary values (nodal displacements of every load step); nodal displacements are used to create the moment-rotation curve in “Design output”</td>
</tr>
</tbody>
</table>

#### Tab. 2.2: VBA-Subroutine

<table>
<thead>
<tr>
<th>Program (Subroutine)</th>
<th>Function</th>
</tr>
</thead>
</table>
2.2.4 Components

The following components are implemented in the program. Detailed explanations of this components can be found in Handbook I in the specific sections. The load deformation behaviour of the anchor plate is considered within the iterative calculation of the load steps.

Tab. 2.3: Components implemented in the calculation program for slim anchor plate

<table>
<thead>
<tr>
<th>Component</th>
<th>Headed stud in tension</th>
<th>Concrete breakout in tension</th>
<th>Stirrups in tension</th>
<th>Pull-out failure of the headed stud</th>
<th>Headed stud in shear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Tab. 2.4: Ultimate limit state (CEN/TS 1992-4-1:2009 4.4.3.1.1 [1])

<table>
<thead>
<tr>
<th>Steel</th>
<th>Anchors tension</th>
<th>Anchors shear</th>
<th>Reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_{Ms} )</td>
<td>( B_{Ms} )</td>
<td>( B_{Ms,ref} )</td>
<td></td>
</tr>
<tr>
<td>( B_{Ms}=1,2* f_{uk} / f_{yk} (\gamma_{Ms} \geq 1,4) )</td>
<td>( B_{Ms}=1,0* f_{uk} / f_{yk} (\gamma_{Ms} \geq 1,25 ( f_{uk} \leq 800 \text{ N/mm}^2 \text{ and } f_{uk} / f_{uk} \leq 0,8)) )</td>
<td>1,15</td>
<td></td>
</tr>
<tr>
<td>( B_{Ms}=1,25 ( f_{uk} &gt; 800 \text{ N/mm}^2 \text{ or } f_{uk} / f_{uk} &gt; 0,8) )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Tab. 2.5: Ultimate limit state (EN 1993-1-8 [9])

<table>
<thead>
<tr>
<th>Steel</th>
<th>Concrete</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steelplate</td>
<td>Cone failure</td>
</tr>
<tr>
<td>( B_{Ms} )</td>
<td>( B_{Mc} )</td>
</tr>
<tr>
<td>1,00</td>
<td>1,5</td>
</tr>
<tr>
<td>(no stability failure)</td>
<td>1,5</td>
</tr>
<tr>
<td></td>
<td>1,5</td>
</tr>
</tbody>
</table>

2.2.5 Safety factors

2.2.6 Boundary conditions

Anchor plates with headed studs at the concrete side and a welded steel profile at the airside do have complex three dimensional load transfer. Under compression forces all sections of the anchor plate are supported in places, where a gap might occur (except in the area of the headed studs) under tensional forces. The web and the flange of the welded steel sections do have a stiffening effect on the anchor plate. Independently from the thickness of the anchor plate the anchor plate is assumed in the stiffened sections as almost completely rigid. Due to this reason the system is assumed as two dimensional continuous beam.
the midsection of the beam the normal and shear forces and the bending moments are acting. Between line 2 and line 3 (see Fig. 2.12) the anchor plate is assumed to be rigid and discretized by a rigid bar. The geometrical cross section of all other bars is formed by the effective width \( b_m \) and the thickness of the anchor plate \( t_{AP} \). As lower limit the effective width \( b_m \) is assumed with \( b_{PR} + 5 \cdot t_{AP} \), as upper limit the entire width of the anchor plate is possible. If plastic hinges in the anchor plate occur the yield lines are assumed as continuous and perpendicular to the axis of the discretized bar (see Fig. 2.12). If this plastic resistance of the anchor plate is larger as if the yield lines would be locally limited due to a triangular shape of the yield lines (see Fig. 2.13). The effective width of the anchor plate is reduced accordingly without falling below the resistance of the lower limit.

**Fig. 2.12:** Static model of the anchor plate yielding lines with yielding lines over the whole width

**Fig. 2.13:** Local rotating yielding lines for cases where \( b_{HS} > b_{PR} \)

The tensional resistance in cases of straight yielding lines (see Fig. 2.12) can be calculated with Equations (2.16) to (2.18).

\[
Z_{Rd} \cdot \delta = m_{p,ld} \cdot b_{AP} \cdot \left( \frac{2 \cdot \delta}{a} + \frac{2 \cdot \delta}{b} \right)
\]

(2.16)

\[
Z_{Rd} = m_{p,ld} \cdot f_{bar}
\]

(2.17)

\[
f_{bar} = \frac{b_{AP} \cdot \left( \frac{z}{a} + \frac{2}{b} \right)}{\delta}
\]

(2.18)

The tensional resistance for local rotating yielding lines (see Fig. 2.14) can be calculated with Equations (2.19) to (2.22).
Fig. 2.14: Geometry for local rotating yielding lines

\[
\begin{align*}
\ell_1 &= c + 2 \cdot d \\
\ell_2 &= ((a + b)^2 + d^2)^{1/2} \\
\ell_3 &= a \\
\ell_4 &= c \\
\ell_5 &= (b^2 + d^2)^{1/2} \\
s_1 &= b \\
s_2 &= \ell_3 \cdot \sin \alpha_{23} \text{ with } \sin \alpha_{23} = d / \ell_2 \\
s_3 &= \ell_3 / \tan \alpha_{23} \text{ with } \tan \alpha_{23} = d / (a + b) \\
s_4 &= \ell_4 \\
s_5-1 &= \ell_5 \cdot \tan \alpha_{15} \text{ with } \tan \alpha_{15} = b / d \\
s_5-2 &= \ell_5 \cdot \tan \alpha_{25} \text{ with } \sin \alpha_{25} = s_2 / \ell_5 \\
\tan \psi_1 &= \delta / s_1 \approx \psi_4 \\
\tan \psi_2 &= \delta / s_2 \approx \psi_2 \\
\tan \psi_3 &= \delta / s_3 \approx \psi_3 \\
\tan \psi_{4,0} &= \delta / \ell_3 + \delta / s_4 \approx \psi_{4,0} \\
\tan \psi_{4,u} &= \delta / \ell_3 \approx \psi_{4,u} \\
\tan \psi_5 &= \delta / s_{5-1} + \delta / s_{5-2} \approx \psi_5 \\
Z_{\text{Rd}} \cdot \delta &= m_{\text{pl,Rd}} \cdot (\ell_1 \cdot \psi_1 + 2 \cdot \ell_2 \cdot \psi_2 + 2 \cdot \ell_3 \cdot \psi_3 + \ell_4 \cdot \psi_{4,0} + \psi_{4,u} + 2 \cdot \ell_5 \cdot \psi_5) \\
f_{\text{local}} &= (\ell_1 \cdot \psi_1 + 2 \cdot \ell_2 \cdot \psi_2 + 2 \cdot \ell_3 \cdot \psi_3 + \ell_4 \cdot (\psi_{4,0} + \psi_{4,u}) + 2 \cdot \ell_5 \cdot \psi_5) / \delta \\
\text{If } f_{\text{local}} < f_{\text{bar}} \text{ the effective width of the bar is calculated with Eq. (2.23).}
\end{align*}
\]

\[
b_m = b_{\text{AP}} \cdot f_{\text{local}} / f_{\text{bar}}
\] (2.23)

If the supplementary reinforcement is located with too large distance from the headed stud or from the concrete surface the anchorage length of the reinforcement within the concrete cone can be too small (see Fig. 2.15). In the worst case the contribution due to the supplementary reinforcement can be neglected. The distances X and Y in Fig. 2.15 have to be minimized.

The design calculations for the connection between the steel profile and the anchor plate are not covered by the design program and have to be done in spate calculations. If steel profiles are not directly welded to the anchor plate and connected by threaded studs and an endplate the dimensions l_{\text{AP}} and b_{\text{AP}} have to be defined analogous independent from the actual dimensions of the steel profile (for example with the distances of the threaded studs l_{\text{AP}} and b_{\text{AP}}). The new components that are used in the program are based on test with large edge distances of the headed studs. Due to this reason the edge distances of Fig. 4.22 are required (see Chapter 4.3.4.4).
2.2.7 **Input mask**

The input sheet “Input + Output” shows on top a sketch of the connection labeling the most important input parameters. In the second part of the worksheet the dimensions, materials and loads on the anchor plate can be entered into the program. With the “Calculation-Button” on the right bottom of the worksheet the non-linear determination of internal forces and the component design will be started. Left beside the “Calculation-Button” the degree of utilization of the main components is shown. In the following the input data is described in particular.

**Steel profile (1. line):** Input of the length $l_{PR}$ [mm] and the width $b_{PR}$ [mm] of the connected profile or steel element to determine the rigid plate area. In case of connections of steel profiles with head plates by threaded studs welded on the anchor plate directly the outer distances of the threaded studs in both directions have to be used for $l_{PR}$ and $b_{PR}$.

**Anchor plate (2. line):** Input of the length $l_{AP}$ [mm], the width $b_{AP}$ [mm] and the thickness $t_{AP}$ [mm] of the anchor plate; the number of headed studs per row (2 or 3); the material of the steel plate (acc. to EN 1993-1-1, EN 10025 [4]).

**Headed studs (3. line):** Input of the distances of the headed studs in longitudinal direction $l_{HS}$ [mm], in cross direction $b_{HS}$ [mm]; the shaft diameter [mm]; the length of the studs $h_{s}$ [mm]; the material of the headed studs (acc. to EN 10025 and EN 10088). In case of $l_{HS} \leq l_{PR}$ the distance $b_{HS}$ of the headed studs has to be equal or smaller than the width $b_{PR}$ plus five times $t_{AP} (b_{HS} \leq b_{PR} + 5 \times t_{AP})$.
Reinforcement (4. line): Input of the diameter $d_5$ [mm] and the material (acc. to DIN 488 [3]) of the reinforcement stirrups. The reinforcement stirrups have to be formed as loops with the smallest admissible bending role diameter. They have to be grouped in pairs close to the shafts of the headed studs with minimum distance to the bottom side of the anchor plate (maximum possible overlapping length of stirrup leg and headed stud).

Concrete member (5. line): Input of the thickness $h_c$ [mm] and the material type (acc. to EN 1992-1-1 [7]) of the concrete member.

Loads (last line): Input of the bending moment $M_{Ed}$ [kNm], the normal force $N_{Ed}$ [kN] and the shear force $V_{Ed}$ [kN] as design loads (ultimate limit state). Design loads have to be determined by the user. Partial factors will not be considered at the load side by the program!
2.2.8 Output mask

The output sheet “Design output” is divided into four parts. The first part gives information about the structural system and the non-linear support conditions (spring models). Results of the non-linear determination of internal forces are shown in the second part. In part 3 the main verifications of the components are given. The last part shows the moment-rotation behaviour of the joint.

Fig. 2.17: Excel worksheet “Design output” page 1/3
### Loads:

<table>
<thead>
<tr>
<th>( M_d )</th>
<th>( N_d )</th>
<th>( V_d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[kNm]</td>
<td>[kN]</td>
<td>[kN]</td>
</tr>
<tr>
<td>32.0</td>
<td>-10.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

\( \Delta M_{Ed} = V_{Ed} \cdot (b_d + d) \)

### Internal forces: Bearing reactions and bending moments caused by \( M_d \) and \( N_d \)

<table>
<thead>
<tr>
<th>node 1</th>
<th>node 2</th>
<th>node 3</th>
<th>node 4</th>
<th>node 5</th>
<th>node 6</th>
<th>node 7</th>
<th>node 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>B_x</td>
<td>0.00</td>
<td>33.31</td>
<td>60.48</td>
<td>0.00</td>
<td>0.00</td>
<td>-161.14</td>
<td>67.34</td>
</tr>
<tr>
<td>M_x</td>
<td>0.00</td>
<td>0.00</td>
<td>2.66</td>
<td>10.49</td>
<td>-13.23</td>
<td>-4.47</td>
<td>3.03</td>
</tr>
</tbody>
</table>

### Bearing reactions caused by \( V_d \) used for concrete design

<table>
<thead>
<tr>
<th>node 1</th>
<th>node 2</th>
<th>node 3</th>
<th>node 4</th>
<th>node 5</th>
<th>node 6</th>
<th>node 7</th>
<th>node 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{d1} )</td>
<td>-15.11</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>-15.11</td>
<td>-</td>
</tr>
<tr>
<td>( V_{d2} )</td>
<td>6.66</td>
<td>10.10</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>13.47</td>
</tr>
</tbody>
</table>

### Bearing reactions caused by \( V_d \) used for steel design

\( V_{Ed,\text{mix}} = \min (\{1-\chi d\} \cdot V_{Ed,\text{mix}} \cdot \varphi_{\text{mix}}; \varphi_{\text{mix}} \cdot V_{Ed,\text{mix}}) = 0 \text{ N} \)

\( V_{Ed,\text{mix}} = V_{Ed,\text{mix}} \cdot \varphi_{\text{mix}} = 0 \text{ N} \)

Statement: \( n_r^2 + n_r = 1 \Rightarrow V_{Ed} \)

### Verifications: Headed studs under tension loads

#### Steel failure of fasteners

Yielding resistance

\[ N_{Ed,\text{mix}} = n_r \cdot A_r \cdot f_{rk} \]

\[ N_{Ed} / N_{Ed,\text{mix}} = 0.69 \]

Ultimate resistance

\[ N_{Ed,\text{mix}} = n_r \cdot A_r \cdot f_{uk} \]

\[ N_{Ed} / N_{Ed,\text{mix}} = 0.63 \]

### Concrete cone failure

\[ N_{Ed,\text{mix}} = N_{\text{conf}} \cdot A_{\text{conf}} / A_{\text{conf}} = 234945 \text{ N} \]

\[ N_{Ed} / N_{Ed,\text{mix}} = 1.03 \]

### Concrete cone failure with reinforcement

Concrete failure

\[ N_{Ed,\text{mix}} = \psi_{\text{conf}} \cdot N_{Ed,\text{mix}} \]

\[ N_{Ed} / N_{Ed,\text{mix}} = 539177 \text{ N} \]

Yielding of reinforcement

\[ N_{Ed,\text{mix}} = \psi_{\text{conf}} \cdot N_{Ed,\text{mix}} \]

\[ N_{Ed} / N_{Ed,\text{mix}} = 403972 \text{ N} \]

Anchorage failure

\[ N_{Ed,\text{mix}} = \psi_{\text{conf}} \cdot N_{Ed,\text{mix}} \]

\[ N_{Ed} / N_{Ed,\text{mix}} = 536888 \text{ N} \]

\[ N_{Ed,\text{mix}} = \min \{ N_{Ed,\text{mix}} / A_{\text{conf}}, N_{Ed,\text{mix}} / A_{\text{conf}}, N_{Ed,\text{mix}} / A_{\text{conf}} \} = 351280 \text{ N} \]

\[ N_{Ed} / N_{Ed,\text{mix}} = 0.46 \]

### Pull-out failure

\[ N_{Ed} = n_r \cdot p_r \cdot A_r \]

\[ N_{Ed} / N_{Ed,\text{mix}} = 746389 \text{ N} \]

\[ N_{Ed,\text{mix}} = 497593 \text{ N} \]

\[ N_{Ed} / N_{Ed,\text{mix}} = 0.32 \]

Design output 2/3

Fig. 2.18: Excel worksheet “Design output” page 2/3
### Headed studs under shear loads

**Steel failure of fasteners**

\[ V_{\text{rel},s} = V_{\text{rel},s}^* \theta^\ast A_s f_{p,k} \]

\[ V_{\text{rel},s} = 335393 \text{ N} \quad V_{\text{rel},s} = 0.00 \text{ N} \]

**Ply-out failure**

\[ V_{\text{rel},cp} = V_{\text{rel},cp} \sqrt{\eta} \]

\[ V_{\text{rel},cp} = 1077299 \text{ N} \quad V_{\text{rel},cp} = 0.00 \text{ N} \]

---

**Headed studs interaction tension / shear**

<table>
<thead>
<tr>
<th>Steel failure</th>
<th>N_{\text{rel}}</th>
<th>V_{\text{rel}}</th>
<th>n_{n1}</th>
<th>n_{n2}</th>
<th>n_{n3}</th>
<th>n_{n4}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>233510 N</td>
<td>168752 N</td>
<td>0.69</td>
<td>0.14</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>N_{\text{rel}}</td>
<td>V_{\text{rel}}</td>
<td>n_{n1}</td>
<td>n_{n2}</td>
<td>n_{n3}</td>
<td>n_{n4}</td>
</tr>
<tr>
<td>Concrete failure</td>
<td>351280 N</td>
<td>538629 N</td>
<td>0.46</td>
<td>0.09</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

\[ n_{n1}^2 + n_{n2}^2 = 0.27 \quad n_{n1}^2 + n_{n2}^2 = 0.02 \]

---

**Concrete member under pression loads**

\[ \sigma_p = \frac{V_{\text{rel}}}{b_0} \text{ N/mm}^2 \]

\[ \sigma_p = 70.0 \text{ N/mm}^2 \text{ (considered by non linear material behaviour)} \]

\[ \epsilon = \frac{V_{\text{rel}}}{V_{\text{rel}}} \text{ and } \frac{V_{\text{rel}}}{V_{\text{rel}}} \text{ N/mm}^2 \]

\[ \epsilon = 0.00350 \quad \epsilon = 0.04 \text{ (considered by non linear material behaviour)} \]

---

**Steel plate under bending moments**

\[ (i=1,3,6 and 7) \quad M_{b,i/R_i} = 447.3 \text{ kNcm} \]

\[ M_{b,i/R_i} = 447.3 \text{ kNcm} \text{ (considered by non linear material behaviour)} \]

---

### Stiffness: Deformation behaviour (element 4)

**Moment-Rotation**

\[ \text{Rotation } (\phi = \arctan(V_2 / V_1)) \times 1000 \text{ [mrad]} \]

**Displacement**

\[ u = (V_1 + V_2) / 2 \]

![Graph showing moment-rotation relationship](image)

Fig. 2.19: Excel worksheet “Design output” page 2/3
2.2.9 Optimization of the joint

Following methods can be applied for increase in loading capacity of the joint. Which one of the following changes should be taken is linked to the individual properties of the joint. Additionally the different methods are interdependent and the optimization of the joint is an iterative process. Within this process the specific component has to be changed until sufficient load carrying capacity is reached, see Chapter 4.4.

For large bending moments $M$ and / or large tensional forces $N$:

- (M1) Arrangement of supplementary reinforcement next to the tensional loaded headed stud row.
- (M2) Enlargement of the distance between the headed studs $l_{HS}$ in the transversal direction.
- (M3) Enlargement of the distance between the headed studs up to $b_{HS} = 3 \times h_{ef}$.
- (M4) Enlargement of the effective height of the headed studs.
- (M5) Enlargement of the diameter of the headed studs.
- (M6) Enlargement of the number of headed studs per row.
- (M7) Choice of different steel properties for the headed studs.
- (M8) Choice of higher concrete strength.
- (M9) Enlargement of the thickness of the anchor plate.
- (M10) Choice of different steel properties for the anchor plate.

For large shear forces $V$:

- (M2a) Enlargement of the distance between the headed studs $l_{HS} = 3 \times h_{ef}$.
- (M3) Enlargement of the distance between the headed studs $b_{HS} = 3 \times h_{ef}$.
- (M4) Enlargement of the effective height of the headed studs.
- (M5) Enlargement of the diameter of the headed studs.
- (M6) Enlargement of the number of headed studs per row.
- (M7) Choice of different steel properties for the headed studs.
- (M8) Choice of higher concrete strength.

For bending- and shear forces the methods as described above might be combined. The following table shows possibilities for optimization of joints for different objectives (see Tab. 2.7).

<table>
<thead>
<tr>
<th>Objectives</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small thickness of the anchor plate</td>
<td>For bending: Arrangement of the headed studs at the edges of the connected steel profile</td>
</tr>
<tr>
<td>High ductility</td>
<td>For bending: Configuration of the components of the joint in a way that the plastic chain becomes the decisive component of the anchor plate. Choice of a ductile steel material of the anchor plate.</td>
</tr>
<tr>
<td>Small length of the headed studs</td>
<td>For bending: Methods M1, M2, M3, M5, M6, (M7), M8, M9, M10; For shear: Methods M2a, M3, M5, M6, (M7), M8</td>
</tr>
<tr>
<td>No supplementary reinforcement</td>
<td>For bending: Methods M2 till M8, (M9), (M10)</td>
</tr>
</tbody>
</table>
2.3 Rigid anchor plate with headed studs – simple joint

2.3.1 General

With the program “Rigid anchor plate with headed studs – simple joint” (Version 2.0) [23] the load carrying capacities of anchor plates with minimum four and maximum six headed studs in two rows under loading in one direction can be calculated (see Fig. 2.20). It is required, that the point of load transfer into the simple joint is defined in the static system as hinged. As this point can’t be assumed directly located at the concrete surface, the eccentricity has to be taken into consideration. As the shear load is applied with some eccentricity also bending moments in the anchor plate have to be considered beside normal and shear forces. In order to increase the tensional resistance of the component of the headed stud, supplementary reinforcement can be used next to the studs. With the supplementary reinforcement high increases in loading of the joint are possible as the load carrying capacity of pure concrete cone failure can be increased by taking the reinforcement into account. The anchor plate is assumed to be rigid without any plastic reserves.

2.3.2 Program structure and static model

The design software is based on the EXCEL table calculation program with the integrated programming language VBA. Within the EXCEL file ten different spreadsheets for the in- and output, for the design of the different components, for the consideration of the joint in the global analysis and for a summary of the joint properties. In a first step the height of the compression zone is assumed. Based on this assumption all unknown forces in Fig. 4.11 can be calculated. Based on moment equilibrium and equilibrium in the vertical direction the assumption can be verified. The shear force $V_{Ed}$ is carried by a frictional part and the two shear components of the headed studs, see Eq. (2.24).

$$V_{Ed} = V_{Ed,2} + V_{Ed,2} + V_f$$  \hspace{1cm} (2.24)

With the equilibrium of moments at the intersection point of the action lines of the concrete force $C_{Ed}$ and the shear components of the headed studs $V_{Ed,2}$ and $V_{Ed,1}$, the formulations in Eq. (2.25) can be obtained for the calculation of the applied normal force in the second stud row. By a vertical equilibrium of forces the assumed height of the compression zone can be verified. In the program the effective compressive height is determined iteratively. For further information see Design Manual I "Design of steel-to-concrete joints", Chapter 5.2.2 [13] and for the calculation of the deformations see Chapter 4.2 and Chapter 4.3.

$$V_{Ed} = V_{Ed,2} \cdot \frac{(e + t + d)}{(z + \mu \cdot d)}$$  \hspace{1cm} (2.25)

2.3.3 EXCEL Worksheets / VBA program

The whole design tool contains 10 Microsoft Excel worksheets. Visible for the user are only the worksheets “Input + Output CM” and “Design output CM”. The following schedule gives a short overview about the function of the different worksheets (see Tab. 2.8).
Tab. 2.8: Overview about the different worksheets

<table>
<thead>
<tr>
<th>Name (Worksheet)</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;Input + Output CM&quot;</td>
<td>Chapter 2.3.7</td>
</tr>
<tr>
<td>&quot;Design output CM&quot;</td>
<td>Chapter 2.3.8</td>
</tr>
<tr>
<td>&quot;Headed studs tension&quot;</td>
<td>Determination of the deformation behaviour and the load bearing capacity of the component &quot;headed studs in tension (considering additional reinforcement)&quot;</td>
</tr>
<tr>
<td>&quot;Headed studs shear&quot;</td>
<td>Determination of the deformation behaviour and the load bearing capacity of the component &quot;headed studs in shear&quot;</td>
</tr>
<tr>
<td>&quot;Headed studs interaction tension-shear&quot;</td>
<td>Determination of the load bearing capacity of headed studs under tension and shear loads</td>
</tr>
<tr>
<td>&quot;Concrete member under compression&quot;</td>
<td>Determination of the deformation behaviour and the load bearing capacity of the component &quot;Concrete member under compression loads&quot;</td>
</tr>
<tr>
<td>&quot;Steel plate bending CM&quot;</td>
<td>Design of the anchor plate under bending moments</td>
</tr>
<tr>
<td>&quot;Calculation core CM&quot;</td>
<td>Calculation of internal forces by equilibrium of forces and moments; iterative determination of the compression zones length</td>
</tr>
<tr>
<td>&quot;Data&quot;</td>
<td>Data schedule for fixed values (materials, dimensions, partial factors, internal control parameters)</td>
</tr>
<tr>
<td>&quot;Data temp&quot;</td>
<td>Data schedule for temporary values (nodal displacements of every load step); nodal displacements are used to create the moment-rotation curve in &quot;Design output&quot;</td>
</tr>
</tbody>
</table>

2.3.4 Components

The following components are implemented in the program (see Tab. 2.9). Detailed explanations of this components can be found in Handbook I in the specific sections. The load deformation behaviour of the anchor plate is considered within the iterative calculation of the load steps.

Tab. 2.9: Components implemented in the calculation program for a rigid anchor plate

<table>
<thead>
<tr>
<th>Component</th>
<th>Headed stud in tension</th>
<th>Concrete breakout in tension</th>
<th>Stirrups in tension</th>
<th>Pull-out failure of the headed stud</th>
<th>Headed stud in shear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure</td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
<td><img src="image5.png" alt="Image" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Component</th>
<th>Friction</th>
<th>Concrete in compression</th>
<th>Threaded studs in tension/shear</th>
<th>Anchor plate in bending and tension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure</td>
<td><img src="image6.png" alt="Image" /></td>
<td><img src="image7.png" alt="Image" /></td>
<td><img src="image8.png" alt="Image" /></td>
<td><img src="image9.png" alt="Image" /></td>
</tr>
</tbody>
</table>

2.3.5 Safety factors

See Chapter 2.2.5
2.3.6 Boundary condition

The calculation of the design resistance of the connection between the steel element and the anchor plate is not covered by the program and has to be done separately by the engineer. If the steel elements or the steel profiles are not directly welded to the anchor plate and connected by threaded studs and an endplate, the dimensions $l_{AP}$ and $b_{AP}$ have to be defined analogous independent from the actual dimensions of the steel profile (for example with the distances of the threaded studs $l_{AP}$ and $b_{AP}$). The new components that are used in the program are based on test with large edge distances of the headed studs. Due to this reason the edge distances described in Chapter 2.2.6 are required. Also requirements for the exact location of the supplementary reinforcement are given there.

2.3.7 Input mask

The input sheet “Input + Output” shows on top a sketch of the connection labeling the most important input parameters (see Fig. 2.21). In the second part of the worksheet the dimensions, materials and loads on the anchor plate can be fed into the program. With the “Calculation-Button” on the right bottom of the worksheet the determination of internal forces and the component design will be started. Left beside the “Calculation-Button” the degree of utilization of the main components is shown. In the following the input data is described in particular.

Steel profile (1. line): Input of the length $l_{PR}$ [mm] and the width $b_{PR}$ [mm] of the connected butt strap.

Anchor plate (2. line): Input of the length $l_{AP}$ [mm], the width $b_{AP}$ [mm] and the thickness $t_{AP}$ [mm] of the anchor plate; the number of headed studs per row (2 or 3); the material of the steel plate (acc. to EN 1993-1-1 [8] and EN 10025 [4]).

Headed studs (3. line): Input of the distances of the headed studs in longitudinal direction $l_{HS}$ [mm], in cross direction $b_{HS}$ [mm]; the shaft diameter [mm]; the length of the studs $h_{s}$ [mm]; the material of the headed studs (acc. to EN 10025 [4]).

Reinforcement (4. line): Input of the diameter $d_{s}$ [mm] and the material (acc. to DIN 488 [3]) of the reinforcement stirrups. The reinforcement stirrups have to be formed as loops with the smallest admissible bending role diameter. They have to be grouped in pairs close to the shafts of the headed studs with minimum distance to the bottom side of the anchor plate (maximum possible overlapping length of stirrup leg and headed stud).

Concrete member (5. line): Input of the thickness $h_{C}$ [mm] and the material type (acc. to EN 1992-1-1 [7]) of the concrete member.

Fig. 2.21: Excel worksheet “Input + Output CM” page 1/1
**Loads (last line):** Input of the shear force \( V_{Ed} \) [kN] and their eccentricity to the anchor plates surface in [cm]. Design loads have to be determined by the user. Partial factors will not be considered at the load side by the program!

### 2.3.8 Output mask

The output sheet “Design output” is divided into three parts. The first part gives information about the structural system (see Fig. 2.21). Results of the static calculation of internal forces are shown in the second part (see Fig. 2.23). In part 3 the main verifications of the components are given (see Fig. 2.23).

**Fig. 2.22:** Excel worksheet “Design output CM”  
**Fig. 2.23:** Excel worksheet “Design output CM”

### 2.3.9 Optimization of the joint

The optimization of the joint can be done according to the optimization of the connection of the slim anchor plate (see Chapter 2.2.9) more information about optimization of simple joints is given in the parameter study for simple joints in Chapter 4.3.
Pomocí karty Domů použijte u textu, který se má zde zobrazit, styl Überschrift 1.
3 Design examples

3.1 Composite beam of a standard office structure connected to reinforced concrete wall

3.1.1 General

3.1.1.1 Depiction of the situation

Multistory office building structures often have a floor modulus of \( n \cdot 1,35 \) m by approx. 7,8 m, which results from the room depth plus corridor. Beneath several variation of concrete slabs with or without beams concrete steel composite beams made of a hot rolled cross section IPE 300 with a semi-finished concrete slab connected by studs can be used to reduce the height of the construction and by this means the total height of each floor. One possibility to design a construction of minimum height properly can be the moment resistant constraint in the wall. The knowledge of rotational behaviour of the connection allows to optimize the connection on behalf of reinforcement and to evaluate the redistribution of forces.

3.1.1.2 Overall structural system

The example shows a concrete steel composite beam made of a hot rolled cross section IPE 300 with a semi-finished concrete slab (total 14 cm) connected by studs. The lateral distance of the beams is \( 2 \cdot 1,35 \) m = 2,70m, the span is 7,8 m. The inner support can be at a reinforced concrete (RC) wall of the building core, the outer support is a façade column (see Fig. 3.1).

- Semi-finished slab (6cm precast concrete) + cast in-situ of altogether 14 cm, continuous system, span 2,70 m each.
- Hot rolled beam IPE 300 S355 JR, \( L = 7,8 \) m; uniformly distributed loading with headed studs.
- Support façade: Steel column spaced 2,7 m.
- Support inner core: Reinforced concrete wall with fully restraint connection by reinforcement and steel/concrete compression contact.

3.1.1.3 Loads

| Own weight slab        | \( g' \) = 1.6 kN/m |
| Own weight slab        | \( g_1 \) = 3.5 kN/m² |
| Dead load screed       | \( g_2 \) = 1.6 kN/m |
| Dead load suspended ceiling + installation | \( g_3 \) = 0.4 kN/m² |
| Dead load (total)      | \( g \) = 5.50 kN/m² |

Live load (B2,C1 acc. DIN 1991-1-1 NA [5])

| q = 3.00 kN/m² |
3.1.2 Execution options

3.1.2.1 Previous realization

To provide a moment resistant connection of composite structures to a concrete wall is not new at all, as it isn’t the separation of tensile forces into the slab’s reinforcement and compression into lower beam flange resp. anchor plate. Nevertheless there have been bolted solutions with fin plates for the shear forces or endplates as an adaption of common steel constructions, which were more costly in terms of manufacturing costs. These solutions with their more complex mechanisms were as well difficult to design effectively and to predict their rotational behaviour. Therefore a larger range of maximum and minimum forces has to be covered, as the redistribution of forces is unknown.

3.1.2.2 Improved implementation

The presented connection of a moment resistant connection of composite structures to a concrete wall provides a solution that is simple feasible on site, because the vertical and horizontal tolerances are relatively high, and the necessary parts are minimized. Forces are strictly separated and transferred by easy mechanisms. Due to this reason the knowledge of the connection behaviour has grown since any of these single shares have been explored further on and the characteristics have been put in a simple component (spring) model. The component method is implemented in Eurocodes, but has been improved by detail research throughout this project. So the stress-strain model of the slab’s reinforcement has been developed with additive tensile stresses in concrete, the displacement of the anchor plate, the slip of the slab studs can now be considered and the contribution of the nonlinear behaviour of the shear panel in the connecting concrete wall has been added.
3.1.3 **Structural analysis of the joint**

3.1.3.1 **Modelling**

The member forces of the structure generally can be calculated with any software which is able to consider ranges of different beam stiffness and rotational springs. As the structure is statically indeterminate the different stiffness of the positive and negative moment range must be taken into account to properly calculate the member and support forces. For this example the software KRASTA [21] for spatial frame analysis has been used. Prior to any calculation we can do a reliable prediction concerning the quality of moment distribution. There will be a maximum negative moment at the moment resistant support, the moment will then be reduced and will cross the zero-line. Afterwards it will drop down to its positive maximum at approx. 5/8 of the span and ending at zero at the hinged support at the end of the beam. The negative range is assumed for the first quarter of span, the positive is set for the rest of span. According to EN 1994-1-1 Cl. 5.4.1.2 [10] the effective width can be calculated with Eq. (3.1).

\[
\text{b}_{\text{eff}} = \text{b}_0 + \sum \text{b}_{e,i}
\]  

(3.1)

In case of equally spaced beams these equation can be calculated in the negative range with Eq. (3.2) and in the positive range with Eq. (3.3) each of them less as the spacing between adjacent beams (270 cm). This means that necessary reinforcement bars in the negative range of the slab must be arranged within the effective width.

\[
\begin{align*}
\text{b}_{\text{eff},2} &= 15 + 2 \times 780 \times 0.25/8 = 63.8 \text{ cm} \\
\text{b}_{\text{eff},1} &= 15 + 2 \times 780 \times 0.75/8 = 161.25 \text{ cm}
\end{align*}
\]  

(3.2)  

(3.3)

The different moments of inertia $I_{\text{pos}}$ are calculated in accordance to common values of creep influence (see Eq. (3.4) to (3.5)). In this example the relation between stiffness shortly after erection and after 1-2 years (means $T=\infty$) is approximately $\frac{3}{4}$. The effect of shrinking (eccentricity of tensional force in slab) is not considered. The moment of inertia for $T=\infty$ will be used with dead load and value for $T=0$ will be used with life load. This will yield the maximum restraint moment and force at support A.

Negative range:  
\[
I_{\text{neg}} \approx 18360 + 15.5 \times (30/2 + 15 - 4)^2 \approx 18000 \text{ cm}^4
\]  

(3.4)

Positive range:  
\[
\begin{align*}
I_{\text{pos},T=0} &\approx 30200 \text{ cm}^4 \\
I_{\text{pos},T=\infty} &\approx 22500 \text{ cm}^4
\end{align*}
\]  

(3.5)

3.1.3.2 **Calculation of forces**

Using the previous mentioned characteristics in the first iteration of forces, the rotation stiffness of the connection is set to infinite, i.e. complete moment resistant restraint. The resultant internal forces for characteristic points of the beam are shown in Fig. 3.3.

The next step will be an assessment of the moment restraint connection and the evaluation of rotation to define a rotational spring characteristic.
The structural design and the evaluation of the connection stiffness will be done by using the program "Restrained connection of composite beams". The model data of the parts which contribute to the connection must be defined in detail. These are geometric data like size, position and thickness of plates with headed studs, the beams cross section, reinforcement in the slab and in the wall. As the rotational characteristics are largely dependent of the slab reinforcement, a contribution of stud slip, the concrete shear panel behaviour and the compressed anchor plate is considered as well. Therefore the user is asked for parameters, which are not important for the connection design but might have influence on the horizontal displacement of the slab. By starting the Excel worksheet all parameters are set by default with a valid set of input data, where an obviously rational result is obtained. It will never the less be a duty for the user to ensure, that all parameters are reasonable, at least under geometrical aspects (spacing of reinforcement and slab studs, enough reinforcement, studs inside plate etc.). These validation results will show up on the right of the input mask.

The slab reinforcement is set to a value a little higher than the minimum – that is due to the assessment of the shear panel resistance, which is amongst others affected by the amount of reinforcement. This area must be built in within the effective width of the negative moment range of 64 cm. The number of studs over the length of tensile action in the slab is chosen as 13, spacing approx. 15 cm.

Though the calculation is executed with an Excel sheet and is therefore directly updating most of the values upon any changed cell input value, there is a Visual-Basic-Macro implemented to iterate depending on the used model. To update all of these characteristics the calculation must be started with pushing the 'calculate' – button in the lower region of the page. Any changes connected with the anchor plate, beginning with wall concrete and reinforcement parameters and the geometry of plate and studs need the use of this updated macro.

After all geometry data and forces have been inserted into the mask, the two main results will be the utilization of the connection, the relation between given force and the resistance of the connection, and secondly the stiffness of the restrained cross section at the edge of wall to generate a new, updated rotational spring. In the Fig. 3.4 see the completely filled input mask and resulting utilization of the connection. In the following figures (see Fig. 3.5 to Fig. 3.7) the complete detailed output with intermediate results of components and the resulting stiffness of the actual constellation is shown.
**Fig. 3.5: Output file with intermediate results (1)**
The rotational stiffness of the connection will be given as a result of the connection assessment, with a secant stiffness of $C_\varphi \approx 93 \text{ MNm/rad}$. If the secant stiffness of $C_\varphi$ is taken as a rotational stiffness of the support, internal forces can be obtained (see Fig. 3.8).
These values will be approximately very close to convergence and are those values to be assessed finally, and in this case of statically indeterminant systems. The reduced moment and shear force acting in the connection will increase the resulting stiffness output to approximately $C_\phi \approx 94$ MNm/rad, which is no remarkable difference to the value considered. Generally with decreasing moments the stiffness value converges to a maximum, the so called initial stiffness, which is 135 MN/m and cannot be exceeded. This limit is connected to steel strain with uncracked concrete contribution. Due to the a possible reduction of the reinforcement grade the resulting stiffness will decrease to 89 MNm/rad. As there is no underestimation of stiffness, higher forces don’t have to be expected in the connection and the connection can be considered safe.

3.1.3.3 Structural analysis

The structural analysis will be done by using the program "Restrained connection of composite beams "(see Fig. 3.9). As mentioned above, the reinforcement grade is possibly reduced according to the moment reduction.

![Diagram: Restrained Connection of Composite Beams](image)

**Fig. 3.9:** Structural analysis of the restrained connection
The bending diameter of the reinforcement in the wall has a significant influence on the model. Even generally allowed with a value of minimum 10 Øs (with higher edge distances) it is strongly recommended to take a value of 20 Øs, because the curvature influences the diagonal concrete strut in size. The larger the diameter of bending is, the larger is the effective concrete area which resists the tensile force within the slab’s reinforcement bars. Definitely this can be the component which limits the resistance of the entire joint. Using the minimum bending diameter one will experience a limitation in the concrete shear panel behaviour which is not appropriate. The increase in the amount of reinforcement or the concrete grade is not a useful option as consequence. In case of statically determined systems the iteration step can obviously be skipped, as the internal forces are not related to the changes of the stiffness in the model.

3.1.4 Conducting

3.1.4.1 Installation

The anchor plate can be installed easily at the inner surface of the formwork because the relatively small headed studs can easily installed on-site within the crosswise placed external reinforcement layer of the concrete wall. The loop-shaped hanger reinforcement will be fixed at the inner reinforcement layer. These must be adjusted after installing the anchor plate, if the distance to the headed studs are too large. The reinforcement in the slab transferring the tensile forces into the wall will be easily mounted by using a rebar splicing system. The screwed joint will be fixed at the formwork. The bar has to have a large bending diameter of 20 Øs as a recommendation to optimize the transfer of the diagonal compression force in the shear panel zone. After removal of formwork the steel bracket will be welded on the anchor plate. In a next step the steel profiles can be mounted, adjusted and the contact element of the formwork for the slab (or semi-finished panels) will be placed and the reinforcement can be screwed into the couplers. After having all reinforcement placed properly the concrete slab can be poured.

3.1.4.2 Tolerances

Deviations of the anchor plate regarding to the longitudinal axis of the beam in horizontal and vertical direction can easily be compensated, because the steel bracket is welded to the anchor plate on-site. If larger tolerances in longitudinal direction of the beam have to be taken into consideration, the beam has to be produced after measurement of the exact distances between supports. Small deviations can be bridged by adapting the steel contact element. The reinforcement connectors placed in the concrete wall may have vertical or horizontal deviations, as far they can be cast in the slab with the necessary concrete cover. The concrete cover should be taken into account sufficiently large not to overrate the inner lever arm of forces.

3.1.4.3 Fire protection

For the structure shown in this example usually the fire resistance R90 has to be fulfilled. The steel structure including its connections must be protected with approved coating systems or plate-shaped panels. As there is no required space for installation within the cross section chambers during erection, the chamber can be filled with concrete as a fire protection. The open bracket at lower flange must nevertheless be protected additionally. This can be assessed by a fire protection expertise. The reinforcement is protected by concrete covert and can be assessed by considering codes.

3.1.4.4 Costs

The ability to calculate the stiffness of the connection with deeper understanding and less uncertainties and therefore getting a more realistic force distribution helps to reduce overall costs of the steel construction. The connection itself can be easily installed by placing the beam on the steel bracket without using any bolts. The use of screwed reinforcement connectors is nevertheless necessary.
3.2 Column base as connection of a safety fence on a car parking deck to a reinforced concrete slab

3.2.1 General

3.2.1.1 Depiction of the situation

A safety fence, consisting of a horizontal beam barrier of two connected hollow sections on vertical columns of rolled sections, is connected at the column bases to a 300 mm thick reinforced concrete slab. Embedded anchor plates with headed studs and welded threaded studs are used to connect the columns base plates with the concrete deck. The distance between the steel columns varies from 1,50 m up to 2,00 m and the centre of the beam barrier is 0,50 m above the concrete surface. The whole construction has to be protected against corrosion, for example by galvanization.

3.2.1.2 Overall structural system

Horizontal beam barrier:
Single span beam L = 1,50 m up to 2,00 m; loaded by a horizontal vehicle impact force.
Vertical column:
Cantilever beam (vertical) L = 0,50 m; loaded by a horizontal vehicle impact force (directly or indirectly by the beam barrier)

3.2.1.3 Loads

Impact passenger car (EN 1991-1-7, Cl. 4.3.1 [6]) \[ F_{dx} = 50,00\, \text{kN} \]
(Load application 0,50 m above street surface)

3.2.1.4 Joint loads

Design load \[ V_{Ed} = F_{dx} = 50,00\, \text{kN} \]
\[ M_{Ed} = 50,00 \times 0,50 = 25,00\, \text{kNm} \]

3.2.2 Execution options

3.2.2.1 Previous realization

![Diagram](image)

Fig. 3.10: Conventional joint solution of the safety fence
In the conventional joint solution the columns base plate is directly bolted to the anchor plate by threaded studs which are welded on the embedded steel plate (see Fig. 3.10). In order to reduce the headed studs tension forces caused by bending, the distance of the studs in load direction and therewith the length of the embedded steel plate have to be large. For this reason the distance between the threaded bolts and headed studs is quite large and high bending moments in the anchor plate are resulting. The anchor plates is designed as thick and rigid in order to consider an elastic approach in the calculation. The dimensions of the anchor plate are given in the following.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Base plate</td>
<td>200 / 200 / 20 mm</td>
<td>S 235</td>
</tr>
<tr>
<td>Threaded bolts</td>
<td>φ 20 mm</td>
<td>$f_{ub} = 500$ N/mm²</td>
</tr>
<tr>
<td>Distance threaded bolts</td>
<td>150 / 150 mm</td>
<td></td>
</tr>
<tr>
<td>Anchor plate</td>
<td>350 / 350 / 30 mm</td>
<td></td>
</tr>
<tr>
<td>Headed studs</td>
<td>22 / 175 mm</td>
<td>S 235</td>
</tr>
<tr>
<td>Distance headed studs</td>
<td>270 / 270 mm</td>
<td></td>
</tr>
</tbody>
</table>

### 3.2.2.2 Improved realization Version 1

Within this modified joint solution the new components of the INFASO [12] project are considered. In this solution the columns base plate is directly bolted to the anchor plate by threaded studs which are welded on the embedded steel plate (see Fig. 3.11). The choice of a quite small anchor plate generates high tension forces in the headed studs caused by external bending moment. So additional hanger reinforcement is fixed very close to the headed studs loaded by tension forces. The distance of the headed studs in load direction is small. Due to the fact that the headed studs and the threaded studs are spaced very close, low bending moments in the anchor plate are resulting. A plastic design of the anchor plate is possible. Thin steel plates can be used. The complete embedded plate is covered by the columns base plate. The dimensions of the anchor plate are given in the following.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Base plate</td>
<td>200 / 200 / 20 mm</td>
<td>S 235</td>
</tr>
<tr>
<td>Threaded bolts</td>
<td>φ 20 mm</td>
<td>$f_{ub} = 500$ N/mm²</td>
</tr>
<tr>
<td>Distance threaded bolts</td>
<td>150 / 150 mm</td>
<td></td>
</tr>
<tr>
<td>Anchor plate</td>
<td>200 / 200 / 12 mm</td>
<td></td>
</tr>
<tr>
<td>Headed studs</td>
<td>22 / 175 mm</td>
<td>S 235</td>
</tr>
<tr>
<td>Distance headed studs</td>
<td>150 / 150 mm</td>
<td></td>
</tr>
</tbody>
</table>
The structural design will be done by using the program "Slim anchor plate with headed studs" (see Fig. 3.12 and Fig. 3.13).

---

**Fig. 3.12: Excel sheet, “Input+Output”-mask for version 1**
3.2.2.3 Improved realization - Version 2

**Description of a modified joint solution considering the new components of WP 1**

Within this modified joint solution the new components of the INFASO [12] project are considered. The columns base plate is directly bolted to the anchor plate by threaded studs which are welded on the embedded steel plate. Except that the anchor plate has only a thickness of 12 mm the construction is the same as shown by structure of the previous realization. The embedded steel plate cannot take the total bending moment. The behaviour of the joint is comparable with a kind of plastic hinge. The joint will be designed by transformation of kinetic energy into plastic deformation energy. The dimensions of the anchor plate are given in the following.

<table>
<thead>
<tr>
<th>Component</th>
<th>Dimensions</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base plate</td>
<td>200 / 200 / 20 mm</td>
<td>S 235</td>
</tr>
<tr>
<td>Threaded bolts</td>
<td>ø 20 mm, f_y = 500 N/mm²</td>
<td></td>
</tr>
<tr>
<td>Distance threaded bolts</td>
<td>150 / 150 mm</td>
<td></td>
</tr>
<tr>
<td>Anchor plate</td>
<td>350 / 350 / 12 mm</td>
<td></td>
</tr>
<tr>
<td>Headed studs</td>
<td>22 / 175 mm</td>
<td>S 235</td>
</tr>
<tr>
<td>Distance headed studs</td>
<td>270 / 270 mm</td>
<td></td>
</tr>
</tbody>
</table>

For structural design information about the bending moment resistance and the deformation behaviour of the joint is needed. The bending moment resistance will be obtained by using the program “Slim anchor plate with headed studs” with stepwise increase of the external forces (see Chapter 2.3). After the last step the moment-rotation curve of the sheet “Design output” can be used (see Fig. 3.15). The verification will be done according to EN 1991-1-7 Annex C 2.2 [6].
The moment resistance, the kinetic energy the deformation energy and the deformation of the joint can be calculated with the Equations (3.6) to (3.9)

\[ M_{Ed} = 17.0 \text{kNm} = M_2 \]
\[ V_{Ed} = 34.0 \text{kN} \]
\[ M_{Ed} + V_{Ed} \cdot (t_{AP} + d_{HS}) = 18.0 \text{kNm} \]
\[ \rightarrow M_1 = 11.0 \text{kNm} \cdot 17/18 = 10.40 \text{kNm} \]

\[ E_{\text{kin}} = 1/2 \cdot m \cdot v^2 = 1/2 \cdot 1500 \text{kg} \cdot (10/3.6 \text{m/s})^2 = 5787 \text{Nm} = 5.787 \text{KNm} \]  
(3.7)

\[ E_{\text{def2}} = 10.4/2 \cdot 2.1/1000 = 0.011 \text{KNm} \]
\[ E_{\text{def2}} = (10.4 \cdot (17 - 10.4)/2) \cdot (13.2 - 2.1)/1000 = 0.152 \text{KNm} \]  
(3.8)

\[ \Delta \Phi_{23} = (5.787 - 0.011 - 0.152)/17 = 0.33 \text{rad} \]
\[ \Delta \Phi_{03} \approx 0.33 \text{rad} = 18.9^\circ \]  
(3.9)

Note 1 The required rotation of 18.9° induces extremely large stretching at the locations with plastic hinges. It has to be checked that the admissible elongation is not exceeded.

Note 2 The component 'Headed studs in tension' is high exploited. So the installation of additional hanger reinforcement is advised to ensure that the component 'Anchor plate in bending' is the decisive component.

Due to the extremely deviation of the necessary rotation from the calculated diagram range, the execution of this version can not be recommended!

3.2.3 Conducting and assessment

3.2.3.1 Installation

In each of both cases the embedded plates can be installed easily. The anchor plate of the previous realization is large and heavy and thus it is not so easy to handle during installation. Much more compact and light is the solution of the improved realization, but additional reinforcement is needed.

3.2.3.2 Tolerances

Deviations of the anchor plate’s centre in any horizontal direction could only be settled by oversized holes, in vertical direction by using filler plates. Normally for more or less rude constructions like guide boards low tolerances are needed.

3.2.3.3 Fire protection

For the structure shown in this example, no requirements relating to fire protection have to be fulfilled. If the classification in a particular fire resistance class should be required in other cases, the steel structure including its connections shall be protected with approved coating systems or plate-shaped panels.

3.2.3.4 Costs

For the improved construction lower material costs can be expected. The advantage of the smallest weight of the anchor plate of the improved realization is a bit compensated by the installation costs of the needed additional reinforcement.
3.3 Connection of a balcony construction on an insulated exterior reinforced concrete wall as simple connection

3.3.1 General

3.3.1.1 Depiction of the situation

Continuous, 3.00 m wide balconies are connected to a thermally insulated reinforced concrete wall, supported at their outer edge at a distance of 6.50 m (see Fig. 3.17). The walk-in area is realized as a 14 cm thick precast concrete slab with surrounding up stand. Paving slabs laid in a gravel bed are laid on top. The load-bearing reinforced concrete plates are supported at their ends and arranged parallel to steel girders of 6.50 meters length. These are connected to interception beams running perpendicular to the wall plane and which are connected to the external wall and the steel columns. Embedded anchor plates are used to fasten the steel girders on the concrete wall. Due to the 22 cm thick thermal insulation composite system of the buildings external wall a joint eccentricity of 30 cm between steel beam and anchor plate has to be considered. Within the insulation, a thermal separation is provided. In order to fulfill the plastering practical and professionally, the intersection of the plaster layer should be done only by simple steel plate. All weathered external components must be galvanized.

Fig. 3.17: Conventional solution and structural system
3.3.1.2 Overall structural system

Precast concrete slab:
Single span slab $L = 3,00$ m (uniaxial load transfer) with uniformly distributed load
Ceiling beams:
Single span beams $L = 6,50$ m with uniformly distributed loading
Interception beams:
Single span beams $L = 3,00$ m (connection eccentricity to the steel column and the anchor plate); single loads $F$ close to the supports (see figure)
Connection adapter anchor plate to interception beam:
Cantilevers $L = 0,30$ m

3.3.1.3 Loads

<table>
<thead>
<tr>
<th>Load Description</th>
<th>$g_1$</th>
<th>$g_2$</th>
<th>$g_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dead load beam construction</td>
<td>0.40 kN/m²</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dead load flooring (gravel and paving slabs)</td>
<td>2.00 kN/m²</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dead load precast concrete slab</td>
<td>3.50 kN/m²</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Load Description</th>
<th>$g$</th>
<th>$q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dead load (total)</td>
<td>5.90 kN/m²</td>
<td>4.00 kN/m²</td>
</tr>
</tbody>
</table>

3.3.1.4 Joint loads

<table>
<thead>
<tr>
<th>Load Description</th>
<th>$F_{g,k}$</th>
<th>$F_{q,k}$</th>
<th>$F_{Ed}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dead load</td>
<td>57.53 kN</td>
<td>39.00 kN</td>
<td>136.17 kN</td>
</tr>
</tbody>
</table>

3.3.2 Execution options

3.3.2.1 Previous realization

Fig. 3.18: Conventional joint solution of the balcony construction

The conventional joint solution consists of two parts with an end plate connection, where the inner part made of a rolled profile segment (IPE 220) is welded directly to the anchor plate. At the other end of the
profile, a welded end plate for a rigid joint to the end plate of the outer connector part is located. This outer part consists of a vertical butt strap for a hinged connection to the web of the interception beam. A compression-proof bearing plate for the thermal separation of the two parts will be placed between the two end plates. Depending on the type and thickness of the separating layer a projecting plate to transfer the shear force without bolts in bending must be welded under the connectors' internal part. The weathered external adapter segment is galvanized. As just the inner part to the anchor plate is welded only coating is planned.

The concrete-casted part of the joint consists of an anchor plate with welded reinforcement and a welded on rolled section. The centrally arranged steel profile is designed to carry the vertical shear force. The location of the load resultant can be assumed approximately in the middle of the shear section. Bending moments caused by outer eccentricity (30 cm) and inner eccentricity (outer edge of the anchor plate up to the mid of the shear section) are taken by a couple of horizontal forces. The pressure force is transferred by contact, the tension force is taken by the welded on reinforcement bars. Due to the relatively low wall thickness, the tensile reinforcement is turned down with large bending roll diameter and overlapping with the vertical reinforcement layer of the walls inner side. The horizontal part of the diagonal, from the point of deflection to the anchor plate's lower pressure point leading strut is at equilibrium with the lower pressure force transferred by contact. The location and size of welded steel profiles have decisive influence on the stiffness of the anchor plate. As the end plate is stiffened by the welded steel profiles, pure bending has to be taken into consideration only in the external sections.

3.3.2.2 Improved realization

![Diagram](image)

**Fig. 3.19: Improved joint solution of the balcony construction**

The steel connection of this version is identical to the previously described solution. The concrete-casted part consists of a 25 mm thick anchor plate with four headed studs 22/150 mm (see Fig. 3.9). Closed to the tensional loaded headed studs two reinforcing loops ø 8 mm are installed. A welding of reinforcement to the anchor plate is not required. The hanger reinforcement is placed next to the reinforcement at the inner side of the wall. The supplementary reinforcement has a large bending roll diameter and overlaps with the vertical reinforcement on the inside of the wall. All four studs are involved in the load transfer of the vertically acting shear force, where only the top couple of headed studs will also be used for carrying the horizontal tensile force resulting from the eccentricity moment. The "concrete cone failure mode" is positively influenced by the slope reinforcement arranged directly parallel to the headed studs. The anchor plate is also stiffened in this connection by the welded steel profile of the docking adapter. The structural design will be done by using the program "Rigid anchor plate with headed studs" (see Fig. 3.20 to Fig. 3.21).
Fig. 3.20: Excel sheet, “Input+Output”-mask of the improved realization of the balcony construction

Fig. 3.21: Excel sheet, “Design output” of the improved realization of the balcony construction
3.3.3 Conducting and Assessment

3.3.3.1 Installation

In the improved realization the anchor plate can be installed easily because the relatively small headed studs have a minimal impact on the crosswise running external reinforcement layer of the concrete wall. The loop-shaped hanger reinforcement can be fixed first on the inner reinforcement layer. These must be adjusted yet after installing the anchor plate, if the distance to the headed studs is too large. Due to welded bars on shear connection in the previous realization and reinforcement the anchor plate is unhandy and the walls reinforcement and their order of installation have to be coordinated to the plate’s anchors.

3.3.3.2 Tolerances

Deviations of the anchor plate’s centre to the longitudinal axis of the docking adapter in horizontal and vertical direction inside the walls plane can easily be absorbed, because the adapter is welded to the anchor plate on-site. If tolerances in longitudinal direction of the adapter have to be taken, the port adapters have to be either manufactured extra-long to be cut to the appropriate size on-site or produced after measurement of the exact location of the anchor plates.

3.3.3.3 Fire protection

For the structure shown in this example, no requirements relating to fire protection have to be fulfilled. If the classification in a particular fire resistance class should be required in other cases, the steel structure including its connections shall be protected with approved coating systems or plate-shaped panels.

3.3.3.4 Costs

The cost advantage of the “improved realization” to the anchor plate with shear section and welded reinforcement mainly results by the simpler manufacturing and installation. The studs are fixed by drawn arc stud welding on the fitting steel plate. This process takes a very short time. Concerning the shear section variant, the steel and the rebar in their position must be fixed first and then circumferential welded by hand. This process takes considerably more time. The same applies also for the installation of the anchor plate, because the relatively large shear section and the welded reinforcement have influence on the assembly of the walls reinforcement and some reinforcing bars can be inserted only after the installation of the anchor plate.
Chyba! Pomocí karty Domů použijte u textu, který se má zde zobrazit, styl Überschrift 1. Chyba! Pomocí karty Domů použijte u textu, který se má zde zobrazit, styl Überschrift 1.
4 Parameter studies

4.1 General

In this parameter studies the three steel-to-concrete connections of the Design Manual I "Design of steel-to-concrete joints" [13] are under examination (see Fig. 1.1). In the parameter study for concrete components in Chapter 4.2 experimentally determined pre-factors are taken into consideration. Their influence on the stiffness of the concrete component is shown. In Chapter 4.3 parameters for geometry and material are changed in order to show their influence on the load-deformation and moment rotational behavior of the simple steel-to-concrete joints. Changes in the thickness of the anchor plate and in steel grade for column bases are explained in Chapter 4.4 where especially this two parameters have a high influence on the structural behavior for this type of connection. Also recommendations for design values are given in this Chapter. In the last parameter study the focus is on composite joints in Chapter 4.5.

4.2 Parameter study on concrete components

4.2.1 General

As discussed in Chapter 3 on concrete components in the Design Manual I "Design of steel-to-concrete joints" [13], the stiffness of an anchorage group is determined by several parameters. Out of these parameters, certain parameters are known with high degree of confidence at the time of analysis such as dimensions of studs, dimensions of reinforcement and further parameters. Certain other parameters, such as concrete material properties, bond strength etc. are known within the statistical range of distribution normally occurring in practice. Certain parameters are used to define the stiffness and are based on empirical studies, which are not known with a high certainty. In this chapter, the sensitivity of the stiffness values towards different parameters has been investigated.

4.2.2 Example considered

A basic example of a single headed stud with supplementary reinforcement subjected to tensile loads is considered in this parameter study. The stud is considered to be far from the edges. All the components are considered while evaluating the stiffness of the anchor. The variation of secant stiffness as a function of anchor displacement is evaluated and plotted. Though the plots are given in terms of absolute values for the anchor stiffness and displacement, the objective of this study is to relatively compare the stiffness variation and verify, principally, the influence of different parameters.

4.2.3 Parameter studied and methodology followed

The stiffness of the anchorage system as a function of anchor displacement was evaluated using the formulations given in Chapter 3 of the Design Manual I "Design of steel-to-concrete joints" [13] and a parameter study was performed to investigate the influence of various parameters on the stiffness of the system. While evaluating the influence of a given parameter, all other parameters were kept constant. These parameters are considered as independent and uncorrelated for this study. The parameters considered for the parameter study along with the range of study are tabulated in Tab. 4.1. This assumption of no correlation between the parameters is valid for all the parameters except the concrete compressive strength and the bond strength between concrete and reinforcement. Therefore, for these two parameters, the dependence of bond strength on concrete compressive strength is considered in this study. As can be seen from Tab. 4.1, a sufficiently wide range is considered for the parameter studies.
Tab. 4.1: Parameters considered for the parameter study

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Units</th>
<th>Recommended value in DM</th>
<th>Range of study</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete Strength</td>
<td>$f_{ck}$</td>
<td>N/mm$^2$ (MPa)</td>
<td></td>
<td>25 – 65</td>
</tr>
<tr>
<td>Factor for concrete breakout in tension</td>
<td>$\alpha_c$</td>
<td>-</td>
<td>-537</td>
<td>250 – 1000 (negative)</td>
</tr>
<tr>
<td>Embedment depth</td>
<td>$h_{ef}$</td>
<td>mm</td>
<td>-</td>
<td>50 – 400</td>
</tr>
<tr>
<td>Shoulder width</td>
<td>$a$</td>
<td>mm</td>
<td>-</td>
<td>0.25 – 4.0</td>
</tr>
<tr>
<td>Pressing relation</td>
<td>$m$</td>
<td>-</td>
<td>9</td>
<td>7 – 12</td>
</tr>
<tr>
<td>Design bond strength</td>
<td>$f_{bd}$</td>
<td>N/mm$^2$ (MPa)</td>
<td></td>
<td>0 – 5</td>
</tr>
<tr>
<td>Diameter of supplementary reinforcement</td>
<td>$d_{cre}$</td>
<td>mm</td>
<td>-</td>
<td>6 – 20</td>
</tr>
<tr>
<td>Descending anchor stiffness for component P</td>
<td>$k_{cp,de}$</td>
<td>N/mm</td>
<td>-10000</td>
<td>5000 – 20000 (negative)</td>
</tr>
</tbody>
</table>

### 4.2.4 Sensitivity to Concrete Strength, $f_{ck}$

The concrete strength influences the concrete cone failure and the pull out failure. The influence of concrete strength, $f_{ck}$ is studied for the cases if supplementary reinforcement is considered or not. In case of plain concrete (no supplementary reinforcement), the concrete strength governs the failure load corresponding to concrete cone failure through Eq. (4.1):

$$N_{Rk,c}^0 = k_1 \cdot h_{ef}^{1.5} \cdot f_{ck}^{0.5} \ [N]$$  \hspace{1cm} (4.1)

The stiffness of the ascending branch of the load-deflection curve is considered as infinite and the failure load is assumed to occur at zero displacement. After the peak load is reached, a linearly degrading softening branch is considered. The concrete strength, $f_{ck}$, governs the stiffness of this descending branch, $k_{c,de}$ through Eq. (4.2) for a single anchor far from edge influences.

$$k_{c,de} = \alpha_c \cdot [f_{ck} \cdot h_{ef}]^{0.5} \ [N/mm]$$  \hspace{1cm} (4.2)

No particular value of $f_{ck}$ is recommended in the design manual. Considering the concrete class in normal strength range, in this study, the sensitivity of stiffness to concrete strength is evaluated for cylindrical concrete strength, $f_{ck}$ within the range of 25 MPa to 65 MPa. Fig. 4.1 shows the influence of concrete strength on the stiffness of the anchorage system in concrete without supplementary reinforcement as a function of displacement. It may be noted that the secant stiffness plotted in Fig. 4.1 is the overall stiffness of the system considering all components and not only the concrete cone component.

![Fig. 4.1: Influence of Concrete Strength $f_{ck}$ on stiffness of anchorage without supplementary reinforcement](image-url)
The secant stiffness gradually reduces with increasing displacement. As expected, a clear but reasonable sensitivity is obtained to concrete strength. At higher displacements, the band of stiffness values gets narrower. For the given range of concrete strength that can be expected in practice from the same concrete mix, the sensitivity to concrete strength can be considered through material safety factor. Therefore, the sensitivity of stiffness on concrete strength can be reasonably considered in the analysis. In case of anchorage with supplementary reinforcement, the ascending stiffness of the component stirrups in tension depends on the concrete strength through Eq. (4.3) up to the failure load:

\[
\kappa_{re} = \frac{\sqrt{n^2 \cdot \alpha_c \cdot f_{ek} \cdot d_C}}{\sqrt{2\delta}} \text{ [N/mm]} \tag{4.3}
\]

Thus, the stiffness of both concrete component and the stirrup component are dependent on concrete strength, \( f_{ek} \). Fig. 4.2 shows the influence of concrete strength on the stiffness of anchorage system with supplementary reinforcement. Again, a similar shaped curve as that obtained for anchorage in plain concrete is obtained. The secant stiffness gradually reduces with increasing displacement and at higher displacements the band of stiffness values gets narrower. Based on the results of calculations, it can be said that the sensitivity of the evaluated stiffness of the anchor to the concrete compressive strength is reasonable. For the given range of concrete strength that can be expected in practice from the same concrete mix, the sensitivity to concrete strength can be considered through material safety factor. Therefore, the sensitivity of stiffness on concrete strength can be reasonably considered in the analysis.

### 4.2.5 Sensitivity to parameter \( \alpha_c \)

The parameter, \( \alpha_c \), is used to determine the stiffness of the linear descending branch in case of concrete breakout in tension (see Eq. (4.2)). Currently, a value of -537 is assigned to the factor \( \alpha_c \). In this study, the influence of variation of this parameter on the secant stiffness of the anchor is considered for \( \alpha_c \) in the range of -250 to -1000. The influence of the factor \( \alpha_c \) on stiffness of the anchorage is displayed in Fig. 4.3. During initial displacement range, the secant stiffness is almost independent of \( \alpha_c \). This is because the stiffness during initial displacements is governed by components other than component C and the factor \( \alpha_c \) governs only the descending stiffness of the component C.

![Fig. 4.2: Influence of Concrete Strength, \( f_{ek} \) on stiffness of anchorage with supplementary reinforcement](image-url)
However, at higher displacements, the descending stiffness of the concrete breakout in tension becomes dominating and as seen in Fig. 4.3, the stiffness of the anchorage system becomes sensitive to the parameter $\alpha_c$. The sensitivity is highest in the range of displacements between 2 to 4 mm. On further increasing of the displacements, the stiffness of the system again becomes less sensitive to the parameter $\alpha_c$ (Fig. 4.3). This is because after certain value of displacement equal to the concrete cone failure load, $N_{Rd,kc}$ divided by descending stiffness, $k_{c,de}$, the component C does not contribute anymore to the anchorage. However, the stiffness variation in the middle range of displacements (2 – 4 mm) is not very high and can be considered through material safety factors.

### 4.2.6 Sensitivity to effective embedment depth $h_{ef}$

For concrete component (component C), the effective embedment depth, $h_{ef}$, is the most important factor, which significantly affects the peak failure load (see Eq. (4.1)) as well as the stiffness (see Eq. (4.2)) of the anchorage system. Further, the effective embedment depth also influences the component RB (bond failure of the stirrups) since the effective bond length, $l_1$, of the stirrups is dependent on $h_{ef}$ (see Fig. 4.4). The failure load corresponding to the bond failure of stirrups is given as

$$N_{Rd,br} = \sum_{n_{s,ef}} \frac{l_1 \cdot \pi \cdot d_{s,ef} \cdot f_{bed}}{\alpha} [N] \quad (4.4)$$

There is no recommended value of $h_{ef}$ given in the design manual, however considering the most used sizes in practice, in this study, a range of $h_{ef}$ from 50 mm to 400 mm is considered. As expected, the stiffness of the anchorage system is strongly influenced by the effective embedment depth, $h_{ef}$ for low displacement levels (see Fig. 4.5).
However, as the displacement level increases and concrete cone breakout occurs and the influence of $h_{ef}$ reduces. Nevertheless, in reality, the effective embedment depth of the headed stud is known a priori almost accurately and therefore, the stiffness can also be estimated with reasonable confidence.

### 4.2.7 Sensitivity to shoulder width $a$

The shoulder width of the headed stud given as $a = 0.5(d_h - d_s)$, where $d_h$ is the head diameter and $d_s$ is the shaft diameter, is an important parameter influencing the stiffness of the anchorage. The stud dimensions and hence the shoulder width is normally well known at the time of the analysis. As the bearing area of the headed stud is proportional to the square of the shoulder width, this factor dominates the stiffness of the component $P$. The load-deflection response for the component $P$ is given by Eq. (4.5) up to the point of failure load corresponding to concrete cone failure and Eq. (4.6) beyond that:

$$\delta_{Rd,p,1} = k_p \cdot \left( \frac{N_{Rd,c}}{A_h \cdot f_{ck} \cdot n_{re}} \right)^2 \text{[mm]}$$

$$\delta_{Rd,p,2} = 2k_p \cdot \left( \frac{N_{Rd,p}}{A_h \cdot f_{ck} \cdot n_{re}} \right)^2 - \delta_{Rd,p,1} \text{[mm]}$$

In expressions (4.5) and (4.6), the shoulder width appears indirectly in the bearing area of the head, $A_h$ as well as through the factor $k_p$. The factor $k_p$ is given by Eq. (4.7).

$$k_p = a_p \cdot \frac{k_a \cdot k_k}{k_2} \text{[-]}$$

In Eq. (4.7), the shoulder width appears indirectly in the expression for $k_a$ (see Eq. (4.8)) and $k_A$ (see Eq. (4.9)).

$$k_a = \sqrt{5/a} \geq 1$$

$$k_A = 0.5 \cdot \sqrt{d^2 + m \cdot (d_h^2 - d_s^2)} - 0.5 \cdot d_h$$

Thus, based on Eq. (4.5) through Eq. (4.9), a significant dependence of the component $P$ on the parameter, $a$, should be expected. No value of the shoulder width is recommended in the design manual, however, considering the normal range of shoulder widths that may be encountered in practice, in this study, the shoulder width is varied between 0.25 mm to 4.0 mm. Fig. 4.6 summarizes the influence of shoulder width on the stiffness of anchorage. As expected, a very high sensitivity of the secant stiffness is obtained on the shoulder width, $a$, especially at lower displacements. However, since the bolt dimensions are known with quite high accuracy while performing the analysis, the estimated stiffness values are not expected to vary significantly from its real value due to shoulder width.

#### 4.2.8 Sensitivity to pressing relation $m$

Pressing relation, $m$, appears in the equation for evaluating stiffness for component ‘$P$’ (see Eq. (4.9)). The value of this factor may not be known exactly and a value of ‘9’ is currently recommended in the design manual. The parameter variation from in the range of 7 to 12 displays little sensitivity of stiffness on this parameter (see Fig. 4.7). Therefore, a value of 9 is reasonable.
For the case of anchorage with supplementary reinforcement, the design bond strength of stirrups, $f_{bd}$ influences the stiffness of the anchorage system. The failure load for the reinforcement due to bond is given by Eq. (4.4). The value of the design bond strength is directly dependent on the concrete strength, $f_{ck}$. No particular value of the design bond strength is recommended in the design manual. For the grades of concrete considered in this study ($f_{ck} = 25 – 65$ MPa), the typical value of the design bond strength may vary approximately between 2 to 5 MPa. In this work, the influence of the design bond strength on the stiffness is evaluated for a range of 0 to 5 MPa. It is observed that the stiffness of the anchorage is not affected by the bond strength at low displacements as shown in Fig. 4.8. Though at higher displacement levels, the estimated stiffness value depends on the $f_{bd}$ value, the variation is within reasonable range that can be accounted for through material safety factor.

**Fig. 4.7:** Influence of pressing relation, $m$ on stiffness of anchorage

**Fig. 4.8:** Influence of design bond strength, $f_{bd}$ on stiffness of anchorage

### 4.2.10 Sensitivity to diameter of supplementary reinforcement $d_{re}$

The diameter of supplementary reinforcement, $d_{re}$ influences the components RS and RB. The failure load corresponding to bond failure of stirrups is given by Eq. (4.4), while the failure load corresponding to the yielding of stirrups in tension is given by Eq. (4.10).
The load-displacement relationship of the anchorage corresponding to the failure of supplementary reinforcement is given by Eq. (4.11).

\[ \delta_{Rd,s,re} = \frac{2N_{Rd,le}^2}{\alpha_s \cdot f_{ck} \cdot d_{s,sn} \cdot n_{re}} \quad [\text{mm}] \]  

In Eq. (4.11), \( N_{Rd,le} \) is equal to the minimum of \( N_{Rd,b,re} \) and \( N_{Rd,s,re} \) (minimum of steel failure and bond failure load of supplementary reinforcement). No particular value of the diameter is recommended for stirrups in the design manual. Considering the normal range of diameters used in practice, the results of the parametric study performed on the anchorage by varying the stirrup diameter in the range of 6 mm to 20 mm are displayed in Fig. 4.9. It can be observed that the stiffness of the anchorage is insensitive to the diameter of supplementary reinforcement at low displacement levels but the variation slightly increases at higher displacement levels. Nevertheless, the diameter of the stirrups is also generally known with good accuracy and therefore the stiffness of the anchorage system can be reasonably accurately estimated.

### 4.2.11 Sensitivity to descending anchor stiffness due to concrete, \( k_{p,de} \)

The stiffness of the descending branch, \( k_{p,de} \) in case of pullout failure of the stud (component P) depends on the failure mode. If the supplementary reinforcement fails by yielding (\( N_{Rd,le} < N_{Rd,p} \)) the recommended value of \( k_{p,de} \) is \(-10^4\) N/mm² (negative due to descending branch). In this parametric study, the value of \( k_{p,de} \) is varied between -5000 to -20,000 N/mm². As shown in Fig. 4.10, the stiffness of the anchorage system is insensitive to this parameter and therefore the value of -10000 can be used with sufficient accuracy.

### 4.2.12 Summary of sensitivity of anchorage stiffness to various parameters

In this study, the sensitivity of the stiffness of an anchorage with supplementary reinforcement towards different parameters is investigated. Table 5.2.2 summarizes the statistical information on the sensitivity to various parameters studied in this chapter. The table gives the values of secant stiffness (kN/mm) corresponding to peak load obtained for various values of parameters governing the stiffness of anchorage system. The values of the stiffness are first arranged in an ascending order in the table followed by the mean and coefficient of variation.
The stiffness of the anchorage is found to be most sensitive to the shoulder width 'a', followed by the embedment depth, 'h_ef'. However, both these parameters are known quite accurately during design, therefore the stiffness can be reasonably accurately determined. The next biggest variation comes through the concrete strength with a reasonable coefficient of variation of 20%. For the given range of concrete strength that can be expected in practice from the same concrete mix, the sensitivity to concrete strength can be considered through material safety factor. The stiffness is sensitive to the parameter, $\alpha_c$ only in the mid-range of displacement (see Figure 5.2.3), while the initial stiffness and the stiffness at large displacements are practically independent of this parameter. The variation in stiffness due to $\alpha_c$ can also be considered through material safety factor. The stiffness is found to be practically insensitive to other parameters listed in Tab. 4.2.

Tab. 4.2: Statistical information on parameter study

<table>
<thead>
<tr>
<th></th>
<th>$f_k$</th>
<th>a</th>
<th>$k_{p,de}$</th>
<th>m</th>
<th>$\alpha_c$</th>
<th>$f_{bd}$</th>
<th>$h_{ef}$</th>
<th>$d_{a,se}$</th>
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<tbody>
<tr>
<td>Min</td>
<td>3.29</td>
<td>1.49</td>
<td>3.3</td>
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<td>2.88</td>
<td>3.5</td>
<td>4.5</td>
<td>3.3</td>
</tr>
<tr>
<td></td>
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</tr>
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<td>3.3</td>
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<tr>
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<td>3.78</td>
<td>3.1</td>
<td>2.56</td>
<td>3.2</td>
</tr>
<tr>
<td>Max</td>
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<td>5.32</td>
<td>3.3</td>
<td>3.1</td>
<td>4.29</td>
<td>3.06</td>
<td>2.0</td>
<td>3.1</td>
</tr>
<tr>
<td>Stabw</td>
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<td>1.539</td>
<td>0.000</td>
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<td>0.522</td>
<td>0.172</td>
<td>0.927</td>
<td>0.084</td>
</tr>
<tr>
<td>Var Koeff</td>
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<td>43%</td>
<td>0%</td>
<td>4%</td>
<td>15%</td>
<td>5%</td>
<td>29%</td>
<td>3%</td>
</tr>
</tbody>
</table>
4.3 Parameter study on simple steel-to-concrete joints

4.3.1 General

The following parameter study is carried out in order to describe the influence by varying different input factors of the simple joint. With the variation of different parameters the load bearing capacity and the rotational stiffness of the whole joint can be influenced to a high extent. The optimization of the simple joint will be aimed to achieve on one hand maximum strength with minimum costs and on the other hand a ductile behaviour of the whole joint. For the concrete component ductility exists if the load can be transferred at the cracked level to other components such as stirrups, as a certain amount of deformation occurs at the maximum level of load. Furthermore the parameter study will demonstrate a range of validity and will point out the possible increase in loading of the joint by changing different parameters.

4.3.2 Validation of the model

Within the INFASO project a design approach based on the component method has been developed for the simple steel-to-concrete joints [12]. Thereby the joint is subdivided into its different components and the deformations can be calculated as each of the component is represented by a spring. The calculation of the load carrying capacity mainly consists of the following procedure:

1. Evaluation of the decisive tension component.
2. Verification of the influence of the compression zone on the tension component.
3. Calculation of the shear resistance of the joint out of the moment equilibrium, see Fig. 4.11.
4. Verification of interaction conditions.

In the Design Manual I "Design of steel-to-concrete joints" [13] this approach is described in more detail based on a flowchart and a worked example. In order to determine the moment-rotation curve of the joint, the deformation of the single components have to be calculated. Therefore load-deformation curves of the single steel and concrete components have been developed. A parameter study on the concrete components and a detailed investigation on the unknown factors of the concrete components can be found in the previous chapter. Fig. 4.11 shows the mechanical model and a simplified joint model if the tensional and the compression components are assembled in one spring.

![Analytic model of the simple steel-to-concrete joint](image)

The deformation of the tensional component can be determined in three ranges (see Fig. 4.11). In the first range the deformations can be calculated according to [12] with Eq. (4.12). For this the deformation due to pull-out failure (see Eq. (4.5)) and the deformation due to elongation of the shaft of the headed stud are added.

\[ \delta_{Range1} = \delta_{Rk,p,1} + \delta_{Rk,s1} \]  

(4.12)

With:
- \( \delta_{Rk,p,1} \) Deformation due to pull-out failure;
- \( \delta_{Rk,s1} \) Deformation due to yielding of the headed stud.
At the end of the first range first cracks might develop from the head of the headed stud into the direction of the concrete surface. If the load carrying capacity at concrete cone failure is reached, the supplementary reinforcement can be activated. In this case the deformations can be calculated up to the ultimate load with Eq. (4.13).

\[
\delta_{\text{Range}2} = \delta_{\text{Rk,p,2}} + \delta_{\text{Rk,s1}} + \delta_{\text{Rk,re}}
\] (4.13)

With:
- \(\delta_{\text{Rk,p,1}}\) Deformation due to pull-out failure;
- \(\delta_{\text{Rk,s1}}\) Deformation due to yielding of the headed stud;
- \(\delta_{\text{Rk,re}}\) Deformation of the supplementary reinforcement in interaction with concrete cone failure.

The deformations in the third range depend on the failure mode of the ultimate load. If yielding of the supplementary reinforcement occurs, a ductile behaviour can be observed in the third range. The deformations in the third range can be calculated with Eq. (4.14).

\[
\delta_{\text{Range}3} = \delta_{\text{Range}2} \left( \frac{N_{\text{Rk,re}}}{k_{c,de}} \right) + k
\] (4.14)

With:
- \(k_{c,de}\) Stiffness of the descending branch see Eq. (4.2);
- \(k = \frac{N_u - N}{10000}\) If yielding of the supplementary reinforcement occurs;
- \(k = 0\) If the failure mode concrete failure and anchorage failure of the stirrups occurs;
- \(N_{\text{Rk,re}}\) Characteristic load carrying capacity of the supplementary reinforcement in interaction with concrete cone failure;
- \(N_u\) Ultimate load carrying capacity.

It has to be considered, that the supplementary reinforcement can only be activated in the optimum case. As the failure modes like, pull-out failure of the headed studs, concrete cone failure between the supplementary reinforcement and steel failure of the headed stud can also occur, scenario distinctions have to be made in all ranges. Within the following parameter study this issue will be highlighted and explained. The rotation of the joint can be determined with Eq. (4.15) according to the spring model of Fig. 4.12.

\[
\Phi_i = \frac{\delta_t - \delta_c}{z}
\] (4.15)

With:
- \(\delta_t\) Deformation of the tensional component;

For the calculation of the full moment-rotation curve of the joint the interaction conditions for steel failure (see Eq. (4.16)) and for concrete failure (see Eq. (4.17)) have to be considered as tension forces and shear forces are acting simultaneously on the simple steel-to-concrete joint.

Steel failure
\[
\left( \frac{N}{N_{\text{Rk,u,s}}} \right)^2 + \left( \frac{V}{V_{\text{Rk,u,s}}} \right)^2 \leq 1
\] (4.16)

Concrete failure
\[
\left( \frac{N}{N_{\text{Rk,c,p}}} \right)^{3/2} + \left( \frac{V}{V_{\text{Rk,c,p}}} \right)^{3/2} \leq 1
\] (4.17)

With:
- \(N_{\text{Rk,u,s}}/V_{\text{Rk,u,s}}\) Characteristic load carrying capacity due to steel-failure of the headed stud;
- \(N_{\text{Rk,c,p}}/V_{\text{Rk,c,p}}\) Characteristic load carrying capacity due to concrete failure modes.

In Fig. 4.13 the validation of the model is shown for the different test specimens [12]. In this figures the moment-rotation curves are compared with the calculated curves according to the developed mechanical joint model. It can be seen, that the model curves fit well to the test curves.
Within the first section the moment-rotational behaviour of the joint is described up to concrete cone failure. Test specimens without additional reinforcement reach their ultimate resistance up to this point. If additional reinforcement is placed next to the stirrups, the load carrying capacity of the tensional component can be increased (see 2nd range for B1-B-R and B2-C-R in Fig. 4.13). In these cases the additional reinforcement is activated and the ultimate load can be increased. In the following parameter study configurations will be shown, where the supplementary reinforcement can be activated and a ductile failure mode can be obtained.

4.3.3 Sensitivity study of major parameters

4.3.3.1 General

Based on the worked example on simple joints of Design Manual I "Design of steel-to-concrete joints" [13] the different parameters were defined and varied in particular in the following. The worked example is used thereby as reference version. Safety factors weren’t considered in this parameter study. Furthermore the absolute terms in the equations for pull-out failure and concrete cone failure were assumed as 12 and 15.5 to reproduce the real load-carrying capacity. Values for application may be taken from the relevant European Technical Specification of the specific headed studs.

4.3.3.2 Overview of the major parameters

The load-carrying capacity and the rotational stiffness depends on different parameters and boundary conditions. The examined parameters are listed in Table 4.1.
Tab. 4.3: Overview of the examined parameters and their influence on the model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Influence</th>
<th>Remarks and brief description</th>
<th>Chapter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective height</td>
<td>++</td>
<td>Influences the ultimate load carrying capacity as well as the load carrying capacity of the concrete cone failure $N_{Rk,c}$</td>
<td>4.3.3.3</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>++</td>
<td>Can have high influence on the moment resistance of the joint within high utilization factors of the interaction equations</td>
<td>4.3.3.5</td>
</tr>
<tr>
<td>Diameter headed studs</td>
<td>+++</td>
<td>High influence on the ductility of the joint.</td>
<td>4.3.3.6</td>
</tr>
<tr>
<td>Diameter stirrups</td>
<td>++</td>
<td>Diameter of the stirrups can increase the overall load-carrying capacity</td>
<td>4.3.3.6</td>
</tr>
<tr>
<td>Number of stirrups</td>
<td>++</td>
<td>If the number of stirrups is increases brittle failure modes can be avoided.</td>
<td>4.3.3.6</td>
</tr>
<tr>
<td>Concrete strength</td>
<td>+++</td>
<td>Concrete strength has influence on all concrete components</td>
<td>4.3.3.7</td>
</tr>
</tbody>
</table>

4.3.3.3 Effective height

The effective height (see Fig. 4.4) can either be varied by changing the thickness of the anchor plate, the overall length of the headed stud or the head height of the anchor. In this study only the anchor length has been modified as this factor has the biggest influence on the final result, see Tab. 4.4.

Tab. 4.4: Analyzed effective heights

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Reference version</th>
<th>Case 3</th>
<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anchor length [mm]</td>
<td>50</td>
<td>100</td>
<td>150</td>
<td>200</td>
<td>250</td>
</tr>
<tr>
<td>Effective height [mm]</td>
<td>65</td>
<td>115</td>
<td>165</td>
<td>215</td>
<td>265</td>
</tr>
</tbody>
</table>

Fig. 4.14 shows the load-displacement curve of the tensile component. The desired behaviour of the simple joint can be monitored, if an effective height of 165 mm is used. By selecting this effective height all three sections $\delta_{Range1}$, $\delta_{Range2}$ and $\delta_{Range3}$ in Fig. 4.14 are clearly evident. By the end of the first section $\delta_{Range1}$ the ultimate load of the concrete cone failure without considering supplementary reinforcement $N_{Rk,c} = 190$ kN is reached for the reference version (see Tab. 4.4). By further increase of the load, the hanger reinforcement is activated and the smallest resistance of steel yielding of the stirrups $N_{Rk,re,1}$, anchorage failure of the stirrups $N_{Rk,re,2}$ or the small concrete cone failure $N_{Rk,cs}$ can be decisive. In the third range $\delta_{Range3}$ the inclination of the load-displacement-curve depends whether brittle failure modes like concrete cone failure or anchorage failure or steel failure becomes crucial. According to Fig. 4.14 ductile behaviour occurs if longer headed studs are used. The headed stud is yielding before other failure modes can be recognized. Fig. 4.14 indicates that the selection of longer headed studs does not increase the load carrying capacity of the tensile component of the joint. In this cases steel failure of the headed studs becomes the decisive failure mode. If headed studs with smaller effective loads are used, brittle failure modes might occur. For effective height of 115 mm concrete cone failure between the supplementary reinforcement with approx. $N_{Rk,cs} = 250$ kN is decisive and for an effective height of 65 mm with approx. $N_{Rk,cs} = 105$ kN is the governing failure mode. In Fig.4.15 the moment-rotation curve by varying the effective length of the headed stud is shown. By changing the effective height the rotational behavior of the joint can be influenced less as if the diameter of the heads stud is changed. Changes of the diameter of the headed stud have higher influence on the stiffness $EA$ of this component (see Eq.(4.12)).
4.3.3.4 Diameter of the headed studs

In Fig. 4.16 the load-displacement curves of the tensional component for different diameters of the headed stud is shown. The diameters of the headed studs are varied according to Tab. 4.5. By using a diameter of 22 mm in the reference version the supplementary reinforcement can be activated. This can be seen in Fig. 4.16 as the load can be increased from approx. $N_{Rk,c} = 190\, kN$ up to $N_{Rk,u} = 350\, kN$ (for definition see Fig. 4.11, right). The overall load carrying capacity cannot be increased by selecting even a higher diameter of the headed stud. As yielding of the supplementary reinforcement becomes the decisive component with $N_{Rk,rel,1} = 350\, kN$ a larger diameter is not more advantageous. If the diameter is reduced, the increase in
loading due to the supplementary reinforcement cannot be taken into account fully. In this case a diameter of 16 mm is too small as the load level of concrete cone failure, $N_{Rk,c} = 190$ kN, cannot be reached. In Fig. 4.17 the moment-rotation curves of this simple steel-to-concrete joint by varying the diameter of the headed stud are shown. The variation of the diameter has also an influence on the interaction equations of the joint. Steel failure might become the decisive interaction equation.

Tab. 4.5: Diameter of the headed studs

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Reference version</th>
<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter headed studs [mm]</td>
<td>13</td>
<td>16</td>
<td>19</td>
<td>22</td>
<td>25</td>
</tr>
</tbody>
</table>

Fig. 4.16: Load-displacement curve of the tensional component for variation of the diameter of the headed studs

Fig. 4.17: Moment-rotation curve of the simple steel-to-concrete joint for variation of the diameter of the headed studs
4.3.3.5 Eccentricity

According to Tab. 4.6 five different cases are considered in order to determine the influence of the eccentricity on the rotational behavior of the joint. Originally there shouldn’t be any effect to the load-carrying capacity of the simple joint since there is no direct relation between the eccentricity and the tensile component. But changes in the eccentricity have influence on the overall load-carrying capacity. As interaction equations (see Eq. (4.16) to (4.17)) have to be considered, increases of the eccentricity can have an influence on the moment resistance of the joint. Either steel failure or concrete failure can be decisive. If joints are designed with higher eccentricities, increases of the tensile components of the joint have to be considered. By varying the eccentricity in this example the interaction equation is not overstepped as the resistances due to friction also increases due to higher normal forces in the joint (see Fig. 4.19). As the normal forces are getting larger the load capacity of the friction component rises and the shear resistance of the headed studs is exceeded. Smaller shear forces are transferred to the second anchor row and possible normal forces can be raised in this row. The particular application must be investigated therefore on case-by-case basis.

Tab. 4.6: Analyzed eccentricities

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Reference version</th>
<th>Case 3</th>
<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eccentricity [mm]</td>
<td>50</td>
<td>75</td>
<td>100</td>
<td>200</td>
<td>250</td>
</tr>
</tbody>
</table>

Fig. 4.18: Moment-rotation curve for variation of the eccentricity

4.3.3.6 Diameter and number of the stirrups

The parameters of the diameter and the number of stirrups have been changed according to Tab. 4.7. If the load carrying capacity of concrete cone failure is reached, further load increases depend on the supplementary reinforcement. Two different failure modes have to be considered if the supplementary reinforcement can be activated. Bond failure according to Chapter 4.2.9 or yielding of the supplementary reinforcement might occur. By increasing the diameter of the stirrup the ultimate resistance of the tensile component can only be increased slightly (see Fig. 4.19). By increasing the diameter of the headed stud, steel failure of the headed studs becomes the decisive component with approx. \( N_{\text{Rk,s}} = 350 \, \text{kN} \). As this is the crucial component increases in the diameter of the supplementary reinforcement are not more advantageous. In a further parameter study the number of stirrups has been varied. If the number of stirrups is reduced brittle failure modes might occur, as the surface area of the supplementary reinforcement is reduced (see Fig. 4.20).
Tab. 4.7: Analyzed diameter of stirrups and analyzed number of stirrups

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case 1</th>
<th>Reference version</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of stirrups [mm]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>Number of stirrups [mm]</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4.3.3.7 Concrete strength

The concrete strength has an influence on all components and therefore a high influence on the load-deformation behavior of the tensile components. The issue of the scattering of this parameter is described in Chapter 4.2.4. In this parameter study this component is varied according to Tab. 4.8. If the concrete strength is reduced the load carrying capacity of the concrete component without considering the additional reinforcement (see Eq. (4.1)) decreases. The supplementary reinforcement cannot be activated fully in two
cases as (Case 1 and Case 2) as pull-out failure is the decisive component $N_{Rk,p} = 280 \text{ kN}$ (Case 1) in this cases.

Tab. 4.8: Variation of concrete strength

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Reference version</th>
<th>Case 3</th>
<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete strength [N/mm²]</td>
<td>C20/25</td>
<td>C25/30</td>
<td>C30/37</td>
<td>C35/45</td>
<td>C40/50</td>
</tr>
</tbody>
</table>

Fig. 4.21: Load-displacement curve for variation of the concrete strength

4.3.4 **Limits of the model and recommendations**
4.3.4.1 General

In the parameter study above the sensitivity of simple steel-to-concrete joints to changes of different parameters is shown. The overall influence on the load-carrying capacity of the tensile components or the moment-rotation curve of the whole joint depends strongly on the design, e.g. geometry, size of anchors etc., of the simple steel-to-concrete joints in particular. Brittle failure modes have to be prevented within case by case studies where the different parameters are changed carefully. A very helpful tool in the design process of the joint can be the use of the design program for “Rigid anchor plate with headed studs – simple joint” (see Chapter 2.3). With the help of this program the failure modes of the specific anchor plate can be determined. By varying the above mentioned parameters ductile behavior of this joint can be achieved. Nevertheless limitations have to be made, if the newly developed INFASO-components are considered. These limitations are described in the following.

4.3.4.2 Edge distances

Within the INFASO project [11] a calculation approach for the tensile concrete components has been developed. Tests with loading perpendicular toward a free edge under shear with consideration of the positive contribution of the supplementary reinforcement have not yet been made. The additional reinforcement can only be taken into account, if the geometric limitations in Fig. 4.22 are taken into consideration. These limitations ensure that there are no edge effects which might lead to different failure modes. Further information will be given in Özbolt [19]. Furthermore conservative calculation approaches for the calculation of the shear resistance due to pry-out failure have to be made, as the calculation of the resistance due to pry-out failure is based on the tensile resistance of the concrete components without considering the additional reinforcement. In future further tests have to be done for this failure mode.

4.3.4.3 Number of headed studs

In the INFASO project [11] tests on anchor plates of simple steel-to-concrete connections with dispositions of 2x3 and 2x2 of the headed studs have been made. Limitations for anchor plates with a larger number of headed studs are given in CEN/TS 1992-4-1 [1]. Anchor plates with more than nine headed studs are not covered by this standard. If the number of headed studs is increased, according to the tests of the INFASO project [11] further considerations have to be made, if the supplementary reinforcement is taken into account.

4.3.4.4 Concrete strength

A relatively low concrete strength of C20/25 [7] has been used for all test specimens to achieve concrete failure modes as lower bound of all failure mechanism. The developed INFASO models [11] are only valid for normal-strength concrete and should not be transferred to high-strength concrete.

4.3.4.5 Number of stirrups

The model of the tensile components has been developed in the INFASO project [11] for one stud row. This model is based on tests of headed studs under pure tension, where supplementary reinforcement is considered. In the test specimens with consideration of supplementary reinforcement two stirrups have been placed next to the headed stud. In total four legs have been considered within the concrete cone of one headed stud. The model of this tensile component has been implemented in the model of the simple joint, where the tensional forces have to be considered in the second row of headed studs (see Fig. 4.11). The
model of this simple steel-to-concrete joint has been validated with good agreement against the test results (see Fig. 4.13) with the exact constellation of supplementary reinforcement as in the component tests under pure tension.

If the new INFASO design approach [11] is transferred to anchor plates with more than two stud rows, the load distribution among the supplementary reinforcement has to be considered in special. If the model is assigned for two stud rows under tension, calculation approaches are given in [16]. If the supplementary reinforcement is placed next to the headed studs according to Fig. 4.24 the concrete cone can be subdivided into the intermediate part with normal concrete break-out and the right side and left side part where the factor $\psi_{supp}$ is considered. The failure load of this component can be calculated with Eq. (4.18):

$$N_{u,\text{max}} = \psi_{supp} \frac{A_{c,N,1}}{A_{c,R,\text{total}}} \cdot N_{u,c} + \frac{A_{c,N,2}}{A_{c,R,\text{total}}} \cdot N_{u,c} + \frac{A_{c,N,3}}{A_{c,R,\text{total}}} \cdot N_{u,c} \quad (4.18)$$

Further investigations on the subject of edge effects, number of stirrups (see [19] and [14]) and number of heads studs (see [15]) are on the way but not yet in a status to be implemented in Eurocodes.

Fig. 4.24: Design approach for more than one headed stud under tension
4.4 Parameter study on column bases

4.4.1 Validation of the model

The analytical component based model of column base with anchor plate was validated on experiments prepared under the project, see Kuhlman et al. [12]. The specimen was consisted of two steel units, see Fig. 4.25. The thin steel anchor plate was $t_{p1} = 10$ mm with welded studs $d = 22$ mm, $h = 150$ mm and with threaded studs $d = 24$ mm, $h = 100$ mm. The thick steel base plate $t_{p2} = 25$ mm was design under the column HE180B, fillet weld $a_w = 6$ mm. The concrete block was made from reinforced concrete $1600 \times 1000 \times 400$ mm, see Tab. 4.9. The test results were published in Ph.D. thesis of Žižka, J. [20].

<table>
<thead>
<tr>
<th>Column</th>
<th>Base plate</th>
<th>Anchor plate</th>
</tr>
</thead>
<tbody>
<tr>
<td>HE180B</td>
<td>S355</td>
<td>S355</td>
</tr>
<tr>
<td>$f_{yk} = 355$ MPa</td>
<td>$f_{uk} = 510$ MPa</td>
<td>$f_{yk} = 510$ MPa</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Threaded studs</th>
<th>Headed studs</th>
<th>Foundation (cracked)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d = 24$ mm; $h = 100$ mm</td>
<td>S355</td>
<td>$d = 22$ mm; $h = 150$ mm</td>
</tr>
<tr>
<td>$f_{yk} = 355$ MPa</td>
<td>$f_{uk} = 510$ MPa</td>
<td>$f_{y,exp} = 444.8$ MPa</td>
</tr>
</tbody>
</table>

The analytical model is described in Design Manual I "Design of steel-to-concrete joints" [13]. For the calculation were taken the measured values of material properties of steel. In the experiments S2-0, S2-5, and S2-30 varies the thickness of the grout, form 0 mm, 5 mm and 30 mm. The experimental moment rotational curves are summarized in Fig. 4.26.

![Moment-rotation curves of three experiments with different position of headed and threaded studs](image)
The differences of experimental results are reported to be due to changes of lever arm during the loading at large deformations, see Fig. 4.27. The vertical deformations were measured at points 1-11, the horizontal ones in 12 and 13, see Fig. 4.27. Results of each experiments were recalculated based on the measured actual acting force, see Fig. 4.28 to Fig. 4.30. The eccentricity is recalculated to the column axes. The comparison of the calculated and measured initial stiffness $S_{i,ini}$ shows a good agreement. The difference is in between a range of 5%. The elastic-plastic stage is affected to the material properties and the development of cracks in concrete block. The modelling respects the engineering level of accuracy in prediction of resistance.

Fig. 4.27: Measured values during the tests

![Fig. 4.27: Measured values during the tests](image)

Fig. 4.28: Comparison of predicted and measured moment rotational relation for experiment S2-0, eccentricity 495 mm

![Fig. 4.28: Comparison of predicted and measured moment rotational relation for experiment S2-0, eccentricity 495 mm](image)

Fig. 4.29: Comparison of predicted and measured moment rotational relation for experiment S2-5, eccentricity 354 mm

![Fig. 4.29: Comparison of predicted and measured moment rotational relation for experiment S2-5, eccentricity 354 mm](image)
4.4.2 Sensitivity study of major parameters

The bending resistance of the base plate with anchor plate is assembled from the tensile and compression resistance of its components. The additional component is the anchor plate in bending and in tension. The procedure for evaluation of the resistance is the same in all connections loaded by bending moment and normal force. The influence of parameters like the base plate thickness, the anchor plate thickness, and the distance between the headed and threaded studs is studied. The study is prepared for column of cross section HE180B, for all plates and cross sections of steel S355 (if not mentioned for all plates and cross sections S235), concrete C25/30, threaded studs M 24, steel S355, and headed studs M22, steel S355. In each normal force moment interaction diagram are marked the important points e. g. the resistance in tension, in maximal bending, in pure bending, and in maximal compression. The Fig. 4.31 and Fig. 4.32 demonstrate the influence of the base plate thickness \( t_{p2} \) for steel S355 and S235.
Fig. 4.32: Moment – normal force interaction diagram for different base plate thickness $t_{p2}$ and steel S235

The parameter study of the anchor plate thickness $t_{p1}$ is influenced by the interaction of acting developed forces in headed studs in shear and in tension. The selected value of effective height of the headed stud is included for each anchor plate thickness. Fig. 4.33 to Fig. 4.36 show the influence of the anchor plate thickness to the column base resistance for two material properties. For headed studs with the effective height 150 mm, only the anchor plate with thickness 10 mm is not affected by the headed stud’s resistance.

Fig. 4.33: Moment – normal force interaction diagram for different anchor plate thickness $t_{p1}$, for the anchor plate steel S355, and the headed stud length $h_{eff} = 150$ mm
Fig. 4.34: Moment – normal force interaction diagram for different anchor plate thickness $t_{p1}$, for the anchor plate steel S235, and the headed stud length $h_{eff} = 150$ mm

Fig. 4.35: Moment – rotation diagram for different anchor plate thickness $t_{p1}$, for the anchor plate steel S355, and the headed stud lengths $h_{eff} = 200$ mm and 350 mm
Fig. 4.36: Moment – rotation interaction diagram for different anchor plate thickness $t_{p1}$, for the anchor plate steel S235, and the headed stud lengths $h_{eff} = 200$ mm and 300 mm.

The influence of the effective length of the headed stud 200 and 350 mm, for steel S355, the plate thickness 25 mm is summarized in a moment – normal force interaction diagram in Fig. 4.37 and Fig. 4.38.

Fig. 4.37: Moment – normal force interaction diagram for different anchor plate thickness $t_{p1}$, for the anchor plate steel S355, and the headed stud lengths $h_{eff} = 200$ mm and 350 mm.
Fig. 4.38: Moment – normal force interaction diagram for different anchor plate thickness $t_{pl}$, for the anchor plate steel S235, and the headed stud lengths $h_{eff} = 200$ mm and 300 mm

Different distance between the headed and threaded studs of the anchor plates and their influence on the moment resistance are shown in Fig. 4.39 to Fig. 4.42 for the anchor plate thickness 10 mm, base plate thickness 25 mm, and steels S355 and S235. The pure bending resistance decreases till the distance between the headed and threaded studs 200 mm, where the base plate resistance is changed.

Fig. 4.39: Moment – normal force interaction diagram for different distances between the headed and threaded studs $m_1$, for the anchor plate steel S355
Fig. 4.40: Moment – normal force interaction diagram for different distances between the headed and threaded studs $m_1$, for the anchor plate steel S235.

Fig. 4.41: Moment – rotation diagram for different distances between the headed and threaded studs $m_1$, for the anchor plate steel S355.
Fig. 4.42: Moment – rotation diagram for different distances between the headed and threaded studs
\( m_1 \), for the anchor plate steel S235

4.4.3 Limits of the model

The analytical design model by component based method of a base plate with an anchor plate offers freedom in selection of material properties and geometries of the threaded and headed studs, the base and anchor plates and the concrete foundation. The limits follows the recommended values in Chapter 2.2. Information about positioning of holes for bolts and rivets is given in EN 1993-1-8 [9]. The symbols used are summarized in Fig. 4.43. These limits may be interpreted for a base plate with an anchor plate in terms of

\[
\begin{align*}
    p_2 &= \min(2.5 \, d_{20}) \\
    e_{b2} &= \min(1.2 \, d_{20}) \\
    m_2 &= \min(1.2 \, d_{20} + a_w \sqrt{2}) \\
    e_{a2} &= \min(1.2 \, d_{20}) \\
    e_{a1} &= \min(1.2 \, d_{10})
\end{align*}
\]

With:

- \( p_2 \) is distance between the threaded studs
- \( d_{10} \) is diameters of headed stud including weld to the anchor plate
- \( d_{20} \) is diameters of threaded stud including weld to the anchor plate
- \( e_{b1} \) is the edge distance the headed stud
- \( e_{b2} \) is the edge distance the threaded stud
- \( m_2 \) is distance between the threaded stud and column cross section
The presented model was validated/verified in limited range of geometry and materials S235 to S355 for:

\[ t_{p1} = 6 \text{ to } 20 \text{ mm} \]
\[ d_1 = 20 \text{ to } 40 \text{ mm} \]
\[ d_2 = \min d_1 \]
\[ h_{\text{eff}} = \min 150 \text{ mm} \]

where

- \( t_{p1} \) is thickness of the anchor plate
- \( t_{b2} \) is thickness of the base plate
- \( d_1 \) is diameters of headed stud
- \( d_2 \) is diameters of threaded stud
- \( h_{\text{eff}} \) is the effective height of the headed stud

For very thin anchor plates \( t_{p1} < 6 \text{ mm} \) and huge headed and threaded studs \( d_1 > 40 \text{ mm} \) too rough simplification of changes in the geometry in the model might occur. The prediction of the tensile resistance of the component anchor plate in bending and in tension should be modelled by iteration, which is described in Žížka, J. [20].

For ductile behaviour of the base plate with anchor plate it is necessary to avoid a brittle failure of the concrete components. The concrete cone failure without or with reinforcement, the pull-out failure of headed studs, the pry-out failure of headed stud, and its interaction. The steel failure of the threaded stud in tension is unacceptable brittle for design of steel structures. In column bases it is approved, that the failure of the anchor bolts is for predominantly static loading of column bases ductile enough. The headed studs with embedded length of at least \( 8 d_1 \) may be expected to present ductile behaviour. The deformation capacity of headed studs with shorter embedded length in the reinforced concrete block should be checked by presented method in Design Manual I "Design of steel-to-concrete joints" [13].

Under serviceability limit state the elastic plastic behaviour without any membrane actions is expected in connections. Column bases with base plate and anchor plate develop the plastic hinge mechanism and the anchor plate due to a tensile bar. This behaviour is ductile but creates large deformations. Hence this method is recommended to limit the serviceability limit state by the creation of the full plastic mechanism only.

### 4.4.4 Recommendation for design values

The column base with base plate in pure compression is not limited by the size of the concrete block for \( a_c = \min 3 a_{p2} \) and \( b_c = \min 3 b_{p2} \). Full resistance of the steel part may be developed, where \( a_c \) and \( b_c \) is the concrete width/length, and \( a_{p2} \) and \( b_{p2} \) is the base plate width/length.

The resistance of column base with anchor plate in pure bending is mostly limited by interaction of tension and shear of headed studs be creating the tensile bar behaviour of the anchor plate. The longer headed studs of higher diameter and better material properties of stirrups allows the development of this ductile behaviour. The contribution of the tensile resistance of the component the anchor plate in bending and in tension is expected \( t_1 \leq \max 0.5 t_2 \), where \( t_1 \) is the anchor plate thickness and \( t_2 \) is the base plate thickness. For typical column cross sections recommended sizes of the column base with anchor plate are summarized in Tab. 4.10. to Tab. 4.15. The table is prepared for all plates and cross sections of steel S355, concrete C25/30, threaded studs M24, steel S355, and headed studs M22 of effective length \( h_{\text{eff}} = 200 \text{ mm} \), steel S355. Stirrups have diameter \( 8 \text{ mm} \), steel B500A, 4 legs for stud. The influence of the weld size on tension part resistance is not taken into account.
The distance between the threaded and headed studs $m_1$ is expected in one direction along the base plate only. For distances in both direction the real distance $m_1$ should be taken into account. The threaded and headed stud’s deviation from the specified location is expected in the calculations as 6 mm. The stud deviation is taken into consideration reducing the lever arm as:

\[
m_1 = \pm 4 \text{ mm}
\]

\[
m_2 = \pm 2 \text{ mm}
\]

Due to loading of the column base with plastic mechanism in the anchor plate internal vertical and horizontal forces in the headed studs from changed geometry have to be considered. In the presented tables this is dissipated till 20% of the horizontal resistance of headed studs. The remaining 80% may carry the acting external shear forces. The symbols used are summarized in Fig. 4.43. By utilization of tables the linear transition for different normal force / bending moment ratio is recommended. The geometry of the column base is defined by following values:

- $a_{p2}$ is width of the base plate
- $d_1$ is diameters of threaded stud
- $d_2$ is diameters of headed stud
- $e_{b1}$ is the edge distance the threaded stud
- $e_{b2}$ is the edge distance the headed stud
- $m_1$ is distance between the threaded and headed studs
- $m_2$ is distance between the threaded stud and column cross section
- $p_1$ is distance between the threaded studs
- $p_2$ is distance between the headed studs
- $t_{p2}$ is thickness of the base plate
- $t_{p1}$ is thickness of the anchor plate

Tabularized are values for:

- $M_{N=0,pl}$ is the bending design resistance of column base under poor bending for SLS, under plastic bending of the anchor plate, see Fig. 4.44.
- $M_{N=0,mem}$ is the bending design resistance of column base under poor bending for ULS, the anchor plate acts as anchor plate
- $M_1$ is the specific design bending resistance of column base for acting normal force $N_1$ for effective cross section under one flange in compression only
- $M_2$ is the maximal design bending resistance of column base for acting normal force $N_2$ for one-half of effective cross section is in compression
- $M_3$ is the specific design bending resistance of column base for acting normal force $N_3$ for effective cross section under the column web and one flange in compression
- $N_{M=0}$ is the design compression resistance of column base under poor compression

The column bases are classified based on its relative stiffness compared to the column stiffness in the terms of column length. Shorter columns with this column base may be design with a rigid connection in bending.
The tables contain the limiting length of columns for the rigid column bases for frames where \( L_{cb} = 8 \frac{E I_c}{S_{ini}} \) and for other frames as \( L_{cb} = 25 \frac{E I_c}{S_{ini}} \).
Chyba! Pomocí karty Domů použijte u textu, který se má zde zobrazit, styl Überschrift 1. Chyba! Pomocí karty Domů
použijte u textu, který se má zde zobrazit, styl Überschrift 1.

Tab. 4.10: Recommended geometry of the column base with anchor plate, its design resistances, stiffness and limiting length for HE160B
Column
HE160B

awf = 6 mm

S355

Foundation
700 x 1200 x 850

C20/25

Base plate
P25 ‐ 200 x 360
ea2 = 50 mm

p2 = 100 mm

ea1 = 60 mm

eb2 = 50 mm

m2 = 50 mm

eb1 = 70 mm

Varying
m1

tp1

[mm]
0

50

100

MN=0,pl

Sj,ini,pl

MN=0,mem

Lcb

[kNm/rad]
15133
15045
14913
6380
8683
10958
494
950
1953

[kNm]

[m]
2.8
2.8
2.8
6.6
4.8
3.8
84.8
44.1
21.4

‐
‐
‐
34
48
65
29
39
58

Column
awf = 6 mm

S355

Foundation

Lco

M1

N1

Sj,ini

tp1

[mm]
0

50

100

Lcb

[m]
[kNm] [kN] [kNm/rad]
8.6
95 700
13939
8.7
100 742
13826
8.8
108 806
13669
20.5
86 767
6301
15.1
98 756
8093
11.9
114 756
9917
265.0
84 784
536
137.7
94 785
930
67.0
112 775
1780

Base plate
P30 ‐ 200 x 360

Threaded studs
Ø 24 mm
S355

p1 = 100 mm

Headed studs
Stirrups
Ø 22 mm
S355 Ø 8 mm
heff = 200 mm

B500A
4 legs for stud

[m]
3.0
3.0
3.1
6.6
5.2
4.2
78.1
45.0
23.5

Lco

ea2 = 50 mm

p2 = 100 mm

ea1 = 60 mm

eb2 = 50 mm

m2 = 50 mm

eb1 = 70 mm

700 x 1200 x 850

C20/25

MN=0,pl

Sj,ini,pl

MN=0,mem

Lcb

[kNm/rad]
16445
16338
16177
6623
9103
11589
493
952
1968

[kNm]

[m]
2.5
2.6
2.6
6.3
4.6
3.6
84.9
44.0
21.3

M2

N2

Sj,ini

[m]
[kNm] [kN] [kNm/rad]
9.4
95 700
13939
9.5
100 742
13826
9.6
108 806
13669
20.8
86 767
6301
16.2
98 756
8093
13.2
114 756
9917
244.2
84 784
536
140.7
94 785
930
73.5
112 775
1780

Anchor plate
P(tp1) ‐ 240 x (380 + 2m1)

S355

Varying
m1

S355

Resistance / Stiffness / Limiting length

[mm] [kNm]
10
53
12
54
15
55
10
31
12
45
15
65
10
24
12
35
15
54

HE160B

Anchor plate
P(tp1) ‐ 240 x (380 + 2m1)

S355

S355

Lcb
[m]
3.0
3.0
3.1
6.6
5.2
4.2
78.1
45.0
23.5

Lco

M3

N3

Sj,ini

[m]
[kNm] [kN] [kNm/rad]
9.4
95 700
13939
9.5
100 742
13826
9.6
108 806
13669
20.8
86 767
6301
16.2
98 756
8093
13.2
114 756
9917
244.2
84 784
536
140.7
94 785
930
73.5
112 775
1780

Threaded studs
Ø 24 mm
S355

p1 = 100 mm

Lcb

Lco

NM=0

[m]
3.0
3.0
3.1
6.6
5.2
4.2
78.1
45.0
23.5

[m]
9.4
9.5
9.6
20.8
16.2
13.2
244.2
140.7
73.5

[kN]
1804
1887
1926
1804
1887
1926
1804
1887
1926

Headed studs
Stirrups
Ø 22 mm
S355 Ø 8 mm
heff = 200 mm

B500A
4 legs for stud

Resistance / Stiffness / Limiting length

[mm] [kNm]
10
55
12
56
15
57
10
33
12
47
15
67
10
25
12
36
15
56

‐
‐
‐
36
49
67
30
41
60

Lco

M1

N1

Sj,ini

[m]
[kNm] [kN] [kNm/rad]
8.0
105 762
15214
8.0
110 805
15090
8.1
119 871
14916
19.8
96 830
6616
14.4
108 820
8540
11.3
125 822
10519
265.4
94 847
544
137.4
105 850
943
66.5
123 841
1803

Lcb
[m]
2.8
2.8
2.8
6.3
4.9
4.0
76.9
44.4
23.2

Lco

M2

N2

Sj,ini

[m]
[kNm] [kN] [kNm/rad]
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105 762
15214
8.7
110 805
15090
8.8
119 871
14916
19.8
96 830
6616
15.3
108 820
8540
12.4
125 822
10519
240.3
94 847
544
138.7
105 850
943
72.6
123 841
1803

Lcb
[m]
2.8
2.8
2.8
6.3
4.9
4.0
76.9
44.4
23.2

Lco

M3

N3

Sj,ini

[m]
[kNm] [kN] [kNm/rad]
8.6
105 762
15214
8.7
110 805
15090
8.8
119 871
14916
19.8
96 830
6616
15.3
108 820
8540
12.4
125 822
10519
240.3
94 847
544
138.7
105 850
943
72.6
123 841
1803

Lcb

Lco

NM=0

[m]
2.8
2.8
2.8
6.3
4.9
4.0
76.9
44.4
23.2

[m]
8.6
8.7
8.8
19.8
15.3
12.4
240.3
138.7
72.6

[kN]
1926
1926
1926
1926
1926
1926
1926
1926
1926

93


### HE180B

#### Foundation
- ea₂ = 50 mm
- p₂ = 100 mm
- εₐ₁ = 60 mm
- p₁ = 100 mm
- εₐ₂ = 80 mm

#### Table 4.11: Recommended geometry of the column base with anchor plate, its design resistances, stiffness and limiting length for HE180B

<table>
<thead>
<tr>
<th>Column</th>
<th>Base plate</th>
<th>Anchor plate</th>
<th>Threaded studs</th>
<th>Headed studs</th>
<th>Stirrups</th>
</tr>
</thead>
<tbody>
<tr>
<td>HE180B</td>
<td>P25 - 220 x 380</td>
<td>S355</td>
<td>P(τ₁) - 260 x (400 + 2m₁)</td>
<td>S355</td>
<td>M24</td>
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</tbody>
</table>

<table>
<thead>
<tr>
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<th>Resistance / Stiffness / Limiting length</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>m₁</td>
<td>l₁</td>
<td>Mₜₛₘₚₚ</td>
</tr>
<tr>
<td>[mm]</td>
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</tr>
<tr>
<td>15</td>
<td>17</td>
<td>67</td>
</tr>
</tbody>
</table>

| 50 | 10 | 37 | 8401 | 40 | 7.7 | 23.9 | 129 | 1173 | 7984 | 8.1 | 25.2 | 129 | 1177 | 7976 | 8.1 | 25.2 | 129 | 1170 | 7976 | 8.1 | 25.2 | 129 | 1170 | 7976 | 8.1 | 25.2 |
| 12 | 54 | 11456 | 56 | 5.6 | 17.6 | 144 | 1174 | 10173 | 6.3 | 19.8 | 144 | 1174 | 10173 | 6.3 | 19.8 | 144 | 1174 | 10173 | 6.3 | 19.8 | 144 | 1174 | 10173 | 6.3 | 19.8 |
| 15 | 77 | 14458 | 77 | 4.5 | 13.9 | 163 | 1197 | 12328 | 5.2 | 16.3 | 163 | 1197 | 12328 | 5.2 | 16.3 | 163 | 1197 | 12328 | 5.2 | 16.3 | 163 | 1197 | 12328 | 5.2 | 16.3 |

| 100 | 10 | 28 | 659 | 46 | 97.7 | 305.3 | 126 | 1193 | 691 | 93.2 | 291.3 | 126 | 1196 | 690 | 93.3 | 291.6 | 126 | 1200 | 689 | 93.4 | 292.0 | 126 | 1200 |
| 12 | 41 | 1272 | 46 | 50.6 | 158.1 | 139 | 1208 | 1188 | 54.2 | 169.2 | 139 | 1208 | 1188 | 54.2 | 169.2 | 139 | 1208 | 1188 | 54.2 | 169.2 | 139 | 1208 |
| 15 | 64 | 2621 | 68 | 24.6 | 76.7 | 161 | 1208 | 2254 | 28.6 | 89.2 | 161 | 1208 | 2254 | 28.6 | 89.2 | 161 | 1208 | 2254 | 28.6 | 89.2 | 161 | 1208 | 2254 | 28.6 |

| 10 | 10 | 37 | 8401 | 40 | 7.7 | 23.9 | 129 | 1173 | 7984 | 8.1 | 25.2 | 129 | 1177 | 7976 | 8.1 | 25.2 | 129 | 1170 | 7976 | 8.1 | 25.2 | 129 | 1170 | 7976 | 8.1 | 25.2 |
| 15 | 77 | 14458 | 77 | 4.5 | 13.9 | 163 | 1197 | 12328 | 5.2 | 16.3 | 163 | 1197 | 12328 | 5.2 | 16.3 | 163 | 1197 | 12328 | 5.2 | 16.3 | 163 | 1197 | 12328 | 5.2 | 16.3 |

Tab. 4.11: Recommended geometry of the column base with anchor plate, its design resistances, stiffness and limiting length for HE180B
<table>
<thead>
<tr>
<th>Varying</th>
<th>Resistance / Stiffness / Limiting length</th>
</tr>
</thead>
<tbody>
<tr>
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<td>tp1</td>
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<tr>
<td>[mm]</td>
<td>[mm]</td>
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<tr>
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<table>
<thead>
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<th>Resistance / Stiffness / Limiting length</th>
</tr>
</thead>
<tbody>
<tr>
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<td>tp1</td>
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<td>[mm]</td>
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<tr>
<td>15</td>
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### Table 4.13: Recommended geometry of the column base with anchor plate, its design resistances, stiffness and limiting length for HE220B

#### HE220B

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<td>[mm]</td>
<td>[mm]</td>
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<tr>
<td>12</td>
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<tr>
<td>15</td>
<td>69</td>
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</tbody>
</table>

#### HE220B

<table>
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<th>Resistance / Stiffness / Limiting length</th>
</tr>
</thead>
<tbody>
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</table>
### Tab. 4.14: Recommended geometry of the column base with anchor plate, its design resistances, stiffness and limiting length for HE240B

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<th>Resistance / Stiffness / Limiting length</th>
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<td>0</td>
<td>10</td>
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<tr>
<td>12</td>
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<tr>
<td>15</td>
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### Tab. 4.14: Recommended geometry of the column base with anchor plate, its design resistances, stiffness and limiting length for HE240B

<table>
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</thead>
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<td>12</td>
<td>47</td>
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<tr>
<td>15</td>
<td>74</td>
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</tbody>
</table>
Tab. 4.15: Recommended geometry of the column base with anchor plate, its design resistances, stiffness and limiting length for HE260B

### HE260B

<table>
<thead>
<tr>
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<th>Resistance / Stiffness / Limiting length</th>
</tr>
</thead>
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<td>L₁₀</td>
</tr>
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<td>[mm]</td>
<td>[m]</td>
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### HE260B

<table>
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<td>[mm]</td>
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<td>12</td>
<td>50</td>
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<tr>
<td>15</td>
<td>78</td>
</tr>
</tbody>
</table>
4.5 Parameter study on composite joints

4.5.1 General

Composite joint behaviour depends on the characteristics of several active components. The hogging moment capacity can be calculated with the hypothesis of failure of the weakest of them, while the total displacement (and relative rotation) can be found considering the contribution of all of them. The following basic components are identified in Design Manual I "Design of steel-to-concrete joints" [13]: i) longitudinal steel reinforcement in the slab; ii) slip of the composite beam; ii) beam web and flange; iv) steel contact plate; v) components activated in the anchor plate connection; vi) the joint link. Among them, the longitudinal steel reinforcement transfers the tension force; the others contribute to the transmission of the compression force. Failure that depends on the steel reinforcement behaviour is ductile, while failure of concrete components is brittle and should be avoided. The aim of this investigation is to evaluate the failure mechanism of the joint in order to ensure a ductile failure. For this reason, a parametric study followed by sensitivity analysis is carried out, taking into account the variation of some parameters that determine the behaviour of basic components.

4.5.2 Parameters Studied and methodology followed

The attention is mainly focused on the behaviour of steel reinforcement and the joint link. The force in the reinforcement is a function of the steel grade and of bars layout. The first aspect concerns the yield strength ($f_{syk}$), the coefficient between the ultimate and yield strength ($k$) and ductility ($\varepsilon_{s,u}$). The second one is characterized by number and diameter of bars and number of layers. In the analysis three values of $f_{syk}$, four values of $k$, three $\varepsilon_{s,u}$ values and four reinforcement layouts are considered. The possibility of development of the strut&tie mechanism in the concrete panel depends on the angle $\theta$. This geometrical quantity is calculated through the ratio between the sum of beam height and slab thickness on the thickness of the wall. In the analysis, six beam profiles, four wall thickness $t_{wall}$ and three slab thickness $s_{slab}$ are considered. Wall concrete properties, i.e. the characteristic compressive cylinder ($f_{ck,cyl}$) and cubic ($f_{ck,cube}$) strength, and secant modulus of elasticity ($E_{cm}$), affect as well the joint link behaviour. In the analysis five concrete grades for wall are considered. The sensitivity analysis compared by 51840 combinations. Tab. 4.16 summarizes the parameters considered for the parameter study.

<table>
<thead>
<tr>
<th>Tab. 4.16: Parameters considered for parameter study</th>
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<tbody>
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<td><strong>Element</strong></td>
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</tr>
<tr>
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</tr>
<tr>
<td>Beam</td>
</tr>
</tbody>
</table>
4.5.3 Failure Mechanism

Considering the simultaneous variation of all parameters, the most common failure type is the joint link (34149 cases of 51840, 65.87%); only in 14493 cases (27.96) a slab reinforcement failure occurs; in few cases (3198, 6.17%) the failure depends on the behaviour of beam. Fig. 4.45 summarizes found failure types in the sensitivity analysis.

![Fig. 4.45: Failure type](image)

4.5.4 Valorization of slab reinforcement properties

The role of slab reinforcement layout is studied, taking into account four bars configurations, according to Tab. 4.16. Fig. 4.46 illustrates the influence. Increasing the reinforcement area, incidence of joint link failure grows significantly, while reinforcement failure decreases. The trend is reversed considering the reinforcements on two layers (Case D). Beam failure is almost absent for low values of steel area, but it assumes a quite relevant rate in Case D.

![Fig. 4.46: Influence of rebar layout](image)

As expected, one of the most influential parameter is the steel grade (see Fig. 4.47). Here, increasing the yield strength, the percentage of joint link failure switches from 49.65 to 75.67, while cases with ductile failure decrease. Variation of ductility of the bar does not lead to changes in failure type distribution.
The coefficient $k$ influence is highlighted in Fig. 4.48. Increasing $k$, joint link failures number rises, while cases of reinforcement failure are approximately halved.

The interaction between the yield and ultimate strength is evaluated in Fig. 4.49. A change of the main failure type is visible for yield strength equal to 400MPa. While the joint link is crucial for high values of $k$ (61% of failures), the longitudinal reinforcement becomes the most important component for a lower value of $k$ (57% of failures). The number of cases with joint link failure grows significantly (+20%) with increasing $k$ for a steel with a low value of yield strength ($f_{syk} = 400$ MPa). The same trend is visible for a steel with greater yield strength ($f_{syk} = 600$ MPa), but the increase is lower (10%).
In order to assess the role of the angle theta, the influence of the individual parameters \((t_{\text{wall}}, t_{\text{slab}}\) and \(h_{\text{beam}}\)) is studied. In addition, the total height \((\text{slab} + \text{beam})\) has been considered. The main parameter that affects the development of the failure mechanism is the wall thickness. For a thickness of 160 mm in 93.45% of cases the failure occurs in the concrete panel. The number of cases of brittle failure drops to 76.04 for a thickness of 200 mm (Fig. 4.50). The ductile failure becomes the main type of failure only for a thickness of 300 mm.

**Fig. 4.49: Interaction between yield and ultimate strength**

### 4.5.5 Variation of angle \(\theta\)

In order to assess the role of the angle theta, the influence of the individual parameters \((t_{\text{wall}}, t_{\text{slab}}\) and \(h_{\text{beam}}\)) is studied. In addition, the total height \((\text{slab} + \text{beam})\) has been considered. The main parameter that affects the development of the failure mechanism is the wall thickness. For a thickness of 160 mm in 93.45% of cases the failure occurs in the concrete panel. The number of cases of brittle failure drops to 76.04 for a thickness of 200 mm (Fig. 4.50). The ductile failure becomes the main type of failure only for a thickness of 300 mm.
Fig. 4.50: Influence of $t_{\text{wall}}$

The influence of the slab thickness is shown in Fig. 4.51. For the three values considered (120, 160, 200), structural failure happens in the concrete panel in most of the cases (variation between 55.09% and 67.34%). The percentage of beam failure does not vary appreciably. The increase in the height of the beam determines a clear trend, as seen in Fig. 4.52. For a height of 240 mm, in about 50% of cases the failure happens for the concrete panel. Here, the number of cases with beam failure is not negligible (22%). With the increase of the height, the possibility of a failure in the beam decreases significantly, increasing sharply the percentage of the failure in the concrete panel. The consideration of the total height (slab + beam) leads to a less clear trend for low values (Fig. 4.53).
Initial beam failure peaks (26.46%, 22.7% and 19.38%) are due to the presence of IPE240. However, joint link represents always the main failure type and the possibility of brittle crisis doubles, moving from a height of 360 mm (41.67%) to 600 mm (82.08).

### 4.5.6 Variation of wall concrete grade

The concrete grade is an important parameter. Fig. 4.54 shows the variation of number of case for each mechanism failure type. For concrete grade C20/25, joint link behaviour represents the limit condition in almost all of the cases (97.25%). The variation is evident. This percentage drops to 36.86 % for concrete C60/75. Brittle failure is the most probable event for $f_{ck,\text{wall}}$ is smaller than 40MPa.
Fig. 4.54: Influence of wall concrete grade

4.5.7 Interaction between wall thickness and concrete grade

The interaction between geometrical properties and material of the wall is studied. For a thickness of 160 mm, joint link determines the failure for all types of concrete considered (Fig. 4.54).

Fig. 4.55: Influence of wall concrete grade for $t_{\text{wall}} = 160$ mm

The percentage drops from 100% (C20/25) to 81.94% (C60/75). The decrease in the percentage of brittle failure is more pronounced for a thickness of 200 mm (Fig. 4.56). In this case for C60/75, ductile failure is more probable than a brittle failure (44.68%).
Fig. 4.56: Influence of wall concrete grade for $t_{\text{wall}} = 200$ mm

With a wall thickness of 240mm (Fig. 4.57), the increase of reinforcement failure is evident. It represents 0.96% of the cases for $f_{\text{ck,Wall}}$ equal to 20 MPa and becomes 70.37 % when $f_{\text{ck,Wall}}$ is 60 MPa. Reduction of joint link failures is noteworthy. From 97.92% (C20/25) to 17.59% (C60/75). Considering a wall thickness of 300 mm (Fig. 4.58), the inversion of most probable failure type (from brittle to ductile) occurs for concrete grade C40/50. A strong change in the trend is visible between C20/25 and C40/50, where brittle failure switches from 91.09 to 25.54 and reinforcement failure switches from 5.44 to 64.12. After this, the change is less marked.

Fig. 4.57: Influence of wall concrete grade for $t_{\text{wall}} = 240$ mm
4.5.8 Summary, Predesign charts for ductile behaviour

The above sensitivity analysis shows the main parameters that affect the failure mode:

**Yield strength:** cases with brittle failure rises from 49.65% (for $f_{syk} = 400$ MPa) to 75.67% (for $f_{syk} = 600$ MPa).

**Wall thickness:** the concrete panel failure occurs in 93.45% of cases for a thickness of 160mm and in 37.06% for a thickness of 300 mm.

**Total height of the composite beam:** for the lowest value (360mm) a ductile failure happens in 59.33% cases, while for the highest height, 19.72% of cases show this failure.

**Concrete grade:** for C20/25, there are 97.25% cases of brittle behaviour, and this percentage drops to 36.86% for concrete C60/75.

For these considerations, a pre design chart (Fig. 4.59, Fig. 4.60 and Fig. 4.61) can be a useful tool in order to lead to a ductile failure. Here, the wall thickness (on the ordinate) is related to the concrete grade (on the abscissa). Separation curves between ductile (top-right) and brittle (bottom-left) failure can be built for nine steel grades (3 $f_{syk}$ and 3 $k$). To take into account the total height of the composite beam, three charts are drawn: Fig. 4.59 represents the pre design chart for a total height between 360mm and 440mm; Fig. 4.60 refers to a range between 440mm and 520mm; finally Fig. 4.61 concerns the behaviour for a total height between 520mm and 600mm. In these figures, black lines refer to $f_{syk} = 400$ MPa, dark grey to $f_{syk} = 500$ MPa and light grey $f_{syk} = 600$ MPa; solid lines refer to $k = 1.05$, dash lines to $k = 1.15$, long dash lines to $k = 1.25$, dash-dot-dot lines to $k = 1.35$. Curves stretches found for regression are shown dotted.

For example, for a total height of 390mm, a wall thickness of 160mm and a concrete characteristic compressive cylinder equal to 50 MPa, the steel yield strength that ensure a ductile behaviour is equal to 400MPa with $k = 1.05$, according to Fig. 4.59.
Fig. 4.59: Pre design chart for ductile behavior in case of total depth of the composite beam between 360 and 440 mm
Fig. 4.60: Pre design chart for ductile behaviour in case of total depth of the composite beam between 440 and 520 mm
Pre design chart for ductile behavior in case of total depth of the composite beam between 520 and 600 mm.
5 Summary

This Design Manual II is based on the Design Manual I "Design of steel-to-concrete joints" [13] which summarizes the reached knowledge in the RFCS Project RFSR-CT-2007-00051 New market Chances for Steel Structures by Innovative Fastening Solutions between Steel and Concrete (INFASO) [12].

Within the INFASO project design programs were developed for three different steel-to-concrete joints. This programs have been revised and updated within INFASO+. In this design manual background information about this design programs is given and the application of the programs is explained in detail (see Chapter 2). This includes following design programs:

- Restrained connection of composite beams (Version 2.0) [21]
- Slim anchor plates with headed studs - bending joints (Version 2.0) [22]
- Rigid anchor plate with headed studs – simple joint (Version 2.0) [23]

Furthermore the transferability of the results to real life is shown within realistic design examples taken from practice where the newly developed design rules are applied (see Chapter 3). In the worked examples common solutions for steel-to-concrete connections are compared with the innovative developed solutions. These connections are compared in terms of calculation approaches, handling, tolerances and behavior under fire conditions. Parameter studies of the components and analytic model of the three different steel-to-concrete joints show the influence of each parameter. Furthermore recommendations for design values and limits of the model are given (see Chapter 4).

The material was prepared in cooperation of two teams of researchers one targeting on fastening technique modelling and other focusing to steel joints design from the Institute of Structural Design and Institute of Construction Materials, University Stuttgart, Department of Steel and Timber structures, Czech Technical University in Prague and practitioners Gabinete de Informática e Projecto Assistido Computador Lda., Coimbra, Goldbeck West GmbH, Bielefeld, stahl+verbundbau GmbH, Dreieich and European Convention for Constructional Steelwork, Bruxelles.
6 References

Standards and guidelines


Textbooks and publications

Software


