6. Modelling and analysis

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Objectives

- This lecture describes principles of modelling and analysis of structures.
- Global analyses distinguishing effects of deformed geometry and material non-linearities are presented.
- Survey of both simple and FEM analyses and modelling are shown.
- Finally some basic examples are presented.

Outline of the lecture

1. Basic requirements
2. Methods of analysis
3. Influence of deformed geometry
4. Influence of material behaviour
5. Simple global modelling of frames, trusses and beams
6. FE modelling
7. Examples
8. Conclusions

1. Basic requirements

- Calculation model should reflect real global and local behaviour of the designed structure (members, cross sections, joints and placement).

- Analysis should correspond to limit states under consideration: ULS (ultimate limit states) or SLS (serviceability limit states), i.e. with appropriate loading, criteria and reliability.

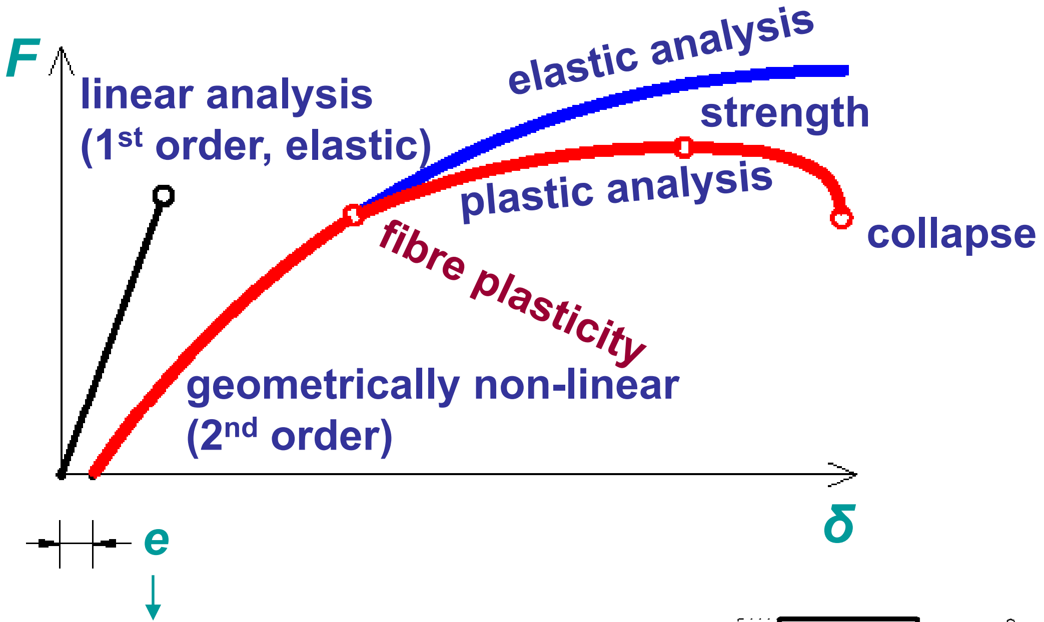
1. Basic requirements

- **Joints** are generally modelled (in accordance with EN 1993-1-8) as:
  - simple (transmitting no bending moments),
  - continuous (with rigidity and resistance providing full continuity of elements),
  - semi-continues (in which the joint behaviour needs to be considered in the global analysis).

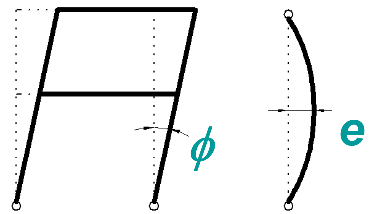
- **Ground-structure interaction** should be considered in case of significant ground support deformation (see EN 1997).

2. Methods of analysis

Simplified scheme of calculation models:



Imperfections (global, local), e.g.:



2. Methods of analysis

General types of analysis:

Elastic

LA: Linear elastic analysis;

LBA: Linear bifurcation analysis;

GNA: Geometrically non-linear analysis.

Non-Elastic

MNA: Materially non-linear analysis;

GMNA: Geometrically and materially non-linear analysis;

GNIA: Geometrically non-linear analysis elastic with imperfections included;

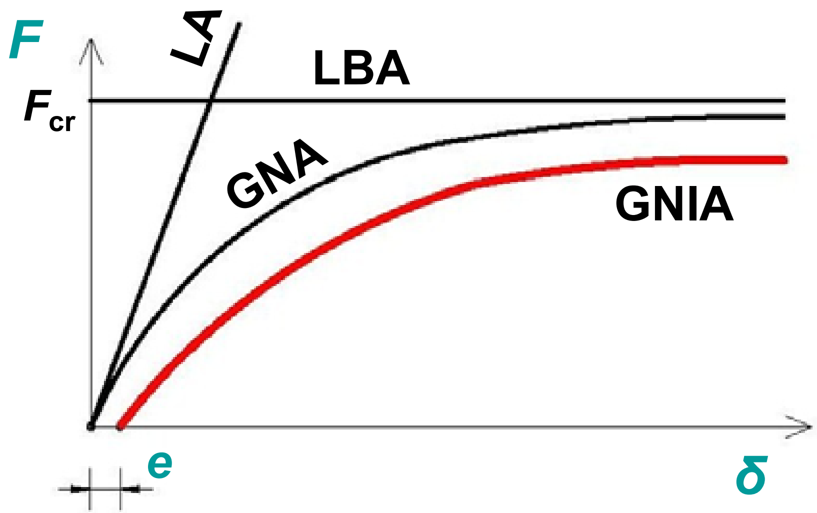
GMNIA: Geometrically and materially non-linear analysis with imperfections included.

2. Methods of analysis

Simplified GNA (using equilibrium equation on deformed structure but the same “small deflections” as in common LA) is called 2^{nd} order analysis. Such analysis is usually sufficient for investigation of buckling in steel frame structures.

3. Influence of deformed geometry

Simplified scheme of elastic analyses:



3. Influence of deformed geometry

- **LA (1st order analysis):**

Benefits:

Superposition valid, easy.

Drawbacks:

Approximate solution, necessary to include imperfections (global, local) and 2nd order effects in other ways (by reduction coefficients for buckling).

3. Influence of deformed geometry

- **LBA (linear bifurcation analysis):**

  This analysis uses 2nd order analysis, introducing, however, zero initial imperfections and zero non-axial loading. The resulting critical forces are expressed in the form

  $N_{cr,i} = \alpha_{cr,i} N_{Ed}$ where $i \in (1; \infty)$

  (N_{Ed} represent initial set of axial forces)

  Note: In **non-linear bifurcation analysis** the GNIA is used and bifurcation occurs by snap-through of initial imperfection shape.

3. Influence of deformed geometry

- **GNA, GNIA (or 2nd order analysis):**

Benefits:

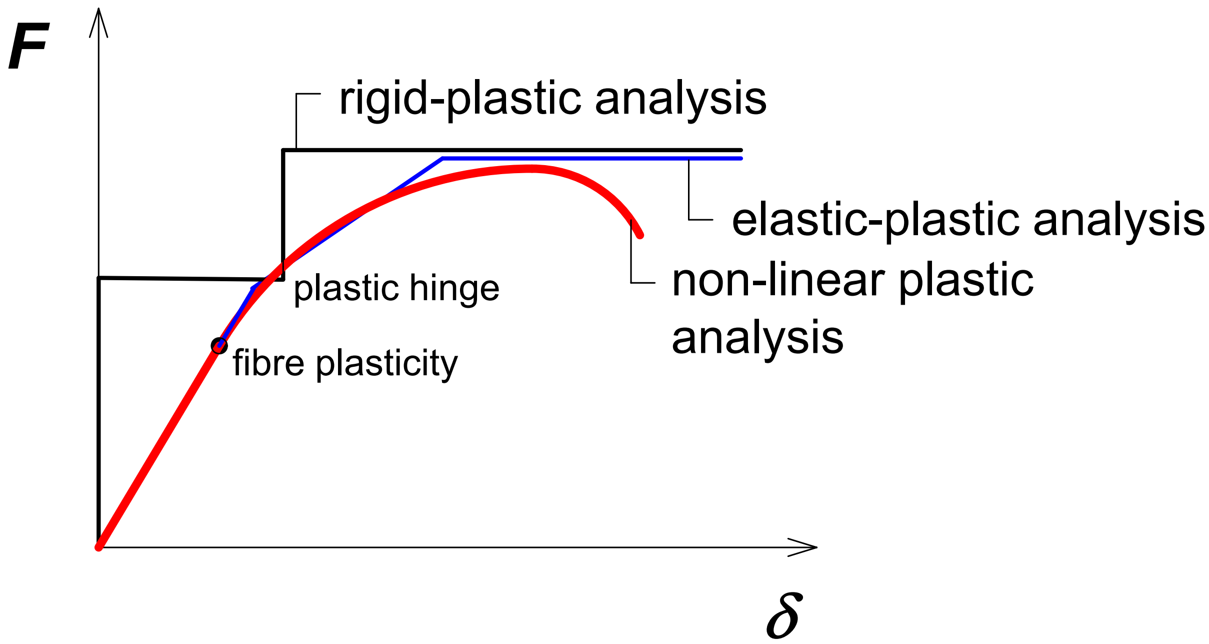
Direct solution of elastic buckling,
covers behaviour of cables.

Drawbacks:

Superposition can not be used,
software necessary.

4. Influence of material behaviour

Simplified scheme of plastic analyses:



4. Influence of material behaviour

- **MNA (plastic analysis):**

Benefits:

Higher strength capacity.

Drawbacks:

May only be used provided that:

- steel is sufficiently ductile ($f_u/f_y \geq 1.1$; $\delta \geq 15$ %; $\varepsilon_u \geq 15\ \varepsilon_y$);
- for global analysis the cross sections are of class 1;
- in global analysis the stability of members at plastic hinges is assured;
- software for plastic global analysis is desirable.

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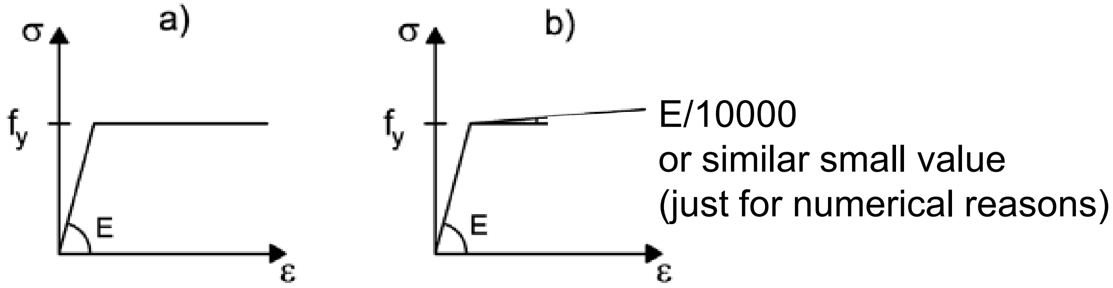
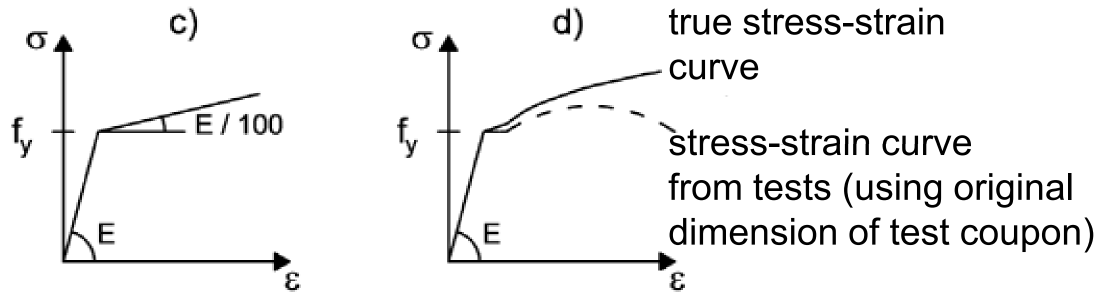
4. Influence of material behaviour

Modelling of material behaviour:

- **without hardening**



- **with hardening**



5. Influence of material behaviour

Plastic global analysis models:

- non-linear plastic analysis considering the partial plastification of members in plastic zones,
- elastic-plastic analysis with plastified sections and/or joints as plastic hinges,
- rigid-plastic analysis neglecting the elastic behaviour between hinges.

Formative Assessment Question 1

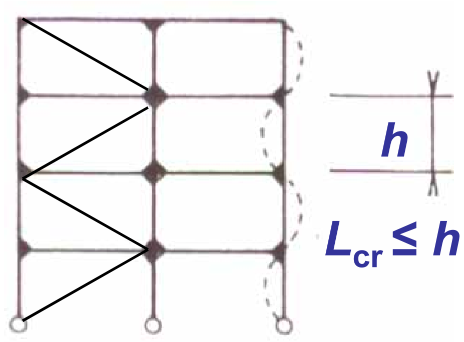
- Describe types of analyses.
- How 2nd order effects in compression members may be covered?
- Describe modelling of material properties.
- What are the limits for using an elastic analysis?
- What are the prerequisites for using a plastic analysis?

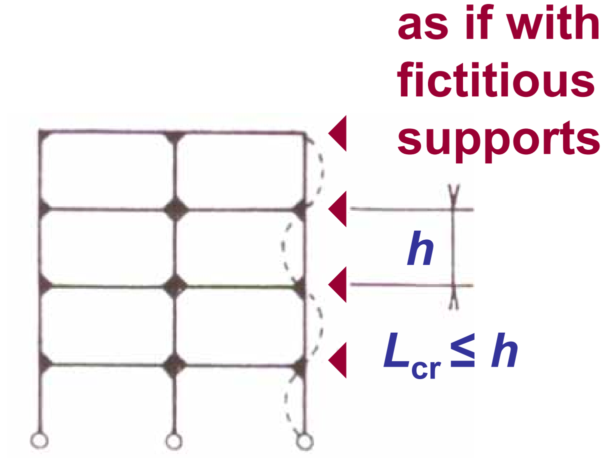
5. Simple modelling of structures

5.1 Frame stability:

- **First order elastic frames**

$$\text{if } \alpha_{cr} = \frac{F_{cr}}{F_{Ed}} \geq 10 \quad \text{(using LBA or approx. formula)}$$



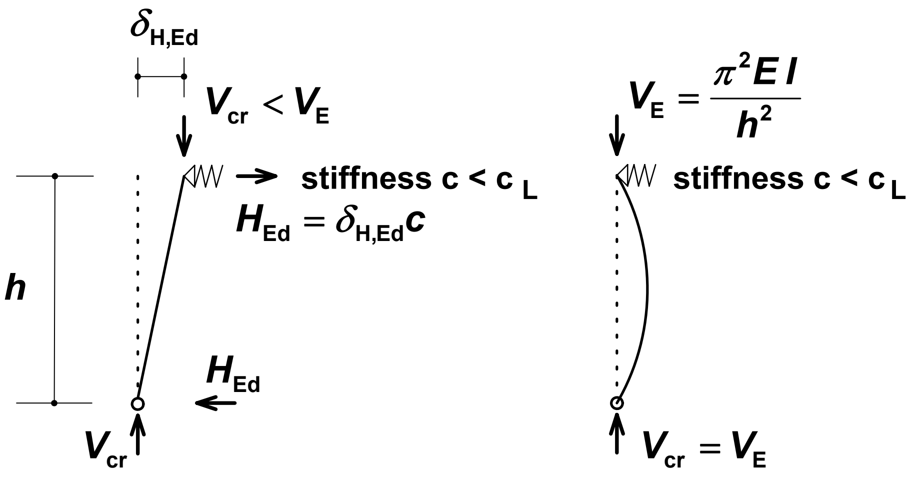


Critical length is lesser than or equal to system length.

5. Simple modelling of structures

Note: Instead of using LBA, approximate value of α_{cr} may be determined from analysis of a compression member with an elastic sway brace:

sway buckling mode | **non-sway buckling mode (Euler's critical load)**



$$\alpha_{cr} = \frac{V_{cr}}{V_{Ed}}$$

For sway buckling mode:

$$V_{cr} \delta_{H,Ed} = H_{Ed} h$$

Therefore, for sway buckling mode: $$\alpha_{cr} = \frac{H_{Ed}}{V_{Ed}} \frac{h}{\delta_{H,Ed}}$$

5. Simple modelling of structures

- **Second order elastic frames**

  if $\alpha_{cr} = \frac{F_{cr}}{F_{Ed}} < 10$ (using LBA or approx. formula)

  Three methods of analysis may be used:

  a) GNIA is generally accepted. If both global and member imperfections are accounted for, no individual stability check for the members is necessary (e.g. compression members are checked for simple resistance to resulting compression without any reduction for buckling).

5. Simple modelling of structures

b) GNA for global analysis of the structure with global imperfections. Member stability checks should be based on buckling lengths equal to the system lengths.

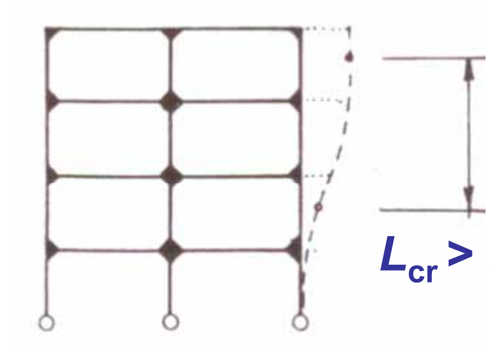
If $3 \leq \alpha_{cr} < 10$ and sway buckling mode is predominant, as a good approximation an **amplified LA** may be used (see Eurocode 3, cl. 5.2.2(5B)) where sway effects (i.e. all horizontal loading) should be increased by a multiple

$$\frac{1}{1-\frac{1}{\alpha_{cr}}}$$

Note: For multi-storey frames this simplification can be used provided they are ”regular” (see Eurocode, cl. 5.2.2(6B)

5. Simple modelling of structures

c) LA for global analysis without considering imperfections. Member stability checks should be based on buckling lengths equal to the global buckling length (received from LBA):



$$\bar{\lambda} = \sqrt{\frac{N_y}{N_{cr}}} = \sqrt{\frac{A f_y}{\alpha_{cr} N_{Ed}}}$$

or

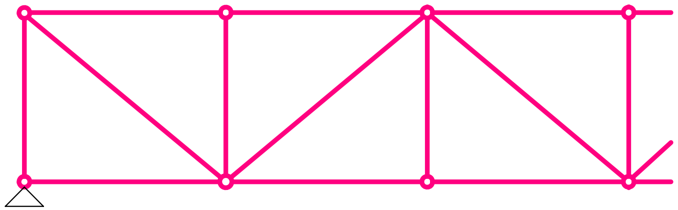
$$L_{cr} = \sqrt{\frac{\pi^2 E I}{N_{cr}}} = \sqrt{\frac{\pi^2 E I_y}{\alpha_{cr} N_{Ed}}}$$

Note: Safe use of this method requires increasing of moments due to sway effects (approx. by 20 %).

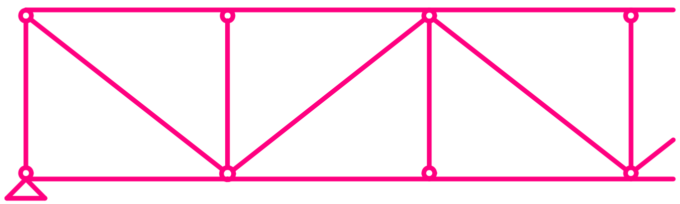
5. Simple modelling of structures

5.2 Trusses – common LA:

- Approximate analysis assuming pin-jointed member ends (secondary moments in members due to stiffness of joints ignored):



- Approximate analysis with continuous chords (usual analysis):



5. Simple modelling of structures

- eccentricity of members in nodes should be limited (see EN 1993-1-8, cl. 5.1.5), otherwise eccentricity moments $\Delta M = (H_1 - H_2)\, e$ shall be distributed to members.

Scheme of truss nodes

- effects of global and local instabilities in trusses in accordance with α_{cr} (global instability is usually negligible, unless slender truss column is analysed).

5. Simple modelling of structures

5.3 Continuous beams

(1st class cross-sections, plastic analysis)

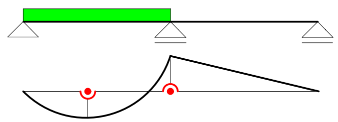
Methods of elastic-plastic or rigid-plastic analysis (leading to complete, overcomplete or partial kinematic mechanism):

- Method of consecutive formation of plastic hinges (used by common software).
- Method of virtual works to form kinematic mechanism.
- Method of moment redistribution (most common).

5. Simple modelling of structures

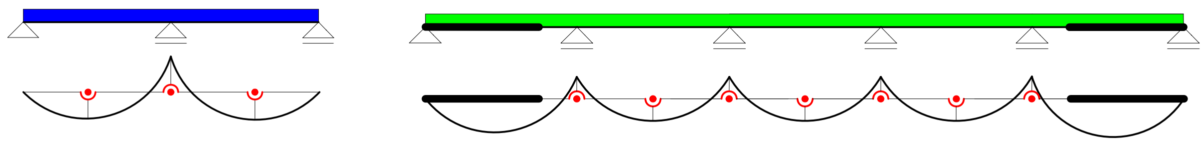
Examples of beams under uniform loadings:

- complete kinematic mechanism



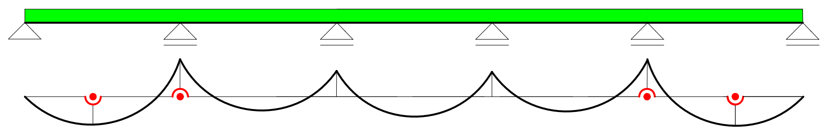
number of hinges = number of statically indeterminates + 1

- hypercomplete kinematic mechanisms



outside spans strengthened

- partial kinematic mechanisms

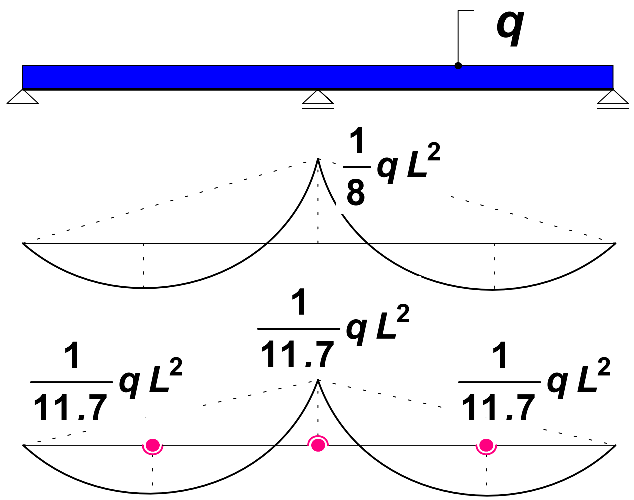


inner spans not fully utilized

5. Simple modelling of structures

Example:

Common analysis of two-span beam under uniform loading (hypercomplete mechanism):



elastic moments

plastic redistribution

6. FE modelling

Requirements for FEM are given in EN 1993-1-5, Annex C. Special care is due to:

- the modelling of the structural component and its boundary conditions;
- the choice of software and documentation;
- the use of imperfections;
- the modelling of material properties;
- the modelling of loads;
- the modelling of limit state criteria;
- the partial factors to be applied.

6. FE modelling

Limit state criteria:

1. For structures susceptible to buckling:
   Attainment of the maximum load.
2. For regions subjected to tensile stresses:
   Attainment of a limiting value of the principal membrane strain (5%).

Other criteria may be used, e.g. attainment of the yielding criterion or limitation of the yielding zone.

6. FE modelling

The design load magnification factor α_u (for simplicity a single design load multiplier) to the ultimate limit state should be sufficient to achieve the required reliability:

$$\alpha_u > \alpha_1 \alpha_2$$

where

- α_1 covers the model uncertainty of the FE-modelling used. It should be obtained from evaluations of test calibrations, see Annex D to EN 1990;
- α_2 covers the scatter of the loading and resistance models. It may be taken as γ_{M1} (= 1.0) if instability governs and γ_{M2} (= 1.25) if fracture governs.

Formative Assessment Question 2

- Describe common calculation models for braced building frame.
- Describe common calculation models for a truss.
- Describe common plastic models for continuous beams under various loadings.
- How ULS may be determined when using various FEM models (ranging from LA up to GMNIA)?

7. Examples

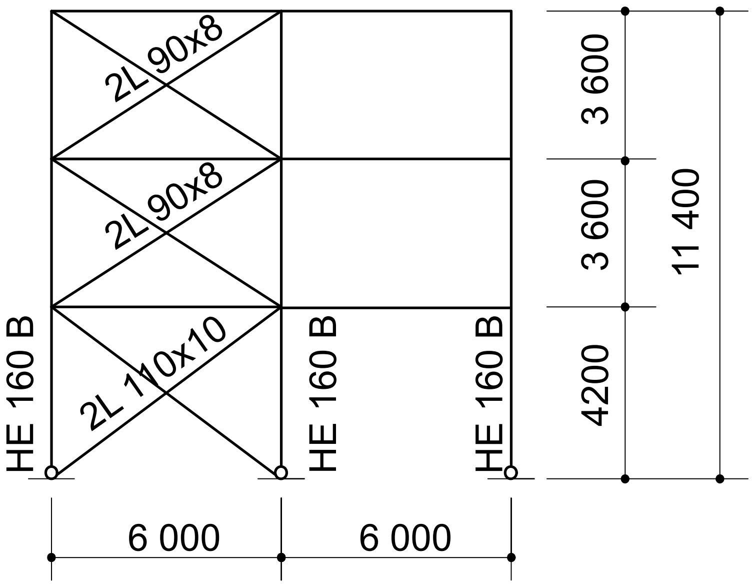
Example 1: Two-bay braced frame

Example 2: Two-hinged arch

Example 3: Plate under uniform loading

7.1 Example 1

Example 1:
Two-bay braced frame



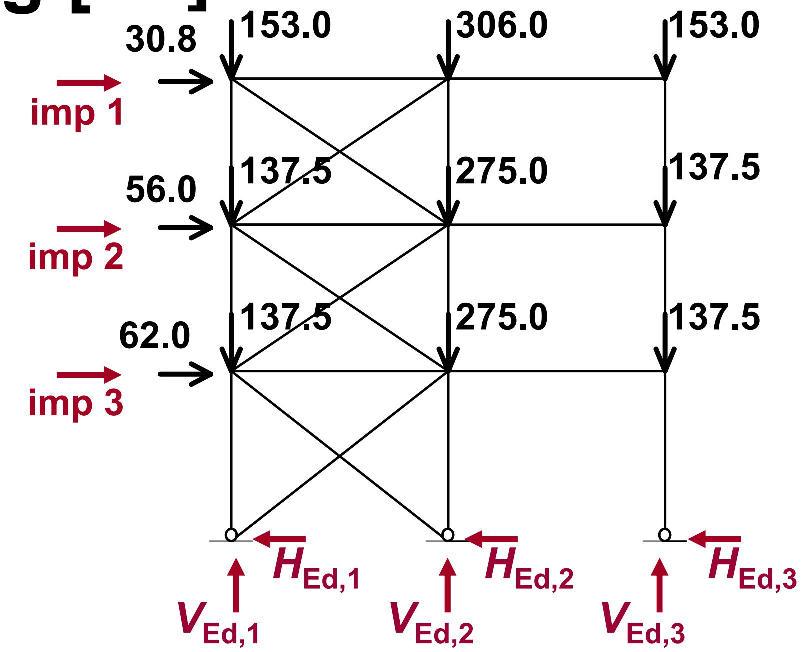
The frames spaced at distance of 6 m, bracing each 12 m.

Geometry and cross sections:

composite floor beams: A= 9345 mm^2, I = 127.4.10^6 mm^4

7.1 Example 1

Loading [kN]



- vertical loading of columns;
- winter loading due to this bracing;
- global imperfections due to this bracing (from 2 cross frames):
  imp 1 = 2×1.6 = 3.2 kN;   imp 2 = imp 3 = 2×1.5 = 3.0 kN.

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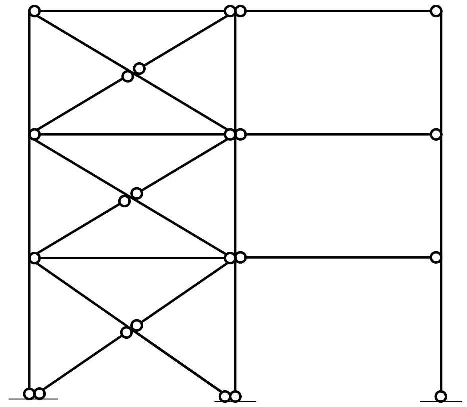
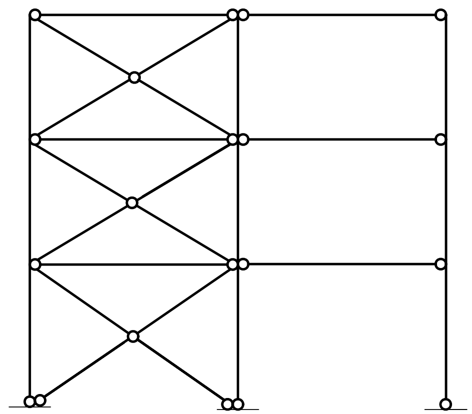
7.1 Example 1

LA - model

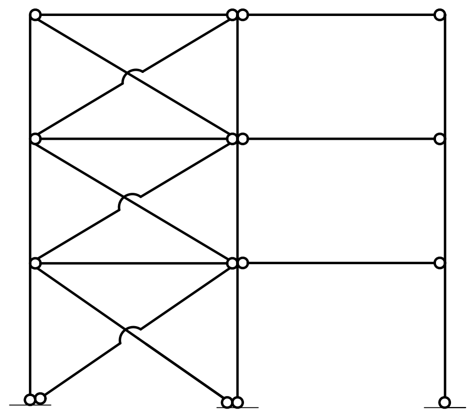
used model:



inappropriate models:



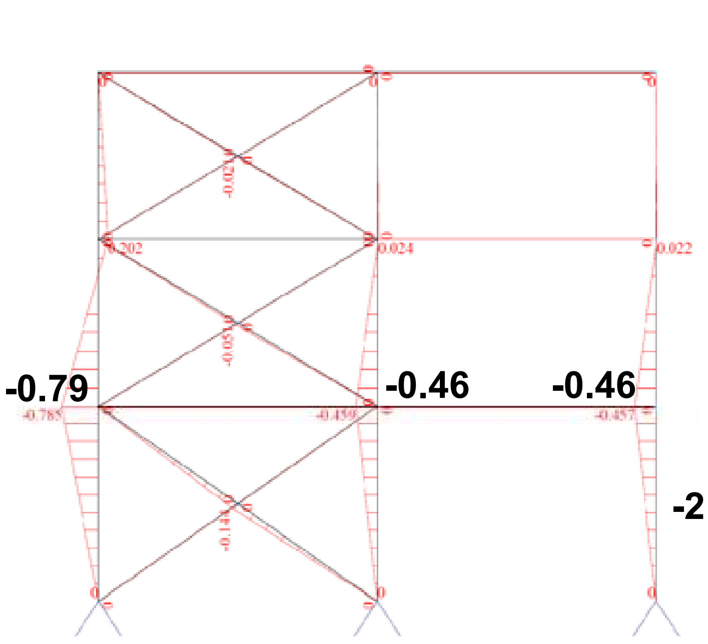
unstable



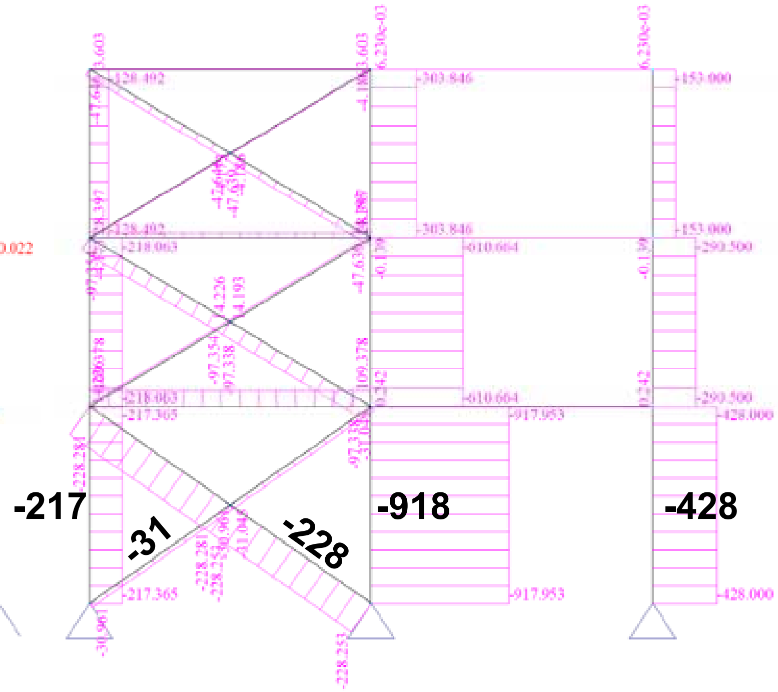
long critical lengths of diagonals

7.1 Example 1

LA - internal forces
(due to loading including global imperfections)



M_{Ed} [kNm]



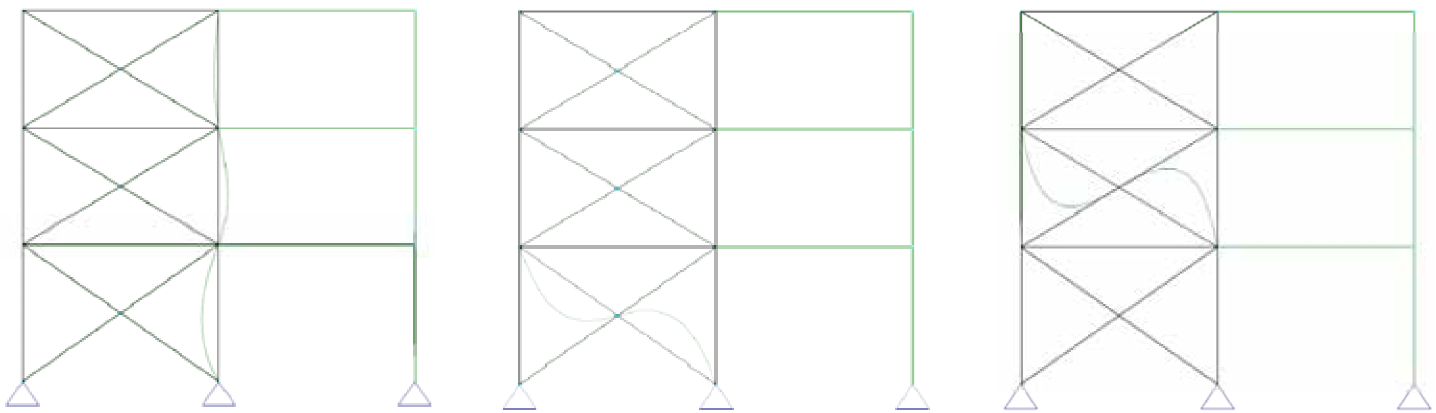
N_{Ed} [kN]

7.1 Example 1

LBA – critical modes

(loading including global imperfections)

6 first critical modes are shown for demonstration:

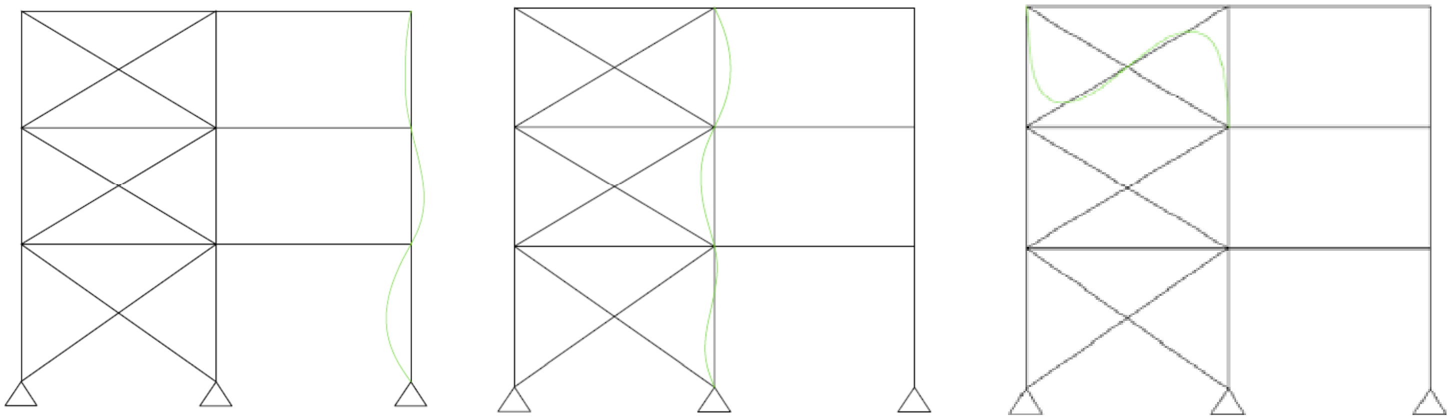


$\alpha_{cr,1}$ = 5.51 (central column) | $\alpha_{cr,2}$ = 7.37 (bottom diagonal) | $\alpha_{cr,3}$ = 8.30 (middle diagonal)

7.1 Example 1

LBA – critical modes

(loading including global imperfections)



$\alpha_{cr,4}$ = 11.75 (right column)

$\alpha_{cr,5}$ = 14.40 (central column)

$\alpha_{cr,6}$ = 16.95 (upper diagonal)

Note: The first sway mode is the 15th, where $\alpha_{cr,15}$ = 144.1;

Using approximate formula: $\alpha_{cr} = \frac{H_{Ed}}{V_{Ed}} \frac{h}{\delta_{H,Ed}} = \frac{162.0}{1712.0} \frac{4200}{2.06} = 192.9$

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7.1 Example 1

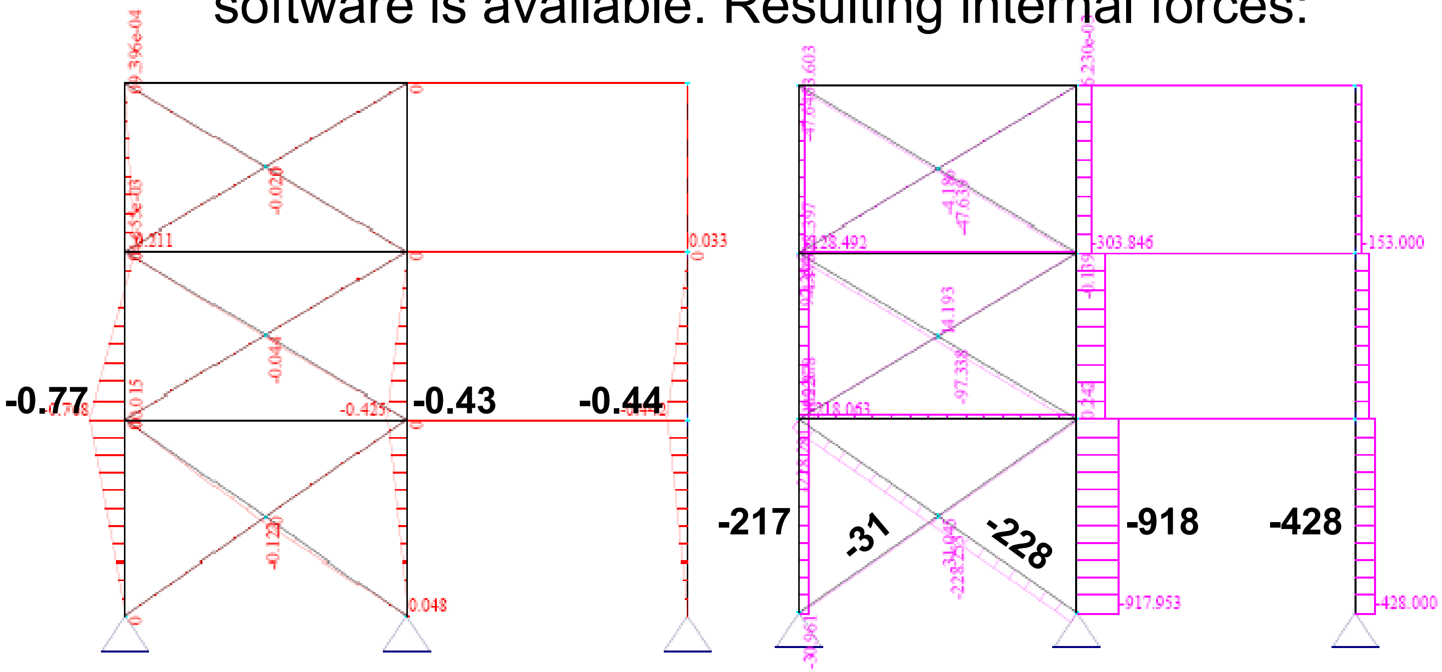
- **Notes for design:**

  - The frame is classified as second order frame ($\alpha_{cr,1}$ = 5.51 < 10). However, the buckling modes are of non-sway character. The possibilities mentioned for analysis (see 5.1) are discussed:

a) **GNIA** may generally be used, where unique imperfection for this frame e_0 = 8.5 mm was determined in Lecture 5. Approximate global imperfections and imperfections of individual elements in accordance with Eurocode 3 or their equivalent transverse loadings may also be used. However, such analyses are generally demanding.

7.1 Example 1

b) **GNA** for global analysis of the structure with global imperfections is simple, provided non-linear software is available. Resulting internal forces:



M_{Ed} [kNm] N_{Ed} [kN]

Compare differences between LA and GNA: **here negligible**.

7.1 Example 1

Member stability checks should be based on buckling lengths equal to the system lengths.

Amplified LA is questioned in spite of $\alpha_{cr,1} = 5.51 > 3$ due to **non-sway buckling character**. Member stability checks should also be based on buckling lengths equal to the system lengths. Details of the method are given for another example in Module 4: Frame Stability.

7.1 Example 1

c) **LBA** gives the first buckling mode which corresponds to buckling of bottom central column, which may be designed for internal forces from **LA** (imperfection may be neglected) and following global slenderness or global buckling length:

$$\bar{\lambda} = \sqrt{\frac{N_y}{N_{cr}}} = \sqrt{\frac{A f_y}{\alpha_{cr} N_{Ed}}} = \sqrt{\frac{5425 \cdot 235}{5.51 \cdot 918 \cdot 10^3}} = 0.50$$

$$L_{cr} = \sqrt{\frac{\pi^2 E I}{N_{cr}}} = \sqrt{\frac{\pi^2 \cdot 210 \cdot 10^3 \cdot 24.92 \cdot 10^6}{5.51 \cdot 918 \cdot 10^3}} = 3195 \text{ mm}$$

7.1 Example 1

The buckling mode is non-sway (see the picture) and due to elastic constraint from upper parts of the columns the L_{cr} < h = 4200 mm.

Note: Other members may be designed conservatively for the same $\alpha_{cr,1}$. Moments due to sway effects (here negligible) should be increased approx. by 20%.

7.1 Example 1

Summary concerning the three approaches:

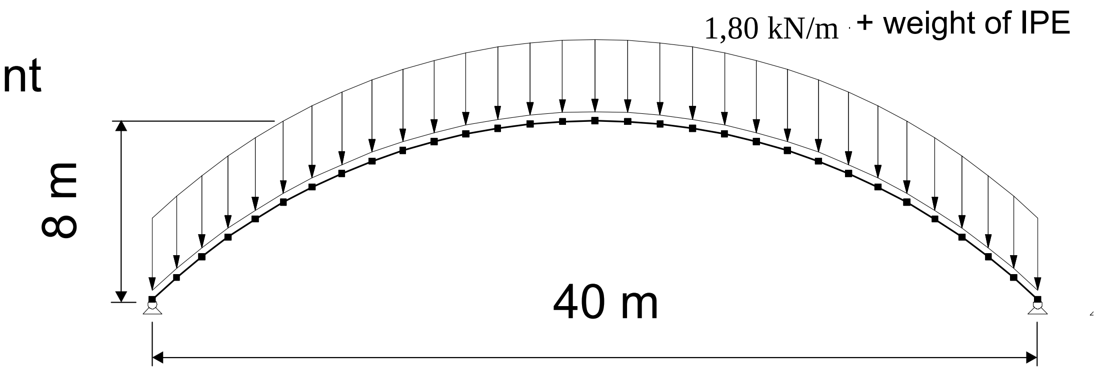
- **GNIA** introducing all kind of imperfections (suitable for all α_{cr}) is demanding, usually not employed.
- **GNA** using global imperfections may be used, followed by member stability checks for system critical lengths. Simplified amplified LA may similarly be used for $\alpha_{cr,1} \geq 3$ but is appropriate for predominantly sway buckling modes.
- **LA** followed by member stability checks using critical lengths from LBA should account for moments due to sway effects.
- Braced multi-storey frames are usually non-sway.

7.2 Example 2

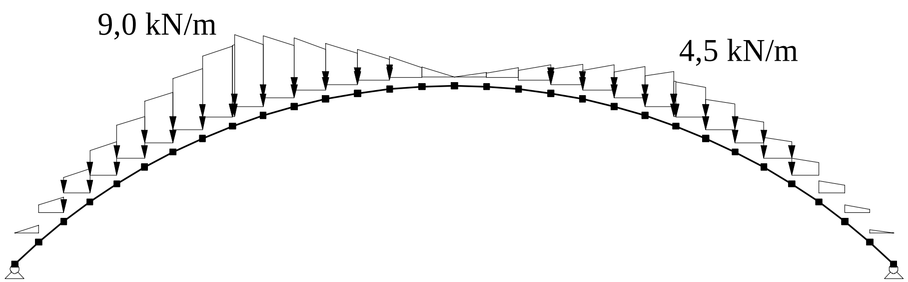
Example 2:

Two-hinged arch (IPE 360, S355)

permanent loading



snow



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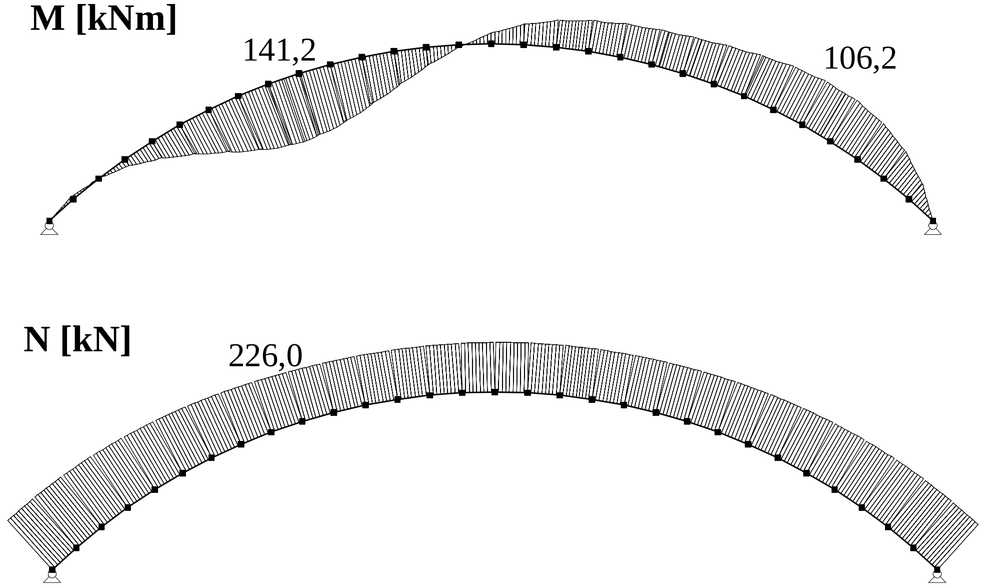
7.2 Example 2

LA - internal forces:

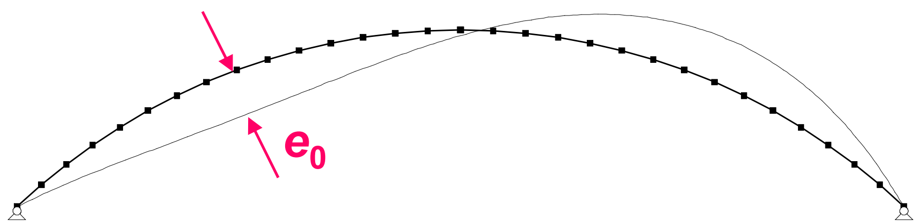


7.2 Example 2

LBA - critical loading:

(α_{cr} = 2.82)

first critical mode



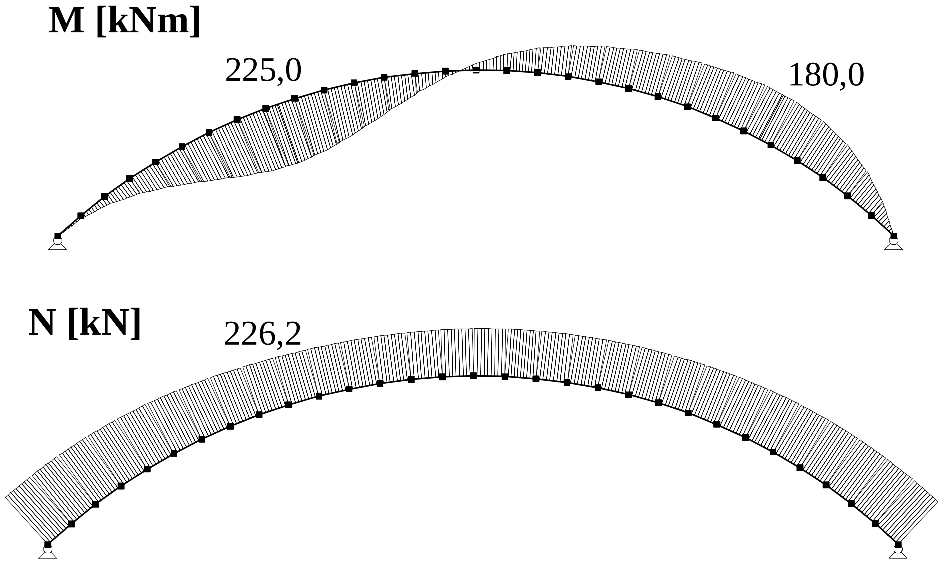
$$\bar{\lambda} = \sqrt{\frac{N_{Rk}}{N_{cr}}} = \sqrt{\frac{N_{Rk}}{\alpha_{cr}\, N_{Ed}}} = \sqrt{\frac{2581 \cdot 10^3}{2,82 \cdot 226 \cdot 10^3}} = 2,01$$

When LA is used, the reduction coefficient $\chi_{y(a)}$ = 0.22.

7.2 Example 2

GNIA - internal forces:

Imperfections in first critical mode, in accordance with Eurocode 3, cl. 5.3.2, e_0 = 53.3 mm.



When GNIA is used, the reduction coefficient χ_y = 1.0.

7.2 Example 2

Summary of the comparison of LA and GNIA for the arch:

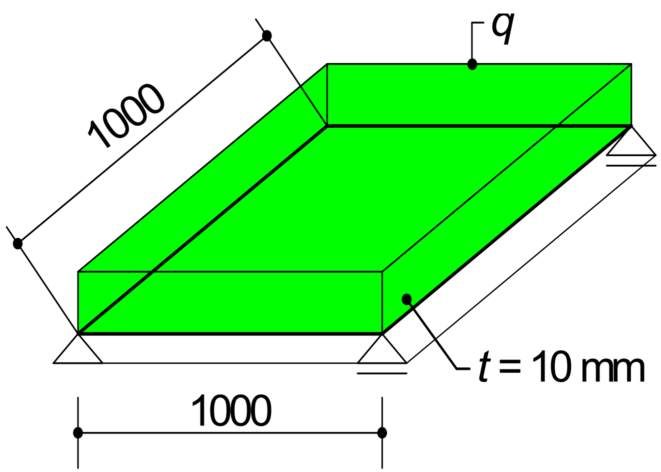
- Using **LA** the reduction coefficient for compression $\chi_{y(a)}$ = 0.22 shall be used in design.
- Using **GNIA** (geometrically non-linear analysis with imperfections) the maximum moment increased about 1.6 times, while reduction coefficient for compression is χ_y = 1.0.
- Comparing resulting cross sections the GNIA is more economic.

7.3 Example 3

Example 3:

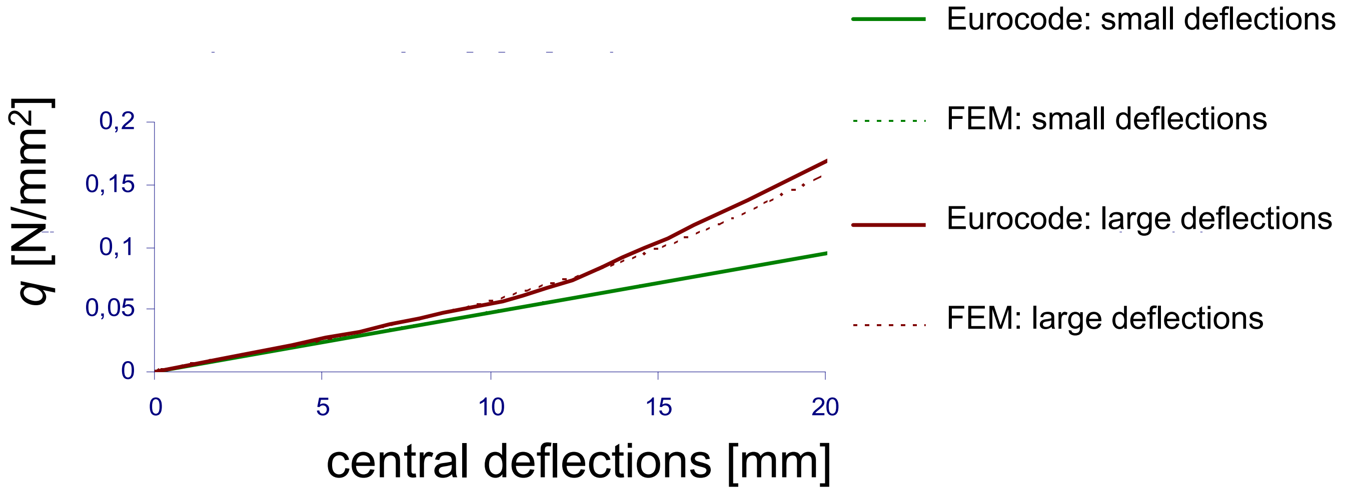
Plate under uniform loading *q*

(edges simply supported with zero membrane stresses)



7.3 Example 3

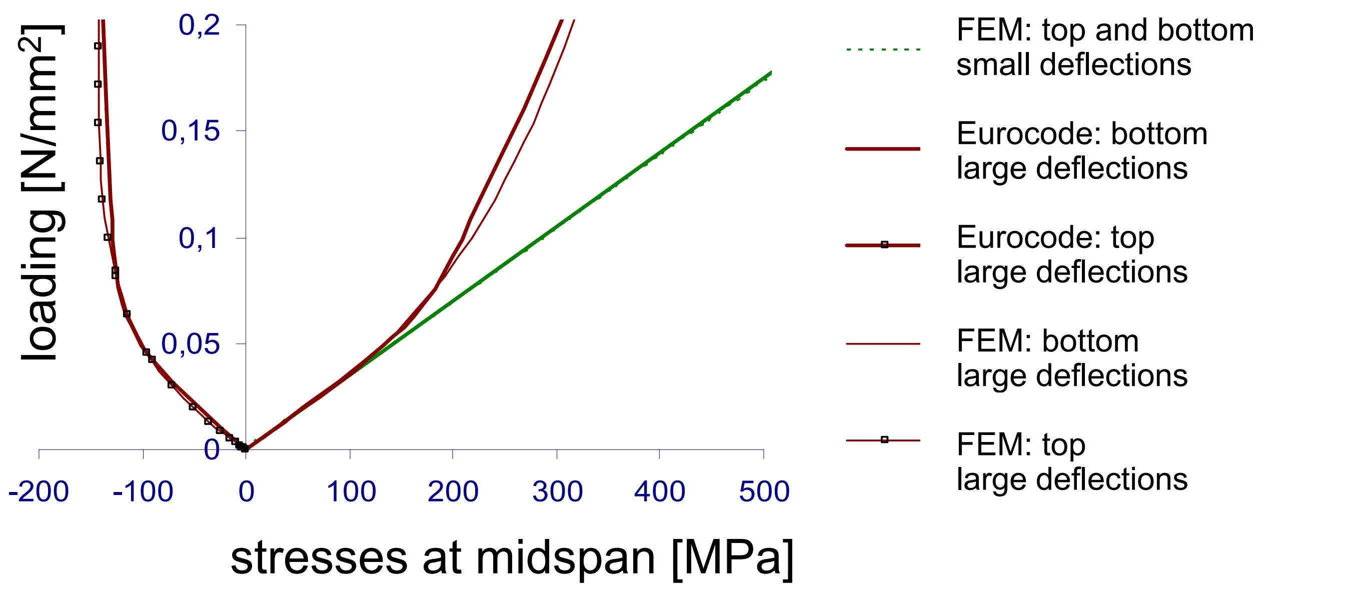
FEM & EN 1993-1-7.
Comparison of LA and GNA:



7.3 Example 3

FEM & EN 1993-1-7.

Comparison of LA and GNA:



7.3 Example 3

Summary of the comparison of LA and GNA for the thin plate in bending:

- Using **GNA** (large deflection theory of plates) the deflections and stresses for the same loading are lower than using LA (small deflection theory of plates).
- **LA** gives uneconomic solution.

8. Conclusions

1) Appropriate models of analysis (from LA up to GMNIA) should be used in accordance with investigated limit state and expected results.

2) Geometrically nonlinear models are essential for structures changing significantly shape under loading (cables, cable nets, structures and members in buckling etc.).

3) Approximate LA may be used provided the influence of deflections and imperfections are adequately covered in both global and member analysis.

8. Conclusions

4) Elastic analysis is always safe but usually uneconomical.

5) Materially nonlinear models may be used as far as special requirements on steel, cross sections and boundary conditions are met.

6) FEM (from LA up to GMNIA) requires special care and factored design loading should achieve at given limit state required reliability.

Notes to Users of the Lecture

- This session is for modelling and analysis of structures and requires about 60 minutes lecturing and 60 minutes tutorial session.
- Within the lecturing, calculation models ranging from LA to GMNIA are described. Influence of deformed geometry and of material behaviour is shown with focus on multi-storey frames and floor elements. Analysis using FEM in general form and use of its results for design with harmony of Eurocodes is also presented.
- Further readings on the relevant documents from website of www.access-steel.com and relevant standards of national standard institutions are strongly recommended.
- Formative questions should be well answered before the summative questions completed within the tutorial session.
- Keywords for the lecture:

  calculation model, linear analysis, non-linear analysis, elastic analysis, plastic analysis, bifurcation analysis, sway frame, frame stability, FE analysis.

Notes for lecturers

- Subject: Modelling and analysis of structures.
- Lecture duration: 60 minutes plus 60 minutes tutorial.
- Keywords: calculation model, linear analysis, non-linear analysis, elastic analysis, plastic analysis, bifurcation analysis, sway frame, frame stability, FE analysis.
- Aspects to be discussed: types of analyses, requirements for use of various analyses, modelling of frames, trusses, beams.
- Within the lecturing, the modelling of multistorey frames incl. stability problems and GNA should be practised. Attention should also be paid to modelling of trusses and beams using LA and MNA.
- Further reading: relevant documents www.access-steel.com and relevant standards of national standard institutions are strongly recommended.
- Preparation for tutorial exercise: see examples within the lecture.